Computer algebra independent integration tests
1_Algebraic_functions/1.2_Trinomial_products/1.2.3General/1.2.3.3(d+ex^n)^q(a+bx^n+c

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

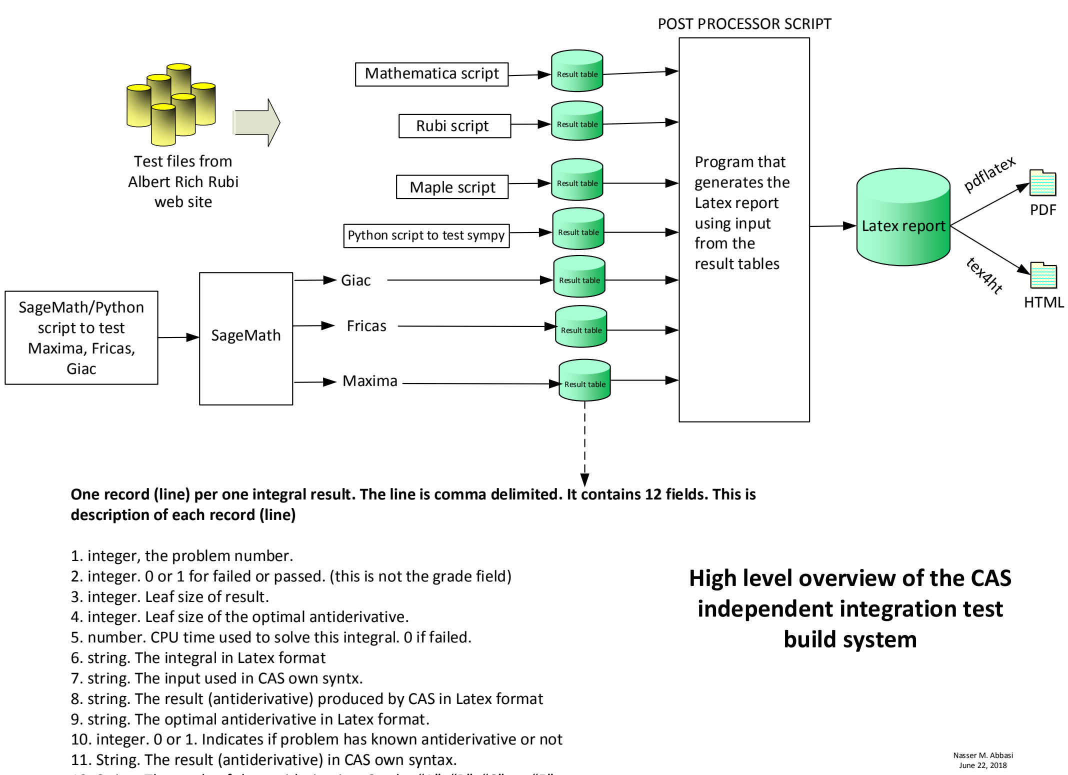
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (96)	% 0. (0)
Rubi in Sympy	% 68.75 (66)	% 31.25 (30)
Mathematica	% 95.83 (92)	% 4.17 (4)
Maple	% 51.04 (49)	% 48.96 (47)
Maxima	% 9.38 (9)	% 90.62 (87)
Fricas	% 46.88 (45)	% 53.12 (51)
Sympy	% 40.62 (39)	% 59.38 (57)
Giac	% 34.38 (33)	% 65.62 (63)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

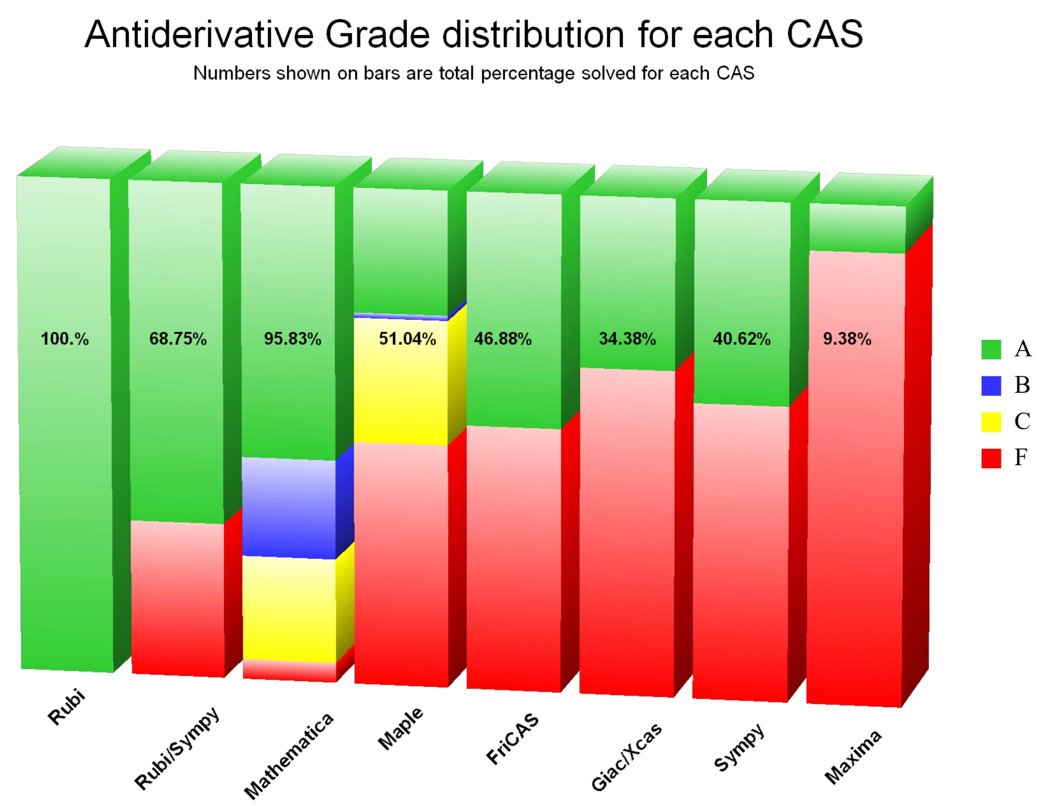
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

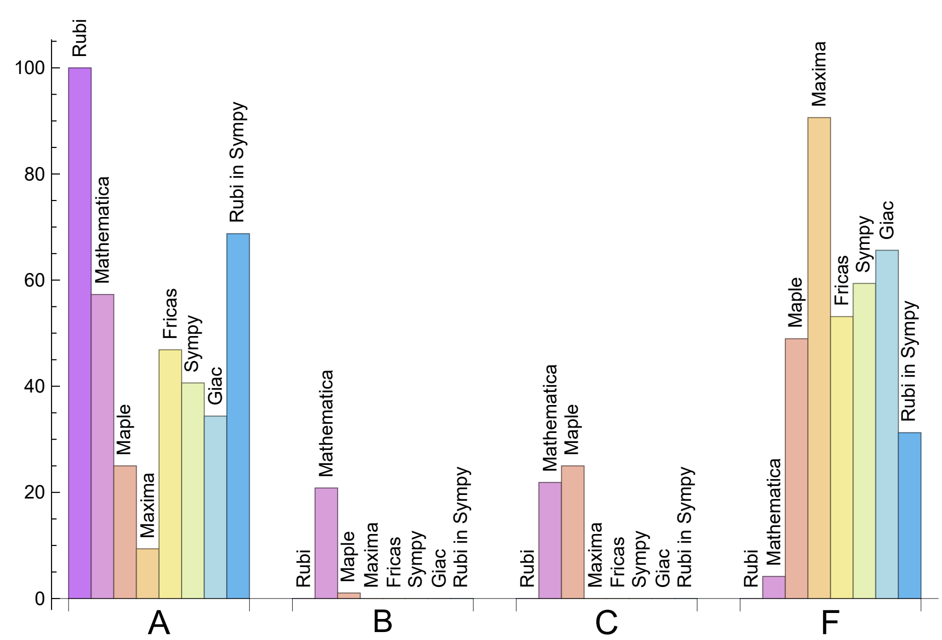
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	68.75	0.	0.	31.25
Mathematica	57.29	20.83	21.88	4.17
Maple	25.	1.04	25.	48.96
Maxima	9.38	0.	0.	90.62
Fricas	46.88	0.	0.	53.12
Sympy	40.62	0.	0.	59.38
Giac	34.38	0.	0.	65.62

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.56	361.82	0.95	288.	1.
Rubi in Sympy	54.12	212.53	0.85	168.	0.88
Mathematica	1.97	2444.28	3.27	153.	0.92
Maple	0.04	89.67	0.51	53.	0.25
Maxima	0.37	29.67	0.6	0.	0.
Fricas	0.35	1681.33	4.9	701.	3.45
Sympy	9.81	199.05	1.49	82.	0.4
Giac	0.28	261.97	1.37	177.	1.14

1.8 list of integrals that has no closed form antiderivative

{59, 90, 94, 95, 96}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {3, 5, 6, 34, 35, 39, 40, 45, 46, 51, 52, 56, 57, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84}

Not solved by Mathematica {58, 63, 64, 65}

Not solved by Maple {42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93}

Not solved by Maxima {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93}

Not solved by Fricas {31, 32, 33, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93}

Not solved by Sympy {32, 33, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96}

Not solved by Giac {5, 6, 7, 8, 9, 10, 12, 17, 18, 20, 21, 23, 28, 29, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {10, 21, 91}

Mathematica {12, 23, 85, 86, 87, 88, 89, 91, 92, 93}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	334	329	0	4134	165	397	309
normalized size	1	1.	1.1	1.08	0.	13.55	0.54	1.3	1.01
time (sec)	N/A	0.559	0.169	0.111	0.	0.359	7.335	0.297	114.599

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	337	380	0	4082	168	424	311
normalized size	1	1.	1.04	1.18	0.	12.64	0.52	1.31	0.96
time (sec)	N/A	0.415	0.189	0.109	0.	0.377	7.569	0.281	75.131

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	534	34	0	3762	199	811	0
normalized size	1	1.	0.71	0.05	0.	4.99	0.26	1.08	0.
time (sec)	N/A	2.794	1.593	0.027	0.	0.486	37.057	0.312	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	425	39	0	3752	202	855	304
normalized size	1	1.	1.29	0.12	0.	11.4	0.61	2.6	0.92
time (sec)	N/A	0.456	0.234	0.02	0.	0.493	36.872	0.311	77.931

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	3082	136	0	0
normalized size	1	1.	0.08	0.07	0.	3.9	0.17	0.	0.
time (sec)	N/A	1.872	0.062	0.059	0.	0.331	20.696	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	67	53	0	3082	136	0	0
normalized size	1	1.	0.08	0.07	0.	3.9	0.17	0.	0.
time (sec)	N/A	1.752	0.048	0.06	0.	0.314	18.967	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	69	55	0	3079	136	0	333
normalized size	1	1.	0.2	0.16	0.	8.82	0.39	0.	0.95
time (sec)	N/A	0.925	0.061	0.041	0.	0.329	20.679	0.	76.343

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	69	55	0	3079	136	0	333
normalized size	1	1.	0.09	0.07	0.	4.1	0.18	0.	0.44
time (sec)	N/A	1.946	0.051	0.043	0.	0.327	19.166	0.	74.154

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	55	42	0	1458	75	0	304
normalized size	1	1.	0.13	0.1	0.	3.55	0.18	0.	0.74
time (sec)	N/A	0.606	0.036	0.064	0.	0.293	7.435	0.	82.881

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	451	55	42	0	1681	24	0	590
normalized size	1	0.96	0.12	0.09	0.	3.58	0.05	0.	1.26
time (sec)	N/A	0.782	0.023	0.01	0.	0.301	3.761	0.	88.896

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	131	73	97	73
normalized size	1	1.	0.75	0.68	1.14	1.54	0.86	1.14	0.86
time (sec)	N/A	0.087	0.031	0.004	0.836	0.288	0.416	0.271	15.586

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	212	190	0	128
normalized size	1	1.	0.96	0.78	0.	1.51	1.36	0.	0.91
time (sec)	N/A	0.185	0.324	0.026	0.	0.292	2.878	0.	28.947

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	258	27	0	1343	19	333	270
normalized size	1	1.	0.74	0.08	0.	3.87	0.05	0.96	0.78
time (sec)	N/A	0.565	0.335	0.009	0.	0.281	4.448	0.299	39.272

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	55	42	0	1048	20	331	270
normalized size	1	1.	0.17	0.13	0.	3.17	0.06	1.	0.82
time (sec)	N/A	0.479	0.024	0.011	0.	0.293	4.851	0.283	43.016

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	42	36	58	26	39	17
normalized size	1	1.	1.15	1.56	1.33	2.15	0.96	1.44	0.63
time (sec)	N/A	0.021	0.021	0.018	0.824	0.268	0.435	0.269	5.046

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	131	96	0	289	49	198	141
normalized size	1	1.	1.	0.73	0.	2.21	0.37	1.51	1.08
time (sec)	N/A	0.181	0.124	0.045	0.	0.313	3.196	0.342	14.183

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	53	40	0	331	24	0	148
normalized size	1	1.	0.34	0.25	0.	2.11	0.15	0.	0.94
time (sec)	N/A	0.177	0.02	0.011	0.	0.288	0.538	0.	17.447

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	55	42	0	701	24	0	168
normalized size	1	1.	0.32	0.25	0.	4.1	0.14	0.	0.98
time (sec)	N/A	0.272	0.021	0.012	0.	0.289	0.563	0.	17.76

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	111	78	0	390	49	166	100
normalized size	1	1.	0.95	0.67	0.	3.33	0.42	1.42	0.85
time (sec)	N/A	0.113	0.073	0.07	0.	0.286	3.171	0.346	10.398

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	57	44	0	1458	76	0	359
normalized size	1	1.	0.11	0.09	0.	2.85	0.15	0.	0.7
time (sec)	N/A	0.875	0.036	0.004	0.	0.292	7.471	0.	119.994

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	411	57	44	0	1007	26	0	590
normalized size	1	0.96	0.13	0.1	0.	2.35	0.06	0.	1.38
time (sec)	N/A	0.705	0.021	0.009	0.	0.295	3.801	0.	85.792

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	68	111	173	82	111	82
normalized size	1	1.	0.93	0.7	1.14	1.78	0.85	1.14	0.85
time (sec)	N/A	0.108	0.111	0.011	0.818	0.266	0.512	0.285	17.103

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	109	0	190	148	0	128
normalized size	1	1.	0.92	0.78	0.	1.36	1.06	0.	0.91
time (sec)	N/A	0.204	0.3	0.019	0.	0.286	2.688	0.	40.927

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	257	29	0	1343	20	333	468
normalized size	1	1.	0.74	0.08	0.	3.87	0.06	0.96	1.35
time (sec)	N/A	0.67	0.289	0.01	0.	0.284	4.305	0.314	68.009

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	1223	26	342	495
normalized size	1	1.	0.16	0.12	0.	3.45	0.07	0.96	1.39
time (sec)	N/A	0.659	0.024	0.01	0.	0.289	4.897	0.286	72.816

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	23	23	17	26	8
normalized size	1	1.	1.92	0.77	1.77	1.77	1.31	2.	0.62
time (sec)	N/A	0.011	0.006	0.002	0.817	0.268	0.354	0.267	3.973

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	129	110	0	401	51	198	141
normalized size	1	1.	1.	0.85	0.	3.11	0.4	1.53	1.09
time (sec)	N/A	0.244	0.122	0.034	0.	0.28	3.187	0.344	13.309

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	55	42	0	581	26	0	168
normalized size	1	1.	0.33	0.25	0.	3.52	0.16	0.	1.02
time (sec)	N/A	0.212	0.021	0.01	0.	0.3	0.584	0.	17.078

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	57	44	0	693	26	0	168
normalized size	1	1.	0.34	0.26	0.	4.1	0.15	0.	0.99
time (sec)	N/A	0.293	0.021	0.01	0.	0.31	0.576	0.	18.865

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	114	90	0	347	51	182	121
normalized size	1	1.	0.91	0.72	0.	2.78	0.41	1.46	0.97
time (sec)	N/A	0.147	0.085	0.034	0.	0.29	3.154	0.347	10.295

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	71	47	0	0	136	144	202
normalized size	1	1.	0.53	0.35	0.	0.	1.01	1.07	1.5
time (sec)	N/A	0.25	0.052	0.076	0.	0.	3.848	0.291	51.762

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	72	62	0	0	0	166	165
normalized size	1	1.	0.44	0.38	0.	0.	0.	1.01	1.01
time (sec)	N/A	0.211	0.053	0.054	0.	0.	0.	0.298	44.585

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	62	0	0	0	177	216
normalized size	1	1.	0.49	0.34	0.	0.	0.	0.98	1.2
time (sec)	N/A	0.276	0.069	0.013	0.	0.	0.	0.29	54.45

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	43	0	1	112	58	0
normalized size	1	1.	1.	0.88	0.	0.02	2.29	1.18	0.
time (sec)	N/A	0.084	0.039	0.006	0.	0.299	1.681	0.268	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	161	0	1	423	115	0
normalized size	1	1.	1.	1.87	0.	0.01	4.92	1.34	0.
time (sec)	N/A	0.184	0.146	0.005	0.	0.268	4.567	0.267	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	293	266	0	1018	109	347	235
normalized size	1	1.	1.16	1.05	0.	4.02	0.43	1.37	0.93
time (sec)	N/A	0.42	0.17	0.007	0.	0.271	3.119	0.273	72.46

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	251	560	0	3429	428	1	214
normalized size	1	1.	1.21	2.69	0.	16.49	2.06	0.	1.03
time (sec)	N/A	1.104	0.325	0.031	0.	0.333	34.703	1.288	76.268

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	346	334	0	4070	167	406	314
normalized size	1	1.	1.11	1.07	0.	13.09	0.54	1.31	1.01
time (sec)	N/A	0.639	0.211	0.07	0.	0.35	8.909	0.325	124.579

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	716	88	67	0	0	0	0	0
normalized size	1	1.	0.12	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	3.498	0.089	0.034	0.	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	753	753	551	45	0	3742	204	873	0
normalized size	1	1.	0.73	0.06	0.	4.97	0.27	1.16	0.
time (sec)	N/A	3.01	2.933	0.007	0.	0.515	47.512	0.311	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	17781	0	0	432
normalized size	1	1.	0.2	0.15	0.	41.06	0.	0.	1.
time (sec)	N/A	2.038	0.105	0.008	0.	3.285	0.	0.	171.85

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	128	0	0	0	0	0	151
normalized size	1	1.	0.91	0.	0.	0.	0.	0.	1.07
time (sec)	N/A	0.289	0.188	0.105	0.	0.	0.	0.	27.745

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	0	0	0	0	0	104
normalized size	1	1.	1.	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.197	0.122	0.088	0.	0.	0.	0.	19.29

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	0	0	60
normalized size	1	1.	0.99	0.	0.	0.	0.	0.	0.72
time (sec)	N/A	0.066	0.044	0.059	0.	0.	0.	0.	9.966

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	131	0	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.153	0.102	0.	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	188	0	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	1.13	0.231	0.	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	0	0	0	0	0	56
normalized size	1	1.	0.99	0.	0.	0.	0.	0.	0.69
time (sec)	N/A	0.07	0.069	0.063	0.	0.	0.	0.	10.371

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	165	0	0	0	0	0	158
normalized size	1	1.	0.57	0.	0.	0.	0.	0.	0.55
time (sec)	N/A	0.522	0.493	0.102	0.	0.	0.	0.	27.384

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	142	0	0	0	0	0	109
normalized size	1	1.	0.7	0.	0.	0.	0.	0.	0.54
time (sec)	N/A	0.345	0.464	0.112	0.	0.	0.	0.	18.857

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	137	0	0	0	0	0	100
normalized size	1	1.	1.02	0.	0.	0.	0.	0.	0.75
time (sec)	N/A	0.126	0.153	0.097	0.	0.	0.	0.	19.185

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	245	0	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.481	0.585	0.187	0.	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	495	0	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.787	1.328	0.21	0.	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	252	0	0	0	0	0	158
normalized size	1	1.	0.59	0.	0.	0.	0.	0.	0.37
time (sec)	N/A	0.829	1.226	0.124	0.	0.	0.	0.	27.846

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	212	0	0	0	0	0	109
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.4
time (sec)	N/A	0.516	0.825	0.13	0.	0.	0.	0.	19.391

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	164	0	0	0	0	0	151
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.216	0.437	0.117	0.	0.	0.	0.	31.452

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	582	582	1031	0	0	0	0	0	0
normalized size	1	1.	1.77	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.941	2.252	0.242	0.	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	701	701	1241	0	0	0	0	0	0
normalized size	1	1.	1.77	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.492	4.606	0.274	0.	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	0	0	0	138
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.425	0.121	0.081	0.	0.	0.	0.	73.619

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.189	0.226	0.	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	213	0	0	0	0	0	240
normalized size	1	1.	0.71	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.319	0.396	0.169	0.	0.	0.	0.	41.21

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	171	0	0	0	0	0	170
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.217	0.207	0.154	0.	0.	0.	0.	29.317

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	110	0	0	0	0	0	104
normalized size	1	1.	0.81	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.129	0.093	0.138	0.	0.	0.	0.	17.075

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	128
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.332	0.07	0.13	0.	0.	0.	0.	70.947

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0	206
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.558	0.117	0.104	0.	0.	0.	0.	137.215

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.795	0.411	0.122	0.	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	66	0	185	656	308	0
normalized size	1	1.	0.92	1.06	0.	2.98	10.58	4.97	0.
time (sec)	N/A	0.083	0.199	0.018	0.	0.272	3.841	0.27	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	138	0	668	3128	1206	0
normalized size	1	1.	0.93	1.05	0.	5.06	23.7	9.14	0.
time (sec)	N/A	0.203	0.364	0.022	0.	0.29	42.678	0.29	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	205	226	0	1632	0	1	0
normalized size	1	1.	0.94	1.04	0.	7.49	0.	0.	0.
time (sec)	N/A	0.378	0.778	0.027	0.	0.279	0.	0.295	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	455	0	0	0	0	0	566
normalized size	1	1.	1.48	0.	0.	0.	0.	0.	1.84
time (sec)	N/A	1.507	3.904	0.07	0.	0.	0.	0.	177.191

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	348	0	0	0	0	0	410
normalized size	1	1.	1.55	0.	0.	0.	0.	0.	1.83
time (sec)	N/A	0.931	1.476	0.058	0.	0.	0.	0.	120.378

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	279	0	0	0	0	0	148
normalized size	1	1.	1.81	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.276	0.827	0.037	0.	0.	0.	0.	32.276

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	379	0	0	0	0	0	0
normalized size	1	1.	1.56	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.829	2.408	0.084	0.	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	2302	0	0	0	0	0	0
normalized size	1	1.	6.26	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.365	6.291	0.176	0.	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	4111	0	0	0	0	0	0
normalized size	1	1.	7.45	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.355	6.474	0.234	0.	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	5537	0	0	0	0	0	0
normalized size	1	1.	7.38	0.	0.	0.	0.	0.	0.
time (sec)	N/A	5.684	6.488	0.097	0.	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	4177	0	0	0	0	0	0
normalized size	1	1.	7.69	0.	0.	0.	0.	0.	0.
time (sec)	N/A	5.109	6.41	0.097	0.	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	328	3152	0	0	0	0	0	0
normalized size	1	0.91	8.71	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.385	6.262	0.09	0.	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	726	726	11767	0	0	0	0	0	0
normalized size	1	1.	16.21	0.	0.	0.	0.	0.	0.
time (sec)	N/A	4.916	7.106	0.262	0.	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1129	1129	16855	0	0	0	0	0	0
normalized size	1	1.	14.93	0.	0.	0.	0.	0.	0.
time (sec)	N/A	9.802	7.577	0.302	0.	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1707	1707	13018	0	0	0	0	0	0
normalized size	1	1.	7.63	0.	0.	0.	0.	0.	0.
time (sec)	N/A	13.928	7.797	0.188	0.	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1191	1191	10910	0	0	0	0	0	0
normalized size	1	1.	9.16	0.	0.	0.	0.	0.	0.
time (sec)	N/A	8.987	6.821	0.156	0.	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	713	8593	0	0	0	0	0	0
normalized size	1	1.	12.05	0.	0.	0.	0.	0.	0.
time (sec)	N/A	4.999	6.678	0.148	0.	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1708	1708	43535	0	0	0	0	0	0
normalized size	1	1.	25.49	0.	0.	0.	0.	0.	0.
time (sec)	N/A	14.907	8.956	0.435	0.	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2446	2446	56566	0	0	0	0	0	0
normalized size	1	1.	23.13	0.	0.	0.	0.	0.	0.
time (sec)	N/A	24.827	9.886	0.608	0.	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	3778	0	0	0	0	0	262
normalized size	1	1.	12.94	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.842	6.259	0.087	0.	0.	0.	0.	80.164

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	10587	0	0	0	0	0	265
normalized size	1	1.	36.01	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.829	6.915	0.092	0.	0.	0.	0.	84.961

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	688	0	0	0	0	0	258
normalized size	1	1.	2.36	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.836	0.861	0.031	0.	0.	0.	0.	89.888

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	3012	0	0	0	0	0	262
normalized size	1	1.	10.11	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.856	6.261	0.02	0.	0.	0.	0.	87.669

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	8781	0	0	0	0	0	262
normalized size	1	1.	29.47	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.843	6.605	0.019	0.	0.	0.	0.	103.334

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.321	0.192	0.	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F(-2)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	606	606	2025	0	0	0	0	0	520
normalized size	1	1.	3.34	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	1.393	24.419	0.161	0.	0.	0.	0.	165.131

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	1522	0	0	0	0	0	382
normalized size	1	1.	3.4	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.993	6.261	0.134	0.	0.	0.	0.	123.616

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	902	0	0	0	0	0	245
normalized size	1	1.	3.13	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.62	1.361	0.134	0.	0.	0.	0.	80.888

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.099	0.122	0.	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.165	0.084	0.	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	1.015	0.11	0.	0.	0.	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [4] had the largest ratio of [0.5556]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	7	1.	17	0.412
2	A	13	7	1.	18	0.389
3	A	19	6	1.	17	0.353
4	A	13	10	1.	18	0.556
5	A	19	6	1.	26	0.231
6	A	19	6	1.	26	0.231
7	A	7	4	1.	27	0.148
8	A	19	6	1.	27	0.222
9	A	19	6	1.	18	0.333
10	A	19	7	0.96	18	0.389
11	A	10	7	1.	18	0.389
12	A	19	6	1.	16	0.375
13	A	19	6	1.	13	0.462
14	A	19	6	1.	18	0.333
15	A	5	5	1.	18	0.278
16	A	7	4	1.	18	0.222
17	A	7	4	1.	18	0.222
18	A	7	4	1.	18	0.222
19	A	7	4	1.	18	0.222
20	A	19	6	1.	20	0.3
21	A	19	7	0.96	20	0.35
22	A	11	8	1.	20	0.4
23	A	19	6	1.	18	0.333
24	A	19	6	1.	15	0.4
25	A	19	6	1.	20	0.3
26	A	5	5	1.	20	0.25
27	A	7	4	1.	20	0.2
28	A	7	4	1.	20	0.2
29	A	7	4	1.	20	0.2
30	A	7	4	1.	20	0.2
31	A	9	6	1.	25	0.24
32	A	9	6	1.	26	0.231
33	A	9	6	1.	33	0.182
34	A	5	5	1.	17	0.294
35	A	6	6	1.	22	0.273
36	A	11	8	1.	17	0.471
37	A	5	4	1.	22	0.182
38	A	14	9	1.	17	0.529
39	A	15	9	1.	22	0.409

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	21	8	1.	17	0.471
41	A	9	6	1.	22	0.273
42	A	5	4	1.	21	0.19
43	A	5	4	1.	21	0.19
44	A	3	3	1.	19	0.158
45	A	6	4	1.	21	0.19
46	A	7	4	1.	21	0.19
47	A	3	3	1.	20	0.15
48	A	9	5	1.	21	0.238
49	A	7	5	1.	21	0.238
50	A	4	4	1.	19	0.21
51	A	10	5	1.	21	0.238
52	A	11	5	1.	21	0.238
53	A	11	5	1.	21	0.238
54	A	8	5	1.	21	0.238
55	A	5	4	1.	19	0.21
56	A	15	5	1.	21	0.238
57	A	16	5	1.	21	0.238
58	A	6	5	1.	23	0.217
59	A	0	0	0.	0	0.
60	A	10	5	1.	21	0.238
61	A	8	5	1.	21	0.238
62	A	6	5	1.	19	0.263
63	A	6	5	1.	21	0.238
64	A	8	5	1.	21	0.238
65	A	10	5	1.	21	0.238
66	A	2	1	1.	22	0.045
67	A	2	1	1.	24	0.042
68	A	2	1	1.	24	0.042
69	A	5	3	1.	26	0.115
70	A	5	3	1.	26	0.115
71	A	3	2	1.	24	0.083
72	A	6	3	1.	26	0.115
73	A	7	3	1.	26	0.115
74	A	8	3	1.	26	0.115
75	A	9	4	1.	26	0.154
76	A	9	5	1.	26	0.192
77	A	4	3	0.91	24	0.125
78	A	10	4	1.	26	0.154
79	A	11	4	1.	26	0.154
80	A	11	4	1.	26	0.154
81	A	11	5	1.	26	0.192
82	A	5	3	1.	24	0.125
83	A	15	4	1.	26	0.154
84	A	16	4	1.	26	0.154
85	A	6	5	1.	26	0.192
86	A	6	5	1.	26	0.192

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
87	A	6	5	1.	26	0.192
88	A	6	5	1.	26	0.192
89	A	6	5	1.	26	0.192
90	A	0	0	0.	0	0.
91	A	10	5	1.	26	0.192
92	A	8	5	1.	26	0.192
93	A	6	5	1.	24	0.208
94	A	0	0	0.	0	0.
95	A	0	0	0.	0	0.
96	A	0	0	0.	0	0.

3 Listing of integrals

3.1 $\int \frac{d+ex^3}{a+cx^6}\,dx$

Optimal. Leaf size=305

$$-\frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+\sqrt{3}\right)}{6a^{5/6}c^{2/3}}+\frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) - ((Sqrt[c]*d
 + Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6
*a^(5/6)*c^(2/3)) + ((Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[
3] + (2*c^(1/6)*x)/a^(1/6)])/(6*a^(5/6)*c^(2/3)) - (e*Log[a^(1/3)
 + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((Sqrt[3]*Sqrt[c]*d - Sqrt
[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(1
2*a^(5/6)*c^(2/3)) + ((Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3)
 + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rubi [A] time = 0.559384, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-\frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+\sqrt{3}\right)}{6a^{5/6}c^{2/3}}+\frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) - ((Sqrt[c]*d
 + Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(6
*a^(5/6)*c^(2/3)) + ((Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[
3] + (2*c^(1/6)*x)/a^(1/6)])/(6*a^(5/6)*c^(2/3)) - (e*Log[a^(1/3)
 + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((Sqrt[3]*Sqrt[c]*d - Sqrt
[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(1
2*a^(5/6)*c^(2/3)) + ((Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3)
 + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rubi in Sympy [A] time = 114.599, size = 309, normalized size = 1.01

$$-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{\frac{2}{3}}}+\frac{\left(\sqrt{a}e-\sqrt{3}\sqrt{c}d\right)\log\left(1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{12a^{\frac{5}{6}}c^{\frac{2}{3}}}+\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\log\left(1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{12a^{\frac{5}{6}}c^{\frac{2}{3}}}+\frac{\left(-\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[6]{a}+\frac{2\sqrt{3}\sqrt[6]{c}x}{3}\right)}{\sqrt[6]{a}}\right)}{6a^{\frac{5}{6}}c^{\frac{2}{3}}}-\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[6]{a}-\frac{2\sqrt{3}\sqrt[6]{c}x}{3}\right)}{\sqrt[6]{a}}\right)}{6a^{\frac{5}{6}}c^{\frac{2}{3}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{\frac{5}{6}}\sqrt[6]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)/(c*x**6+a),x)
```

```
[Out] -e*log(a**(1/3) + c**(1/3)*x**2)/(6*a**(1/3)*c**(2/3)) + (sqrt(a)
*e - sqrt(3)*sqrt(c)*d)*log(1 + c**(1/3)*x**2/a**(1/3) - sqrt(3)*
c**(1/6)*x/a**(1/6))/(12*a**(5/6)*c**(2/3)) + (sqrt(a)*e + sqrt(3
)*sqrt(c)*d)*log(1 + c**(1/3)*x**2/a**(1/3) + sqrt(3)*c**(1/6)*x/
a**(1/6))/(12*a**(5/6)*c**(2/3)) + (-sqrt(3)*sqrt(a)*e + sqrt(c)*
d)*atan(sqrt(3)*(a**(1/6) + 2*sqrt(3)*c**(1/6)*x/3)/a**(1/6))/(6*
a**(5/6)*c**(2/3)) - (sqrt(3)*sqrt(a)*e + sqrt(c)*d)*atan(sqrt(3)
*(a**(1/6) - 2*sqrt(3)*c**(1/6)*x/3)/a**(1/6))/(6*a**(5/6)*c**(2/
3)) + d*atan(c**(1/6)*x/a**(1/6))/(3*a**(5/6)*c**(1/6))
```

Mathematica [A] time = 0.169395, size = 334, normalized size = 1.1

$$-\frac{\left(\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12ac^{2/3}}-\frac{\left(-a^{2/3}e-\sqrt{3}\sqrt[6]{a}\sqrt{c}d\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12ac^{2/3}}+\frac{\left(\sqrt{3}a^{2/3}e+\sqrt[6]{a}\sqrt{c}d\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}x-\sqrt{3}\sqrt[6]{a}}{\sqrt[6]{a}}\right)}{6ac^{2/3}}+\frac{\left(\sqrt[6]{a}\sqrt{c}d-\sqrt{3}a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}}+\frac{d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)/(a + c*x^6),x]
```

```
[Out] (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) + ((a^(1/6)*S
qrt[c]*d + Sqrt[3]*a^(2/3)*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1
/6)*x)/a^(1/6)])/(6*a*c^(2/3)) + ((a^(1/6)*Sqrt[c]*d - Sqrt[3]*a^
(2/3)*e)*ArcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^
(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((S
qrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e)*Log[a^(1/3) - Sqrt[3]*a^(1/
6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(2/3)) - ((-(Sqrt[3]*a^(1/6)
*Sqrt[c]*d) - a^(2/3)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x
+ c^(1/3)*x^2])/(12*a*c^(2/3))
```

Maple [A] time = 0.111, size = 329, normalized size = 1.1

$$\frac{c\sqrt{3}d}{12\,a^{2}}\left(\frac{a}{c}\right)^{\frac{7}{6}}\ln\left(x^{2}+\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)+\frac{e}{12\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^{2}+\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)+\frac{d}{6\,a}\sqrt[6]{\frac{a}{c}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}+\sqrt{3}\right)-\frac{\sqrt{3}e}{6\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}+\sqrt{3}\right)-\frac{e}{6\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^{2}+\sqrt[3]{\frac{a}{c}}\right)+\frac{d}{3\,a}\sqrt[6]{\frac{a}{c}}\arctan\left(x\frac{1}{\sqrt[6]{\frac{a}{c}}}\right)+\frac{e}{12\,a}\ln\left(x^{2}-\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}-\frac{\sqrt{3}d}{12\,a}\ln\left(x^{2}-\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)\sqrt[6]{\frac{a}{c}}+\frac{\sqrt{3}e}{6\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}-\sqrt{3}\right)+\frac{d}{6\,a}\sqrt[6]{\frac{a}{c}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)/(c*x^6+a),x)
```

```
[Out] 1/12*c*(1/c*a)^(7/6)/a^2*ln(x^2+3^(1/2)*(1/c*a)^(1/6)*x+(1/c*a)^(
1/3))*3^(1/2)*d+1/12*(1/c*a)^(2/3)/a*ln(x^2+3^(1/2)*(1/c*a)^(1/6)
*x+(1/c*a)^(1/3))*e+1/6*(1/c*a)^(1/6)/a*arctan(2*x/(1/c*a)^(1/6)+
3^(1/2))*d-1/6*(1/c*a)^(2/3)/a*arctan(2*x/(1/c*a)^(1/6)+3^(1/2))*
3^(1/2)*e-1/6*(1/c*a)^(2/3)/a*e*ln(x^2+(1/c*a)^(1/3))+1/3*(1/c*a)
^(1/6)/a*d*arctan(x/(1/c*a)^(1/6))+1/12/a*ln(x^2-3^(1/2)*(1/c*a)^
(1/6)*x+(1/c*a)^(1/3))*(1/c*a)^(2/3)*e-1/12/a*ln(x^2-3^(1/2)*(1/c
*a)^(1/6)*x+(1/c*a)^(1/3))*3^(1/2)*(1/c*a)^(1/6)*d+1/6/a*(1/c*a)^
(2/3)*arctan(2*x/(1/c*a)^(1/6)-3^(1/2))*3^(1/2)*e+1/6/a*(1/c*a)^(
1/6)*arctan(2*x/(1/c*a)^(1/6)-3^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/(c*x^6 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.358673, size = 4134, normalized size = 13.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/(c*x^6 + a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*arctan(-(sq
rt(3)*a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(
a^5*c^3)) + sqrt(3)*(a*c^2*d^4 - 3*a^2*c*d^2*e^2))*((a^2*c^2*sqrt
```

```
(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*
e - a*e^3)/(a^2*c^2))^(1/3)/(2*(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d
*e^4)*x + 2*(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*sqrt(((c^3*d^
7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x^2 + (2*a^5*c
^3*d*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3))
 + a^2*c^3*d^5 - 4*a^3*c^2*d^3*e^2 + 3*a^4*c*d*e^4)*((a^2*c^2*sqr
t(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2
*e - a*e^3)/(a^2*c^2))^(2/3) - ((a^4*c^3*d^2*e + a^5*c^2*e^3)*x*s
qrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + (a*c^
3*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*((a^2*c^2*sqrt(-(
c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e -
 a*e^3)/(a^2*c^2))^(1/3))/(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*
e^4 - 3*a^3*d*e^6)) - (a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) + a*c^2*d^4 - 3*a^2*c*d^2*e^2)*((a^2*c
^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3
*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))) - 1/3*sqrt(3)*(-(a^2*c^2*sqr
t(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2
*e + a*e^3)/(a^2*c^2))^(1/3)*arctan(-(sqrt(3)*a^4*c^2*e*sqrt(-(c^
2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - sqrt(3)*(a*c^
2*d^4 - 3*a^2*c*d^2*e^2))*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e
^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1
/3)/(2*(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 2*(c^2*d^5 - 2
*a*c*d^3*e^2 - 3*a^2*d*e^4)*sqrt(((c^3*d^7 - a*c^2*d^5*e^2 - 5*a^
2*c*d^3*e^4 - 3*a^3*d*e^6)*x^2 - (2*a^5*c^3*d*e*sqrt(-(c^2*d^6 -
6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a^2*c^3*d^5 + 4*a^3*c
^2*d^3*e^2 - 3*a^4*c*d*e^4)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4
*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^
(2/3) + ((a^4*c^3*d^2*e + a^5*c^2*e^3)*x*sqrt(-(c^2*d^6 - 6*a*c*d
^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (a*c^3*d^6 - 2*a^2*c^2*d^4*e
^2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^
2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/
3))/(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)) +
(a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c
^3)) - a*c^2*d^4 + 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6
*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^
2*c^2))^(1/3))) - 1/12*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*
log(-(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x^
2 - (2*a^5*c^3*d*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4
)/(a^5*c^3)) + a^2*c^3*d^5 - 4*a^3*c^2*d^3*e^2 + 3*a^4*c*d*e^4)*(
(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3
)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) + ((a^4*c^3*d^2*e + a^5*
c^2*e^3)*x*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c
^3)) + (a*c^3*d^6 - 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*((a^2
*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) +
 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)) - 1/12*(-(a^2*c^2*sqrt(-(c^
2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a
*e^3)/(a^2*c^2))^(1/3)*log(-(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^
3*e^4 - 3*a^3*d*e^6)*x^2 + (2*a^5*c^3*d*e*sqrt(-(c^2*d^6 - 6*a*c*
d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a^2*c^3*d^5 + 4*a^3*c^2*d^3
*e^2 - 3*a^4*c*d*e^4)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(2/3)
- ((a^4*c^3*d^2*e + a^5*c^2*e^3)*x*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2
 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (a*c^3*d^6 - 2*a^2*c^2*d^4*e^2 - 3
*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*
a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)) +
1/6*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^
5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2*a
*c*d^3*e^2 - 3*a^2*d*e^4)*x - (a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d
^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a*c^2*d^4 - 3*a^2*c*d^2*e^2)
*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c
^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)) + 1/6*(-(a^2*c^2*sqrt
(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*
e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2
*d*e^4)*x + (a^4*c^2*e*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2
*e^4)/(a^5*c^3)) - a*c^2*d^4 + 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt(-
(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e
+ a*e^3)/(a^2*c^2))^(1/3))
```

Sympy [A] time = 7.3355, size = 165, normalized size = 0.54

$$\operatorname{RootSum}\left(46656t^6a^5c^4 + t^3\left(432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\log\left(x + \frac{-1296t^4a^4c^2e}{3a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)/(c*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*a**5*c**4 + _t**3*(432*a**4*c**2*e**3 - 1296*
a**3*c**3*d**2*e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**
4*e**2 + c**3*d**6, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*
e - 6*_t*a**3*e**4 + 36*_t*a**2*c*d**2*e**2 - 6*_t*a*c**2*d**4)/(
3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c**2*d**5))))
```

GIAC/XCAS [A] time = 0.29698, size = 397, normalized size = 1.3

$$\frac{\left(ac^5\right)^{\frac{1}{6}}d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} - \frac{\left(ac^5\right)^{\frac{2}{3}}|c|e\ln\left(x^2 + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,ac^5} + \frac{\left(\left(ac^5\right)^{\frac{1}{6}}c^3d - \sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\left(ac^5\right)^{\frac{1}{6}}c^3d + \sqrt{3}\left(ac^5\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{2x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}c^3d + \left(ac^5\right)^{\frac{2}{3}}e\right)\ln\left(x^2 + \sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}} + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}c^3d - \left(ac^5\right)^{\frac{2}{3}}e\right)\ln\left(x^2 - \sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}} + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/(c*x^6 + a),x, algorithm="giac")
```

```
[Out] 1/3*(a*c^5)^(1/6)*d*arctan(x/(a/c)^(1/6))/(a*c) - 1/6*(a*c^5)^(2/
3)*abs(c)*e*ln(x^2 + (a/c)^(1/3))/(a*c^5) + 1/6*((a*c^5)^(1/6)*c^
3*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))
/(a/c)^(1/6))/(a*c^4) + 1/6*((a*c^5)^(1/6)*c^3*d + sqrt(3)*(a*c^5
)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4
) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d + (a*c^5)^(2/3)*e)*ln(x^2 +
 sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(3)*(a*
c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*ln(x^2 - sqrt(3)*x*(a/c)^(1/6
) + (a/c)^(1/3))/(a*c^4)
```

3.2 $\int \frac{d+ex^3}{a-cx^6} \, dx$

Optimal. Leaf size=323

$$\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}}$$

```
[Out] -((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(a^(1/6) - 2*c^(1/6)*x)/(Sqrt[
3]*a^(1/6))])/(2*Sqrt[3]*a^(5/6)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]
*e)*ArcTan[(a^(1/6) + 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/(2*Sqrt[3]
*a^(5/6)*c^(2/3)) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/6) - c^(1/6
)*x])/(6*a^(5/6)*c^(2/3)) + ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/6
) + c^(1/6)*x])/(6*a^(5/6)*c^(1/6)) - ((d - (Sqrt[a]*e)/Sqrt[c])*
Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(1/
6)) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x +
c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rubi [A] time = 0.415444, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$

$$\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] -((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[(a^(1/6) - 2*c^(1/6)*x)/(Sqrt[
3]*a^(1/6))])/(2*Sqrt[3]*a^(5/6)*c^(1/6)) + ((Sqrt[c]*d + Sqrt[a]
*e)*ArcTan[(a^(1/6) + 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/(2*Sqrt[3]
*a^(5/6)*c^(2/3)) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/6) - c^(1/6
)*x])/(6*a^(5/6)*c^(2/3)) + ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/6
) + c^(1/6)*x])/(6*a^(5/6)*c^(1/6)) - ((d - (Sqrt[a]*e)/Sqrt[c])*
Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(1/
6)) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x +
c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rubi in Sympy [A] time = 75.1308, size = 311, normalized size = 0.96

$$-\frac{\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{\frac{5}{6}}c^{\frac{2}{3}}}+\frac{\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{c}x^{2}-\sqrt{a}\sqrt[6]{c}x\right)}{12a^{\frac{5}{6}}c^{\frac{2}{3}}}+\frac{\sqrt{3}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[6]{a}}{3}-\frac{2\sqrt[6]{c}x}{3}\right)}{\sqrt[6]{a}}\right)}{6a^{\frac{5}{6}}c^{\frac{2}{3}}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{\frac{5}{6}}c^{\frac{2}{3}}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{c}x^{2}+\sqrt{a}\sqrt[6]{c}x\right)}{12a^{\frac{5}{6}}c^{\frac{2}{3}}}+\frac{\sqrt{3}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[6]{a}}{3}+\frac{2\sqrt[6]{c}x}{3}\right)}{\sqrt[6]{a}}\right)}{6a^{\frac{5}{6}}c^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)/(-c*x**6+a),x)
```

```
[Out] -(sqrt(a)*e - sqrt(c)*d)*log(a**(1/6) + c**(1/6)*x)/(6*a**(5/6)*c
**(2/3)) + (sqrt(a)*e - sqrt(c)*d)*log(a**(2/3) + a**(1/3)*c**(1/
3)*x**2 - sqrt(a)*c**(1/6)*x)/(12*a**(5/6)*c**(2/3)) + sqrt(3)*(s
qrt(a)*e - sqrt(c)*d)*atan(sqrt(3)*(a**(1/6)/3 - 2*c**(1/6)*x/3)/
a**(1/6))/(6*a**(5/6)*c**(2/3)) - (sqrt(a)*e + sqrt(c)*d)*log(a**
(1/6) - c**(1/6)*x)/(6*a**(5/6)*c**(2/3)) + (sqrt(a)*e + sqrt(c)*
d)*log(a**(2/3) + a**(1/3)*c**(1/3)*x**2 + sqrt(a)*c**(1/6)*x)/(1
2*a**(5/6)*c**(2/3)) + sqrt(3)*(sqrt(a)*e + sqrt(c)*d)*atan(sqrt(
3)*(a**(1/6)/3 + 2*c**(1/6)*x/3)/a**(1/6))/(6*a**(5/6)*c**(2/3))
```

Mathematica [A] time = 0.189456, size = 337, normalized size = 1.04

$$-2\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)-\sqrt{c}d\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^{2}\right)+\sqrt{c}d\log\left(\sqrt[6]{a}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)/(a - c*x^6),x]
```

```
[Out] (-2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (2*c^(1/6)*x)/a^(
1/6))/Sqrt[3]] + 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(1 + (2
*c^(1/6)*x)/a^(1/6))/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)
*x] - 2*Sqrt[a]*e*Log[a^(1/6) - c^(1/6)*x] + 2*Sqrt[c]*d*Log[a^(1
/6) + c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/6) + c^(1/6)*x] - Sqrt[c]
*d*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log
[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[c]*d*Log[a^(1/
3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) + a
^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Maple [A] time = 0.109, size = 380, normalized size = 1.2

$$\frac{e}{12a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2-\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)-\frac{e\sqrt{3}}{6a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x\sqrt{3}}{3}\frac{1}{\sqrt[6]{\frac{a}{c}}}-\frac{\sqrt{3}}{3}\right)$$
$$-\frac{d}{12a}\sqrt[6]{\frac{a}{c}}\ln\left(x^2-\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)+\frac{d\sqrt{3}}{6a}\sqrt[6]{\frac{a}{c}}\arctan\left(\frac{2x\sqrt{3}}{3}\frac{1}{\sqrt[6]{\frac{a}{c}}}-\frac{\sqrt{3}}{3}\right)-\frac{e}{6c}\ln\left(x+\sqrt[6]{\frac{a}{c}}\right)\frac{1}{\sqrt[3]{\frac{a}{c}}}$$
$$+\frac{d}{6c}\ln\left(x+\sqrt[6]{\frac{a}{c}}\right)\left(\frac{a}{c}\right)^{-\frac{5}{6}}-\frac{e}{6c}\ln\left(-x+\sqrt[6]{\frac{a}{c}}\right)\frac{1}{\sqrt[3]{\frac{a}{c}}}-\frac{d}{6c}\ln\left(-x+\sqrt[6]{\frac{a}{c}}\right)\left(\frac{a}{c}\right)^{-\frac{5}{6}}$$
$$+\frac{e}{12a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2+\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)+\frac{e\sqrt{3}}{6a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x\sqrt{3}}{3}\frac{1}{\sqrt[6]{\frac{a}{c}}}+\frac{\sqrt{3}}{3}\right)$$
$$+\frac{d}{12a}\sqrt[6]{\frac{a}{c}}\ln\left(x^2+\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)+\frac{d\sqrt{3}}{6a}\sqrt[6]{\frac{a}{c}}\arctan\left(\frac{2x\sqrt{3}}{3}\frac{1}{\sqrt[6]{\frac{a}{c}}}+\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)/(-c*x^6+a),x)
```

```
[Out] 1/12/a*(1/c*a)^(2/3)*e*ln(x^2-(1/c*a)^(1/6)*x+(1/c*a)^(1/3))-1/6/
a*(1/c*a)^(2/3)*e*3^(1/2)*arctan(2/3*3^(1/2)/(1/c*a)^(1/6)*x-1/3*
3^(1/2))-1/12/a*d*(1/c*a)^(1/6)*ln(x^2-(1/c*a)^(1/6)*x+(1/c*a)^(1
/3))+1/6/a*d*(1/c*a)^(1/6)*3^(1/2)*arctan(2/3*3^(1/2)/(1/c*a)^(1/
6)*x-1/3*3^(1/2))-1/6/c/(1/c*a)^(1/3)*ln(x+(1/c*a)^(1/6))*e+1/6/c
/(1/c*a)^(5/6)*ln(x+(1/c*a)^(1/6))*d-1/6/c/(1/c*a)^(1/3)*ln(-x+(1
/c*a)^(1/6))*e-1/6/c/(1/c*a)^(5/6)*ln(-x+(1/c*a)^(1/6))*d+1/12/a*
(1/c*a)^(2/3)*e*ln(x^2+(1/c*a)^(1/6)*x+(1/c*a)^(1/3))+1/6/a*(1/c*
a)^(2/3)*e*3^(1/2)*arctan(2/3*3^(1/2)/(1/c*a)^(1/6)*x+1/3*3^(1/2)
)+1/12/a*d*(1/c*a)^(1/6)*ln(x^2+(1/c*a)^(1/6)*x+(1/c*a)^(1/3))+1/
6/a*d*(1/c*a)^(1/6)*3^(1/2)*arctan(2/3*3^(1/2)/(1/c*a)^(1/6)*x+1/
3*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^3 + d)/(c*x^6 - a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.377304, size = 4082, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^3 + d)/(c*x^6 - a),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2
*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*arctan(-(s
qrt(3)*a^4*c^2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(
a^5*c^3)) - sqrt(3)*(a*c^2*d^4 + 3*a^2*c*d^2*e^2))*(-(a^2*c^2*sqr
t((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*
e + a*e^3)/(a^2*c^2))^(1/3)/(2*(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d
*e^4)*x + 2*(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*sqrt(((c^3*d^
7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - (2*a^5*c
^3*d*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3))
- a^2*c^3*d^5 - 4*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4)*(-(a^2*c^2*sqr
t((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*
e + a*e^3)/(a^2*c^2))^(2/3) + ((a^4*c^3*d^2*e - a^5*c^2*e^3)*x*sq
rt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (a*c^3*
d^6 + 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt((c^
2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a
*e^3)/(a^2*c^2))^(1/3))/(c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^
4 + 3*a^3*d*e^6)) + (a^4*c^2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*
a^2*d^2*e^4)/(a^5*c^3)) - a*c^2*d^4 - 3*a^2*c*d^2*e^2)*(-(a^2*c^2
*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*
d^2*e + a*e^3)/(a^2*c^2))^(1/3))) + 1/3*sqrt(3)*((a^2*c^2*sqrt((c
^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e -
a*e^3)/(a^2*c^2))^(1/3)*arctan(-(sqrt(3)*a^4*c^2*e*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + sqrt(3)*(a*c^2*d^4
+ 3*a^2*c*d^2*e^2))*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a
^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)/(2*(
c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 2*(c^2*d^5 + 2*a*c*d^3
*e^2 - 3*a^2*d*e^4)*sqrt(((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*
e^4 + 3*a^3*d*e^6)*x^2 + (2*a^5*c^3*d*e*sqrt((c^2*d^6 + 6*a*c*d^4
*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a^2*c^3*d^5 + 4*a^3*c^2*d^3*e^
2 + 3*a^4*c*d*e^4)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^
2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) - ((a
^4*c^3*d^2*e - a^5*c^2*e^3)*x*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a
^2*d^2*e^4)/(a^5*c^3)) + (a*c^3*d^6 + 2*a^2*c^2*d^4*e^2 - 3*a^3*c
*d^2*e^4)*x)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))/(c^3*d^7 +
 a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)) - (a^4*c^2*e*sqr
t((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a*c^2*d^
4 + 3*a^2*c*d^2*e^2)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*
a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))) -
 1/12*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(
a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log((c^3*d^7 + a*
c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^2 - (2*a^5*c^3*d*e
*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - a^2*
c^3*d^5 - 4*a^3*c^2*d^3*e^2 - 3*a^4*c*d*e^4)*(-(a^2*c^2*sqrt((c^2
*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*
e^3)/(a^2*c^2))^(2/3) + ((a^4*c^3*d^2*e - a^5*c^2*e^3)*x*sqrt((c^
2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - (a*c^3*d^6 +
2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*(-(a^2*c^2*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/
(a^2*c^2))^(1/3)) - 1/12*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2
+ 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)
*log((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x^
2 + (2*a^5*c^3*d*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)
/(a^5*c^3)) + a^2*c^3*d^5 + 4*a^3*c^2*d^3*e^2 + 3*a^4*c*d*e^4)*((
a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3))
 - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(2/3) - ((a^4*c^3*d^2*e - a^5*c^
2*e^3)*x*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)
) + (a*c^3*d^6 + 2*a^2*c^2*d^4*e^2 - 3*a^3*c*d^2*e^4)*x)*((a^2*c^
2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c
*d^2*e - a*e^3)/(a^2*c^2))^(1/3)) + 1/6*(-(a^2*c^2*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/
(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x +
 (a^4*c^2*e*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c
^3)) - a*c^2*d^4 - 3*a^2*c*d^2*e^2)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*
a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2
*c^2))^(1/3)) + 1/6*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a
^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(
-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x - (a^4*c^2*e*sqrt((c^2
*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + a*c^2*d^4 + 3*
a^2*c*d^2*e^2)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^
2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3))
```

Sympy [A] time = 7.56856, size = 168, normalized size = 0.52

$$-\operatorname{RootSum}\left(46656t^6a^5c^4 + t^3\left(-432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 - 3a^2cd^2e^4 + 3ac^2d^4e^2 - c^3d^6, \left(t \mapsto t\log\left(x + \frac{-1296t^4a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)/(-c*x**6+a),x)
```

```
[Out] -RootSum(46656*_t**6*a**5*c**4 + _t**3*(-432*a**4*c**2*e**3 - 129
6*a**3*c**3*d**2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d
**4*e**2 - c**3*d**6, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**
2*e + 6*_t*a**3*e**4 + 36*_t*a**2*c*d**2*e**2 + 6*_t*a*c**2*d**4)
/(3*a**2*d*e**4 - 2*a*c*d**3*e**2 - c**2*d**5))))
```

GIAC/XCAS [A] time = 0.281019, size = 424, normalized size = 1.31

$$\frac{\left(-ac^5\right)^{\frac{1}{6}} d \arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} - \frac{\left(-ac^5\right)^{\frac{2}{3}} |c| e \ln\left(x^2 + \left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,ac^5} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3 d - \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right) \arctan\left(\frac{2\,x + \sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3 d + \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right) \arctan\left(\frac{2\,x - \sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3 d + \left(-ac^5\right)^{\frac{2}{3}} e\right) \ln\left(x^2 + \sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}} + \left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3 d - \left(-ac^5\right)^{\frac{2}{3}} e\right) \ln\left(x^2 - \sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}} + \left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^3 + d)/(c*x^6 - a),x, algorithm="giac")
```

```
[Out] 1/3*(-a*c^5)^(1/6)*d*arctan(x/(-a/c)^(1/6))/(a*c) - 1/6*(-a*c^5)^
(2/3)*abs(c)*e*ln(x^2 + (-a/c)^(1/3))/(a*c^5) + 1/6*((-a*c^5)^(1/
6)*c^3*d - sqrt(3)*(-a*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(-a/c)
^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/6*((-a*c^5)^(1/6)*c^3*d + sqrt(
3)*(-a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(-a/c)^(1/6))/(-a/c)^(
1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d + (-a*c^5)^(2/
3)*e)*ln(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4) - 1
/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d - (-a*c^5)^(2/3)*e)*ln(x^2 - sq
rt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

3.3 $\int \frac{d+ex^4}{a+cx^8}\,dx$

Optimal. Leaf size=754

$$\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}-\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}}-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}}-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}+\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}}+\frac{\left(-\frac{\sqrt{a}e}{\sqrt{c}}+\sqrt{2}d+d\right)\log\left(\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}$$

```
[Out] -(Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[
(Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1
/8))])/(8*a^(7/8)*c^(5/8)) + (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8
)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(7/8)*c^(5/8)) + (Sqrt[2
- Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[2 -
 Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8
*a^(7/8)*c^(5/8)) - (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d -
 Sqrt[a]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqr
t[2 - Sqrt[2]]*a^(1/8))])/(8*a^(7/8)*c^(5/8)) + (((1 - Sqrt[2])*S
qrt[c]*d - Sqrt[a]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(
1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(7/8)*c^(5/8))
- (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/4) + Sqrt[2 - S
qrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])
]*a^(7/8)*c^(5/8)) - (((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log[a
^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*S
qrt[2*(2 + Sqrt[2])]*a^(7/8)*c^(5/8)) + ((d + Sqrt[2]*d - (Sqrt[a
]*e)/Sqrt[c])*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x +
 c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(7/8)*c^(1/8))
```

Rubi [A] time = 2.79363, antiderivative size = 754, normalized size of antiderivative = 1., number of

steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} - \frac{\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{7/8}c^{5/8}} - \frac{\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}c^{5/8}} - \frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} + \frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} + \frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} - \frac{\sqrt{2+\sqrt{2}}\left(\left(1-\sqrt{2}\right)\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{7/8}c^{5/8}} + \frac{\left(-\frac{\sqrt{a}e}{\sqrt{c}}+\sqrt{2}d+d\right)\log\left(\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{7/8}\sqrt[8]{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(a + c*x^8),x]
```

```
[Out] -(Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[
(Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1
/8))])/(8*a^(7/8)*c^(5/8)) + (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) - 2*c^(1/8
)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(7/8)*c^(5/8)) + (Sqrt[2
- Sqrt[2]]*((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[2 -
 Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1/8))])/(8
*a^(7/8)*c^(5/8)) - (Sqrt[2 + Sqrt[2]]*((1 - Sqrt[2])*Sqrt[c]*d -
 Sqrt[a]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqr
t[2 - Sqrt[2]]*a^(1/8))])/(8*a^(7/8)*c^(5/8)) + (((1 - Sqrt[2])*S
qrt[c]*d - Sqrt[a]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^(1/8)*c^(
1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(7/8)*c^(5/8))
- (((1 - Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log[a^(1/4) + Sqrt[2 - S
qrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])
]*a^(7/8)*c^(5/8)) - (((1 + Sqrt[2])*Sqrt[c]*d - Sqrt[a]*e)*Log[a
^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*S
qrt[2*(2 + Sqrt[2])]*a^(7/8)*c^(5/8)) + ((d + Sqrt[2]*d - (Sqrt[a
]*e)/Sqrt[c])*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x +
 c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(7/8)*c^(1/8))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/(c*x**8+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.59334, size = 534, normalized size = 0.71

$$2\tan^{-1}\left(\frac{\sqrt[8]{c}x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}-\tan\left(\frac{\pi}{8}\right)\right)\left(\sqrt[8]{a}\sqrt{c}d\cos\left(\frac{\pi}{8}\right)-a^{5/8}e\sin\left(\frac{\pi}{8}\right)\right)+2\tan^{-1}\left(\frac{\sqrt[8]{c}x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}+\tan\left(\frac{\pi}{8}\right)\right)\left(\sqrt[8]{a}\sqrt{c}d\cos\left(\frac{\pi}{8}\right)-a^{5/}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(a + c*x^8),x]
```

```
[Out] (-2*a^(1/8)*ArcTan[Cot[Pi/8] - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(Sq
rt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + 2*a^(1/8)*ArcTan[Cot[P
i/8] + (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt
[c]*d*Sin[Pi/8]) - a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)*
c^(1/8)*x*Sin[Pi/8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8])
+ a^(1/8)*Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^(1/8)*x*Sin[Pi/
8]]*(Sqrt[a]*e*Cos[Pi/8] + Sqrt[c]*d*Sin[Pi/8]) + a^(1/8)*Log[a^(
1/4) + c^(1/4)*x^2 - 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*
Cos[Pi/8]) + Sqrt[a]*e*Sin[Pi/8]) - a^(1/8)*Log[a^(1/4) + c^(1/4)
*x^2 + 2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(-(Sqrt[c]*d*Cos[Pi/8]) + S
qrt[a]*e*Sin[Pi/8]) + 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Ta
n[Pi/8]]*(a^(1/8)*Sqrt[c]*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]) + 2*
ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(1/8)*Sqrt[c
]*d*Cos[Pi/8] - a^(5/8)*e*Sin[Pi/8]))/(8*a*c^(5/8))
```

Maple [C] time = 0.027, size = 34, normalized size = 0.1

$$\frac{1}{8c}\sum_{\_R=RootOf(c\_Z^8+a)}\frac{(\_R^4e+d)\ln(x-\_R)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/(c*x^8+a),x)
```

```
[Out] 1/8/c*sum((_R^4*e+d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^4+d}{cx^8+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(c*x^8 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(c*x^8 + a), x)
```

Fricas [A] time = 0.485574, size = 3762, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(c*x^8 + a),x, algorithm="fricas")
```

```
[Out] -1/2*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^
4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*
d*e^3)/(a^3*c^2))^(1/4)*arctan((a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c
^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^
7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2
*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*
c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2
))^(1/4)/((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)
*x + (c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*sqrt
(((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e
^6 + a^4*e^8)*x^2 + (2*a^6*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*
e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))
 + a^2*c^4*d^6 - 7*a^3*c^3*d^4*e^2 + 7*a^4*c^2*d^2*e^4 - a^5*c*e^
6)*sqrt(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*
d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*
a*d*e^3)/(a^3*c^2)))/(c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*
e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)))) + 1/2*((a^3*c^2*sqrt(-(c^4*d^
8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^
4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*arcta
n((a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e
^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c
^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6
*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)
) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)/((c^3*d^6 - 5*a*c^2*d
^4*e^2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x + (c^3*d^6 - 5*a*c^2*d^4*e^
2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(((c^4*d^8 - 4*a*c^3*d^6*e^2 -
 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x^2 - (2*a^6*c^4
*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a^2*c^4*d^6 + 7*a^3*c^3*d^4
*e^2 - 7*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(c^4*d^
8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^
4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2)))/(c^4*d^8 -
 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8
)))) + 1/8*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*
c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e
- 4*a*d*e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 - 5*
a^2*c*d^2*e^4 + a^3*e^6)*x + (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*c^3
*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*
c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2*s
qrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*
d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))
^(1/4)) - 1/8*(-(a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3
*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2 -
 5*a^2*c*d^2*e^4 + a^3*e^6)*x - (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*a*
c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a
^7*c^5)) + a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^
2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3
*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^
2))^(1/4)) - 1/8*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38
*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d
^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^2
 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x + (a^5*c^3*e*sqrt(-(c^4*d^8 - 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/
(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c
^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^
3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*c
^2))^(1/4)) + 1/8*((a^3*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 3
8*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*
d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)*log((c^3*d^6 - 5*a*c^2*d^4*e^
2 - 5*a^2*c*d^2*e^4 + a^3*e^6)*x - (a^5*c^3*e*sqrt(-(c^4*d^8 - 12
*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)
/(a^7*c^5)) - a*c^3*d^5 + 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*
c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e + 4*a*d*e^3)/(a^3*
c^2))^(1/4))
```

Sympy [A] time = 37.0573, size = 199, normalized size = 0.26

$$\text{RootSum}\left(16777216t^8a^7c^5 + t^4\left(-32768a^5c^3de^3 + 32768a^4c^4d^3e\right) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8, \left(t \mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/(c*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**7*c**5 + _t**4*(-32768*a**5*c**3*d*e**3
 + 32768*a**4*c**4*d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a
**2*c**2*d**4*e**4 + 4*a*c**3*d**6*e**2 + c**4*d**8, Lambda(_t, _
t*log(x + (-32768*_t**5*a**5*c**3*e + 40*_t*a**3*c*d*e**4 - 80*_t
*a**2*c**2*d**3*e**2 + 8*_t*a*c**3*d**5)/(a**3*e**6 - 5*a**2*c*d*
*2*e**4 - 5*a*c**2*d**4*e**2 + c**3*d**6))))
```

GIAC/XCAS [A] time = 0.31174, size = 811, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(c*x^8 + a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c
)^(1/8))*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt
(2) + 2)*(a/c)^(1/8)))/a - 1/8*(sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e
- d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(-sqrt(2) +
2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/a + 1/8*(sqrt(sq
rt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arct
an((2*x + sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c
)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt
(2) + 2)*(a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) + 2)*(a/c)^(1/8)
)/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/a - 1/16*(sqrt(-sqrt(2) + 2)*
(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*ln(x^2 + x*sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a + 1/16*(sqrt(-sqrt(2) +
 2)*(a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*ln(x^2 - x*s
qrt(sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a + 1/16*(sqrt(sqrt(2
) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*ln(x^2 +
 x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a - 1/16*(sqrt(s
qrt(2) + 2)*(a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*ln(
x^2 - x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/a
```

3.4 $\int \frac{d+ex^4}{a-cx^8}\,dx$

Optimal. Leaf size=329

$$\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

```
[Out] ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*
c^(5/8)) - ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 - (Sqrt[2]*c^(1/8)
*x)/a^(1/8)])/(4*Sqrt[2]*a^(7/8)*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqr
t[c])*ArcTan[1 + (Sqrt[2]*c^(1/8)*x)/a^(1/8)])/(4*Sqrt[2]*a^(7/8)
*c^(1/8)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTanh[(c^(1/8)*x)/a^(1/8)]
)/(4*a^(7/8)*c^(5/8)) - ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) -
Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)*c^(1
/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c
^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)*c^(1/8))
```

Rubi [A] time = 0.456416, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556

$$\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tanh^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{7/8}c^{5/8}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2}a^{7/8}\sqrt[8]{c}}-\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}+\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}+1\right)}{4\sqrt{2}a^{7/8}\sqrt[8]{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(a - c*x^8),x]
```


```
[Out] ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*a^(7/8)*
c^(5/8)) - ((d - (Sqrt[a]*e)/Sqrt[c])*ArcTan[1 - (Sqrt[2]*c^(1/8)
*x)/a^(1/8)])/(4*Sqrt[2]*a^(7/8)*c^(1/8)) + ((d - (Sqrt[a]*e)/Sqr
t[c])*ArcTan[1 + (Sqrt[2]*c^(1/8)*x)/a^(1/8)])/(4*Sqrt[2]*a^(7/8)
*c^(1/8)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTanh[(c^(1/8)*x)/a^(1/8)]
)/(4*a^(7/8)*c^(5/8)) - ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) -
Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)*c^(1
/8)) + ((d - (Sqrt[a]*e)/Sqrt[c])*Log[a^(1/4) + Sqrt[2]*a^(1/8)*c
^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a^(7/8)*c^(1/8))
```

Rubi in Sympy [A] time = 77.9307, size = 304, normalized size = 0.92

$$\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^{2}\right)}{16a^{\frac{7}{8}}c^{\frac{5}{8}}}-\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^{2}\right)}{16a^{\frac{7}{8}}c^{\frac{5}{8}}}+\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{8a^{\frac{7}{8}}c^{\frac{5}{8}}}-\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{8a^{\frac{7}{8}}c^{\frac{5}{8}}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{\frac{7}{8}}c^{\frac{5}{8}}}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atanh}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4a^{\frac{7}{8}}c^{\frac{5}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/(-c*x**8+a),x)
```

```
[Out] sqrt(2)*(sqrt(a)*e - sqrt(c)*d)*log(-sqrt(2)*a**(1/8)*c**(1/8)*x
+ a**(1/4) + c**(1/4)*x**2)/(16*a**(7/8)*c**(5/8)) - sqrt(2)*(sqr
t(a)*e - sqrt(c)*d)*log(sqrt(2)*a**(1/8)*c**(1/8)*x + a**(1/4) +
c**(1/4)*x**2)/(16*a**(7/8)*c**(5/8)) + sqrt(2)*(sqrt(a)*e - sqrt
(c)*d)*atan(1 - sqrt(2)*c**(1/8)*x/a**(1/8))/(8*a**(7/8)*c**(5/8)
) - sqrt(2)*(sqrt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/8)*x/a
**(1/8))/(8*a**(7/8)*c**(5/8)) + (sqrt(a)*e + sqrt(c)*d)*atan(c**
(1/8)*x/a**(1/8))/(4*a**(7/8)*c**(5/8)) + (sqrt(a)*e + sqrt(c)*d)
*atanh(c**(1/8)*x/a**(1/8))/(4*a**(7/8)*c**(5/8))
```

Mathematica [A] time = 0.233701, size = 425, normalized size = 1.29

$$\frac{\left(a^{5/8}e-\sqrt[8]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^{2}\right)}{8\sqrt{2}ac^{5/8}}-\frac{\left(a^{5/8}e-\sqrt[8]{a}\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^{2}\right)}{8\sqrt{2}ac^{5/8}}-\frac{\left(a^{5/8}e+\sqrt[8]{a}\sqrt{c}d\right)\log\left(\sqrt[8]{a}-\sqrt[8]{c}x\right)}{8ac^{5/8}}-\frac{\left(-a^{5/8}e-\sqrt[8]{a}\sqrt{c}d\right)\log\left(\sqrt[8]{a}+\sqrt[8]{c}x\right)}{8ac^{5/8}}+\frac{\left(a^{5/8}e+\sqrt[8]{a}\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt[8]{c}x}{\sqrt[8]{a}}\right)}{4ac^{5/8}}-\frac{\left(a^{5/8}e-\sqrt[8]{a}\sqrt{c}d\right)\tan^{-1}\left(\frac{2\sqrt[8]{c}x-\sqrt{2}\sqrt[8]{a}}{\sqrt{2}\sqrt[8]{a}}\right)}{4\sqrt{2}ac^{5/8}}-\frac{\left(a^{5/8}e-\sqrt[8]{a}\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2}\sqrt[8]{a}}\right)}{4\sqrt{2}ac^{5/8}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(a - c*x^8),x]
```

```
[Out] ((a^(1/8)*Sqrt[c]*d + a^(5/8)*e)*ArcTan[(c^(1/8)*x)/a^(1/8)])/(4*
a*c^(5/8)) - ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*ArcTan[(-(Sqrt[2
]*a^(1/8)) + 2*c^(1/8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sqrt[2]*a*c^(5/8
)) - ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*ArcTan[(Sqrt[2]*a^(1/8)
+ 2*c^(1/8)*x)/(Sqrt[2]*a^(1/8))])/(4*Sqrt[2]*a*c^(5/8)) - ((a^(1
/8)*Sqrt[c]*d + a^(5/8)*e)*Log[a^(1/8) - c^(1/8)*x])/(8*a*c^(5/8)
) - ((-(a^(1/8)*Sqrt[c]*d) - a^(5/8)*e)*Log[a^(1/8) + c^(1/8)*x])
/(8*a*c^(5/8)) + ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4)
- Sqrt[2]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a*c^(5/8))
 - ((-(a^(1/8)*Sqrt[c]*d) + a^(5/8)*e)*Log[a^(1/4) + Sqrt[2]*a^(1
/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2]*a*c^(5/8))
```

Maple [C] time = 0.02, size = 39, normalized size = 0.1

$$\frac{1}{8c} \sum_{\_R = RootOf(c\_Z^8 - a)} \frac{(-\_R^4 e - d)\ln(x - \_R)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/(-c*x^8+a),x)
```

```
[Out] 1/8/c*sum((-_R^4*e-d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^4 + d}{cx^8 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^4 + d)/(c*x^8 - a),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^4 + d)/(c*x^8 - a), x)
```

Fricas [A] time = 0.492967, size = 3752, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^4 + d)/(c*x^8 - a),x, algorithm="fricas")
```

```
[Out] -1/2*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*
e^3)/(a^3*c^2))^(1/4)*arctan(-(a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3
*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*
c^5)) - a*c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sq
rt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^
2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(
1/4)/((c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*x +
 (c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(((c
^4*d^8 + 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 +
 a^4*e^8)*x^2 - (2*a^6*c^4*d*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a^
2*c^4*d^6 - 7*a^3*c^3*d^4*e^2 - 7*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sq
rt((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3
)/(a^3*c^2)))/(c^4*d^8 + 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 + 4
*a^3*c*d^2*e^6 + a^4*e^8)))) + 1/2*(-(a^3*c^2*sqrt((c^4*d^8 + 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/
(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)*arctan(-(a^5
*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12
*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c^3*d^5 + 6*a^2*c^2*d^3*
e^2 + a^3*c*d*e^4)*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c
*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)/((c^3*d^6 + 5*a*c^2*d^4*e^2
- 5*a^2*c*d^2*e^4 - a^3*e^6)*x + (c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a
^2*c*d^2*e^4 - a^3*e^6)*sqrt(((c^4*d^8 + 4*a*c^3*d^6*e^2 - 10*a^2
*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*x^2 + (2*a^6*c^4*d*e*sq
rt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^
2*e^6 + a^4*e^8)/(a^7*c^5)) + a^2*c^4*d^6 + 7*a^3*c^3*d^4*e^2 + 7
*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt((c^4*d^8 + 12*a
```

```
*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(
a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2)))/(c^4*d^8 + 4*a*c^3
*d^6*e^2 - 10*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)))) + 1
/8*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^
3)/(a^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2
*e^4 - a^3*e^6)*x + (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*
c^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt((c^4*d^
8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^
4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1/
8*((a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3
)/(a^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*
e^4 - a^3*e^6)*x - (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - a*c
^3*d^5 - 6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*((a^3*c^2*sqrt((c^4*d^8
 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4
*e^8)/(a^7*c^5)) + 4*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))^(1/4)) - 1/8
*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4
 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3
)/(a^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2*
e^4 - a^3*e^6)*x + (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*c
^3*d^5 + 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2*sqrt((c^4*d^
8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^
4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4)) + 1/
8*(-(a^3*c^2*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^
4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^
3)/(a^3*c^2))^(1/4)*log(-(c^3*d^6 + 5*a*c^2*d^4*e^2 - 5*a^2*c*d^2
*e^4 - a^3*e^6)*x - (a^5*c^3*e*sqrt((c^4*d^8 + 12*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + a*
c^3*d^5 + 6*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*(-(a^3*c^2*sqrt((c^4*d
^8 + 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a
^4*e^8)/(a^7*c^5)) - 4*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))^(1/4))
```

Sympy [A] time = 36.8716, size = 202, normalized size = 0.61

$$-\operatorname{RootSum}\left(16777216t^8a^7c^5+t^4\left(-32768a^5c^3de^3-32768a^4c^4d^3e\right)-a^4e^8+4a^3cd^2e^6-6a^2c^2d^4e^4+4ac^3d^6e^2-c^4d^8,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/(-c*x**8+a),x)
```

```
[Out] -RootSum(16777216*_t**8*a**7*c**5 + _t**4*(-32768*a**5*c**3*d*e**
3 - 32768*a**4*c**4*d**3*e) - a**4*e**8 + 4*a**3*c*d**2*e**6 - 6*
a**2*c**2*d**4*e**4 + 4*a*c**3*d**6*e**2 - c**4*d**8, Lambda(_t,
_t*log(x + (-32768*_t**5*a**5*c**3*e + 40*_t*a**3*c*d*e**4 + 80*_
t*a**2*c**2*d**3*e**2 + 8*_t*a*c**3*d**5)/(a**3*e**6 + 5*a**2*c*d
**2*e**4 - 5*a*c**2*d**4*e**2 - c**3*d**6))))
```

GIAC/XCAS [A] time = 0.310602, size = 855, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^4 + d)/(c*x^8 - a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(-a
/c)^(1/8))*arctan((2*x + sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(s
```

```
qrt(2) + 2)*(-a/c)^(1/8)))/a - 1/8*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/
8)*e - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8))*arctan((2*x - sqrt(-sqrt
(2) + 2)*(-a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(-a/c)^(1/8)))/a + 1/8*
(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2) + 2)*(-a/c)^(
1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(-a/c)^(1/8))/(sqrt(-sqrt(2
) + 2)*(-a/c)^(1/8)))/a + 1/8*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e +
 d*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8))*arctan((2*x - sqrt(sqrt(2) +
2)*(-a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(-a/c)^(1/8)))/a - 1/16*(sqr
t(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(-a/c)^(1/8)
)*ln(x^2 + x*sqrt(sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1/4))/a + 1
/16*(sqrt(-sqrt(2) + 2)*(-a/c)^(5/8)*e - d*sqrt(sqrt(2) + 2)*(-a/
c)^(1/8))*ln(x^2 - x*sqrt(sqrt(2) + 2)*(-a/c)^(1/8) + (-a/c)^(1/4
))/a + 1/16*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-sqrt(2) +
 2)*(-a/c)^(1/8))*ln(x^2 + x*sqrt(-sqrt(2) + 2)*(-a/c)^(1/8) + (-
a/c)^(1/4))/a - 1/16*(sqrt(sqrt(2) + 2)*(-a/c)^(5/8)*e + d*sqrt(-
sqrt(2) + 2)*(-a/c)^(1/8))*ln(x^2 - x*sqrt(-sqrt(2) + 2)*(-a/c)^(
1/8) + (-a/c)^(1/4))/a
```

3.5 $\int \frac{d+ex^4}{d^2+bx^4+e^2x^8}\,dx$

Optimal. Leaf size=791

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}$$

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x
)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*S
qrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt
[e] + Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - S
qrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2
*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] + 2*
Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]
*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqr
t[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sq
rt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sq
rt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b
 + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - S
qrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-
b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] -
Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[
-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] +
 Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt
[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e]
+ Sqrt[-b + 2*d*e]])
```

Rubi [A] time = 1.87183, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-b}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-b}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x
)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*S
qrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt
[e] + Sqrt[-b + 2*d*e]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - S
qrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2
*d*e]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b + 2*d*e]] + 2*
Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]
*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[-b + 2*d*e]]) + ArcTan[(Sqrt[2*Sqr
t[d]*Sqrt[e] + Sqrt[-b + 2*d*e]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sq
rt[e] - Sqrt[-b + 2*d*e]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sq
rt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-b
 + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - S
qrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[-
b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] -
Sqrt[-b + 2*d*e]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[
-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] +
 Sqrt[-b + 2*d*e]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt
[-b + 2*d*e]]*x + Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e]
+ Sqrt[-b + 2*d*e]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/(e**2*x**8+b*x**4+d**2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0623148, size = 67, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2+\#1^4b+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(d^2 + b*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 + b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1
]*#1^4)/(b*#1^3 + 2*e^2*#1^7) & ]/4
```

Maple [C] time = 0.059, size = 53, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(e^2\_Z^8+b\_Z^4+d^2\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{2\_R^7e^2+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/(e^2*x^8+b*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^
2+_Z^4*b+d^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 + bx^4 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2), x)
```

Fricas [A] time = 0.330535, size = 3082, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)
/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*arctan(-1/2*(2*d*e - (4*d^4*
e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^
6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4
*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d
^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^
2*d^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e^2*x^2 + sqrt(1/2)*(2*b*d*e
+ b^2 - (8*d^5*e^3 + 12*b*d^4*e^2 + 6*b^2*d^3*e + b^3*d^2)*sqrt(-
(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)))*
sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7
*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4
*b*d^3*e + b^2*d^2)))/e^2))) - sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 +
4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2
+ 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))
)*arctan(1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)
*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)
/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e
^2*x^2 + sqrt(1/2)*(2*b*d*e + b^2 + (8*d^5*e^3 + 12*b*d^4*e^2 + 6
*b^2*d^3*e + b^3*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2
 + 6*b^2*d^5*e + b^3*d^4)))*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^
2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^
3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))/e^2))) + 1/4*sqr
t(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e
 - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4
*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e
^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6
*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*
e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^
6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2
*d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2
*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e +
 b^3*d^4)) + b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2
*(2*d*e - (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*
d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)*sqrt(sqrt(1
/2)*sqrt(-((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8
*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) + b)/(4*d^4*e^2
 + 4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2
+ 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^
2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2
)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt
(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4))
 + b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt
```

```
(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4))
 - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*s
qrt(((4*d^4*e^2 + 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e
^3 + 12*b*d^6*e^2 + 6*b^2*d^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b
*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 + 4*b*d^3*e
 + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d
^5*e + b^3*d^4)) + b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*b*d^3*e
 + b^2*d^2)*sqrt(-(2*d*e - b)/(8*d^7*e^3 + 12*b*d^6*e^2 + 6*b^2*d
^5*e + b^3*d^4)) - b)/(4*d^4*e^2 + 4*b*d^3*e + b^2*d^2))))
```

Sympy [A] time = 20.6955, size = 136, normalized size = 0.17

$$\text{RootSum}\left(t^8\left(65536b^4d^2 + 524288b^3d^3e + 1572864b^2d^4e^2 + 2097152bd^5e^3 + 1048576d^6e^4\right) + t^4\left(256b^3 + 1024b^2de + 1024b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/(e**2*x**8+b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 + 524288*b**3*d**3*e + 1572864*b**
2*d**4*e**2 + 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(2
56*b**3 + 1024*b**2*d*e + 1024*b*d**2*e**2) + e**2, Lambda(_t, _t
*log(x + (1024*_t**5*b**2*d**2 + 4096*_t**5*b*d**3*e + 4096*_t**5
*d**4*e**2 + 4*_t*b + 4*_t*d*e)/e)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 + bx^4 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + b*x^4 + d^2), x)
```

3.6 $\int \frac{d+ex^4}{d^2+fx^4+e^2x^8}\,dx$

Optimal. Leaf size=791

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}$$

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)
/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqr
t[d]*Sqrt[e] + Sqrt[2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]
) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] + 2*Sqrt[e]
*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt
[e] + Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sq
rt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e -
f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f
]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x +
 Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]
]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]
) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + S
qrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])
```

Rubi [A] time = 1.75234, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de-f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de-f}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)
/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqr
t[d]*Sqrt[e] + Sqrt[2*d*e - f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[2*d*e - f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]
) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]] + 2*Sqrt[e]
*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt
[e] + Sqrt[2*d*e - f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sq
rt[2*d*e - f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e -
f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f
]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]]*x +
 Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e - f]
]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]
) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]]*x + S
qrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e - f]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/(e**2*x**8+f*x**4+d**2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0476941, size = 67, normalized size = 0.08

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2+\#1^4f+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2+\#1^3f}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(d^2 + f*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 + f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1
]*#1^4)/(f*#1^3 + 2*e^2*#1^7) & ]/4
```

Maple [C] time = 0.06, size = 53, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(e^2\_Z^8+f\_Z^4+d^2\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{2\_R^7e^2+\_R^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/(e^2*x^8+f*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2+_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^
2+_Z^4*f+d^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 + fx^4 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2), x)
```

Fricas [A] time = 0.313566, size = 3082, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)
/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*arctan(-1/2*(2*d*e - (4*d^4*
e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*
e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4
*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6
*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^
2*f^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e^2*x^2 + sqrt(1/2)*(2*d*e*f
+ f^2 - (8*d^5*e^3 + 12*d^4*e^2*f + 6*d^3*e*f^2 + d^2*f^3)*sqrt(-
(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)))*
sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7
*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4
*d^3*e*f + d^2*f^2)))/e^2))) - sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 +
4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f
+ 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2))
)*arctan(1/2*(2*d*e + (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)
*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)
/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e
^2*x^2 + sqrt(1/2)*(2*d*e*f + f^2 + (8*d^5*e^3 + 12*d^4*e^2*f + 6
*d^3*e*f^2 + d^2*f^3)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f
 + 6*d^5*e*f^2 + d^4*f^3)))*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^
2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^
4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))/e^2))) + 1/4*sqr
t(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e
 - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4
*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e - (4*d^4*e
^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e
^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*
e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*
e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2
*f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2
*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 +
 d^4*f^3)) + f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2
*(2*d*e - (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*
d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1
/2)*sqrt(-((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8
*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) + f)/(4*d^4*e^2
 + 4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2
+ 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*
f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2
)))*log(e*x + 1/2*(2*d*e + (4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt
(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3))
 + f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt
(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3))
```

```
 - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*s
qrt(((4*d^4*e^2 + 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e
^3 + 12*d^6*e^2*f + 6*d^5*e*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d
^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e + (4*d^4*e^2 + 4*d^3*e*f
 + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e
*f^2 + d^4*f^3)) + f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 + 4*d^3*e*f
 + d^2*f^2)*sqrt(-(2*d*e - f)/(8*d^7*e^3 + 12*d^6*e^2*f + 6*d^5*e
*f^2 + d^4*f^3)) - f)/(4*d^4*e^2 + 4*d^3*e*f + d^2*f^2))))
```

Sympy [A] time = 18.9669, size = 136, normalized size = 0.17

$$\operatorname{RootSum}\left(t^{8}\left(1048576d^{6}e^{4}+2097152d^{5}e^{3}f+1572864d^{4}e^{2}f^{2}+524288d^{3}ef^{3}+65536d^{2}f^{4}\right)+t^{4}\left(1024d^{2}e^{2}f+1024def^{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/(e**2*x**8+f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 + 2097152*d**5*e**3*f + 1572864*
d**4*e**2*f**2 + 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(1
024*d**2*e**2*f + 1024*d*e*f**2 + 256*f**3) + e**2, Lambda(_t, _t
*log(x + (4096*_t**5*d**4*e**2 + 4096*_t**5*d**3*e*f + 1024*_t**5
*d**2*f**2 + 4*_t*d*e + 4*_t*f)/e)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 + fx^4 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 + f*x^4 + d^2), x)
```

3.7 $\int \frac{d+ex^4}{d^2-bx^4+e^2x^8}\,dx$

Optimal. Leaf size=349

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

```
[Out] -((Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt
[b + 2*d*e]]])/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sq
rt[b + 2*d*e]])) - (Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[
b - 2*d*e] + Sqrt[b + 2*d*e]]])/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqr
t[b - 2*d*e] + Sqrt[b + 2*d*e]]) - (Sqrt[e]*ArcTanh[(Sqrt[2]*Sqrt
[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]])/(Sqrt[2]*Sqrt[b
- 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]) - (Sqrt[e]*ArcT
anh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]])
/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]
)
```

Rubi [A] time = 0.924635, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{\sqrt{2}\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] -((Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt
[b + 2*d*e]]])/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sq
rt[b + 2*d*e]])) - (Sqrt[e]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[
b - 2*d*e] + Sqrt[b + 2*d*e]]])/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqr
t[b - 2*d*e] + Sqrt[b + 2*d*e]]) - (Sqrt[e]*ArcTanh[(Sqrt[2]*Sqrt
[e]*x)/Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]])/(Sqrt[2]*Sqrt[b
- 2*d*e]*Sqrt[Sqrt[b - 2*d*e] - Sqrt[b + 2*d*e]]) - (Sqrt[e]*ArcT
anh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]])
/(Sqrt[2]*Sqrt[b - 2*d*e]*Sqrt[Sqrt[b - 2*d*e] + Sqrt[b + 2*d*e]]
)
```

Rubi in Sympy [A] time = 76.3425, size = 333, normalized size = 0.95

$$-\frac{\sqrt{2}\sqrt{e}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{2\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{2}\sqrt{e}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}\right)}{2\sqrt{b-2de}\sqrt{\sqrt{b-2de}+\sqrt{b+2de}}}-\frac{\sqrt{2}\sqrt{e}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{2\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}-\frac{\sqrt{2}\sqrt{e}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}\right)}{2\sqrt{b-2de}\sqrt{\sqrt{b-2de}-\sqrt{b+2de}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/(e**2*x**8-b*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(e)*atan(sqrt(2)*sqrt(e)*x/sqrt(sqrt(b - 2*d*e) + sq
rt(b + 2*d*e)))/(2*sqrt(b - 2*d*e)*sqrt(sqrt(b - 2*d*e) + sqrt(b
+ 2*d*e))) - sqrt(2)*sqrt(e)*atanh(sqrt(2)*sqrt(e)*x/sqrt(sqrt(b
- 2*d*e) + sqrt(b + 2*d*e)))/(2*sqrt(b - 2*d*e)*sqrt(sqrt(b - 2*d
*e) + sqrt(b + 2*d*e))) - sqrt(2)*sqrt(e)*atan(sqrt(2)*sqrt(e)*x/
sqrt(sqrt(b - 2*d*e) - sqrt(b + 2*d*e)))/(2*sqrt(b - 2*d*e)*sqrt(
sqrt(b - 2*d*e) - sqrt(b + 2*d*e))) - sqrt(2)*sqrt(e)*atanh(sqrt(
2)*sqrt(e)*x/sqrt(sqrt(b - 2*d*e) - sqrt(b + 2*d*e)))/(2*sqrt(b -
 2*d*e)*sqrt(sqrt(b - 2*d*e) - sqrt(b + 2*d*e)))
```

Mathematica [C] time = 0.0612332, size = 69, normalized size = 0.2

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2-\#1^4b+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2-\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(d^2 - b*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 - b*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1
]*#1^4)/(-(b*#1^3) + 2*e^2*#1^7) & ]/4
```

Maple [C] time = 0.041, size = 55, normalized size = 0.2

$$\frac{1}{4}\sum_{\_R=RootOf\left(e^2\_Z^8-b\_Z^4+d^2\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{2\_R^7e^2-\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/(e^2*x^8-b*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*e^
2-_Z^4*b+d^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^4+d}{e^2x^8-bx^4+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2), x)
```

Fricas [A] time = 0.328652, size = 3079, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)
/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*arctan(1/2*(2*d*e + (4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6
*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6
*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*
d^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e^2*x^2 - sqrt(1/2)*(2*b*d*e -
b^2 - (8*d^5*e^3 - 12*b*d^4*e^2 + 6*b^2*d^3*e - b^3*d^2)*sqrt(-(2
*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)))*sq
rt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^
3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*
d^3*e + b^2*d^2)))/e^2))) + sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*
b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 +
6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*
arctan(-1/2*(2*d*e - (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d
*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*
sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*
d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)
/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e
^2*x^2 - sqrt(1/2)*(2*b*d*e - b^2 + (8*d^5*e^3 - 12*b*d^4*e^2 + 6
*b^2*d^3*e - b^3*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2
 + 6*b^2*d^5*e - b^3*d^4)))*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d
^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b
^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))/e^2))) + 1/4*sq
rt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e
 + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4
*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6
*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e
^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6
*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*
d^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d
^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b
^3*d^4)) + b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))*log(e*x - 1/2*(
2*d*e + (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^
7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2
)*sqrt(((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^
7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) + b)/(4*d^4*e^2 -
4*b*d^3*e + b^2*d^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 -
4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2
+ 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2))
)*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-
(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) -
 b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(
-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4))
- b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)))) - 1/4*sqrt(sqrt(1/2)*sq
rt(-((4*d^4*e^2 - 4*b*d^3*e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e
^3 - 12*b*d^6*e^2 + 6*b^2*d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b
*d^3*e + b^2*d^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 - 4*b*d^3*e
 + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*d
^5*e - b^3*d^4)) - b)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*b*d^3*
e + b^2*d^2)*sqrt(-(2*d*e + b)/(8*d^7*e^3 - 12*b*d^6*e^2 + 6*b^2*
d^5*e - b^3*d^4)) - b)/(4*d^4*e^2 - 4*b*d^3*e + b^2*d^2))))
```

Sympy [A] time = 20.6786, size = 136, normalized size = 0.39

$$\operatorname{RootSum}\left(t^8\left(65536b^4d^2 - 524288b^3d^3e + 1572864b^2d^4e^2 - 2097152bd^5e^3 + 1048576d^6e^4\right) + t^4\left(-256b^3 + 1024b^2de - 102\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/(e**2*x**8-b*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(65536*b**4*d**2 - 524288*b**3*d**3*e + 1572864*b**
```

```
2*d**4*e**2 - 2097152*b*d**5*e**3 + 1048576*d**6*e**4) + _t**4*(-
256*b**3 + 1024*b**2*d*e - 1024*b*d**2*e**2) + e**2, Lambda(_t, _
t*log(x + (1024*_t**5*b**2*d**2 - 4096*_t**5*b*d**3*e + 4096*_t**
5*d**4*e**2 - 4*_t*b + 4*_t*d*e)/e)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 - bx^4 + d^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - b*x^4 + d^2), x)
```

3.8 $\int \frac{d+ex^4}{d^2-fx^4+e^2x^8}\,dx$

Optimal. Leaf size=751

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)
/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqr
t[d]*Sqrt[e] + Sqrt[2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]
) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] + 2*Sqrt[e]
*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt
[e] + Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sq
rt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e +
f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f
]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x +
 Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]
]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]
) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + S
qrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])
```

Rubi [A] time = 1.94605, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{\log\left(-x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\log\left(x\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}-\frac{\log\left(-x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\log\left(x\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+\sqrt{d}+\sqrt{e}x^2\right)}{8\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}-2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}-2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}+2\sqrt{e}x}{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}\right)}{4\sqrt{d}\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2de+f}+2\sqrt{d}\sqrt{e}}+2\sqrt{e}x}{\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}\right)}{4\sqrt{d}\sqrt{2\sqrt{d}\sqrt{e}-\sqrt{2de+f}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)
/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqr
t[d]*Sqrt[e] + Sqrt[2*d*e + f]]) - ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e]
 + Sqrt[2*d*e + f]] - 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[
2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]
) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]] + 2*Sqrt[e]
*x)/Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*
Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]) + ArcTan[(Sqrt[2*Sqrt[d]*Sqrt
[e] + Sqrt[2*d*e + f]] + 2*Sqrt[e]*x)/Sqrt[2*Sqrt[d]*Sqrt[e] - Sq
rt[2*d*e + f]]]/(4*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e +
f]]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x
+ Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f
]]) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]]*x +
 Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] - Sqrt[2*d*e + f]
]) - Log[Sqrt[d] - Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x +
Sqrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]
) + Log[Sqrt[d] + Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]]*x + S
qrt[e]*x^2]/(8*Sqrt[d]*Sqrt[2*Sqrt[d]*Sqrt[e] + Sqrt[2*d*e + f]])
```

Rubi in Sympy [A] time = 74.1544, size = 333, normalized size = 0.44

$$-\frac{\sqrt{2}\sqrt{e}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{-2de+f}+\sqrt{2de+f}}}\right)}{2\sqrt{-2de+f}\sqrt{\sqrt{-2de+f}+\sqrt{2de+f}}}-\frac{\sqrt{2}\sqrt{e}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{-2de+f}+\sqrt{2de+f}}}\right)}{2\sqrt{-2de+f}\sqrt{\sqrt{-2de+f}+\sqrt{2de+f}}}-\frac{\sqrt{2}\sqrt{e}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{-2de+f}-\sqrt{2de+f}}}\right)}{2\sqrt{-2de+f}\sqrt{\sqrt{-2de+f}-\sqrt{2de+f}}}-\frac{\sqrt{2}\sqrt{e}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{\sqrt{-2de+f}-\sqrt{2de+f}}}\right)}{2\sqrt{-2de+f}\sqrt{\sqrt{-2de+f}-\sqrt{2de+f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/(e**2*x**8-f*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(e)*atan(sqrt(2)*sqrt(e)*x/sqrt(sqrt(-2*d*e + f) + s
qrt(2*d*e + f)))/(2*sqrt(-2*d*e + f)*sqrt(sqrt(-2*d*e + f) + sqrt
(2*d*e + f))) - sqrt(2)*sqrt(e)*atanh(sqrt(2)*sqrt(e)*x/sqrt(sqrt
(-2*d*e + f) + sqrt(2*d*e + f)))/(2*sqrt(-2*d*e + f)*sqrt(sqrt(-2
*d*e + f) + sqrt(2*d*e + f))) - sqrt(2)*sqrt(e)*atan(sqrt(2)*sqrt
(e)*x/sqrt(sqrt(-2*d*e + f) - sqrt(2*d*e + f)))/(2*sqrt(-2*d*e +
f)*sqrt(sqrt(-2*d*e + f) - sqrt(2*d*e + f))) - sqrt(2)*sqrt(e)*at
anh(sqrt(2)*sqrt(e)*x/sqrt(sqrt(-2*d*e + f) - sqrt(2*d*e + f)))/(
2*sqrt(-2*d*e + f)*sqrt(sqrt(-2*d*e + f) - sqrt(2*d*e + f)))
```

Mathematica [C] time = 0.0509845, size = 69, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8e^2-\#1^4f+d^2\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7e^2-\#1^3f}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(d^2 - f*x^4 + e^2*x^8),x]
```

```
[Out] RootSum[d^2 - f*#1^4 + e^2*#1^8 & , (d*Log[x - #1] + e*Log[x - #1
]*#1^4)/(-(f*#1^3) + 2*e^2*#1^7) & ]/4
```

Maple [C] time = 0.043, size = 55, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(e^2\_Z^8-f\_Z^4+d^2\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{2\_R^7e^2-\_R^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(e^2*x^8-f*x^4+d^2),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*e^2-_R^3*f)*ln(x-_R),_R=RootOf(_Z^8*e^
2-_Z^4*f+d^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^4+d}{e^2x^8-fx^4+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

Fricas [A] time = 0.327354, size = 3079, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)
/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*arctan(1/2*(2*d*e + (4*d^4*e
^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e
^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e
^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e
^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*
f^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e^2*x^2 - sqrt(1/2)*(2*d*e*f -
f^2 - (8*d^5*e^3 - 12*d^4*e^2*f + 6*d^3*e*f^2 - d^2*f^3)*sqrt(-(2
*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)))*sq
rt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^
3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^
3*e*f + d^2*f^2)))/e^2))) + sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*
d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f +
6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*
arctan(-1/2*(2*d*e - (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d
*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*
sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*
d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)
/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))/(e*x + sqrt(1/2)*e*sqrt((2*e
^2*x^2 - sqrt(1/2)*(2*d*e*f - f^2 + (8*d^5*e^3 - 12*d^4*e^2*f + 6
*d^3*e*f^2 - d^2*f^3)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f
 + 6*d^5*e*f^2 - d^4*f^3)))*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f
^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d
^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))/e^2))) + 1/4*sq
rt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e
 + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4
*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x + 1/2*(2*d*e + (4*d^4*e
^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e
```

```
^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(((4*d^4*e
^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e
^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*
f^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f
^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d
^4*f^3)) + f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))*log(e*x - 1/2*(
2*d*e + (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^
7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2
)*sqrt(((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^
7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) + f)/(4*d^4*e^2 -
4*d^3*e*f + d^2*f^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 -
4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f
+ 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2))
)*log(e*x + 1/2*(2*d*e - (4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-
(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) -
 f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(
-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3))
- f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)))) - 1/4*sqrt(sqrt(1/2)*sq
rt(-((4*d^4*e^2 - 4*d^3*e*f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e
^3 - 12*d^6*e^2*f + 6*d^5*e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d
^3*e*f + d^2*f^2)))*log(e*x - 1/2*(2*d*e - (4*d^4*e^2 - 4*d^3*e*f
 + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*e
*f^2 - d^4*f^3)) - f)*sqrt(sqrt(1/2)*sqrt(-((4*d^4*e^2 - 4*d^3*e*
f + d^2*f^2)*sqrt(-(2*d*e + f)/(8*d^7*e^3 - 12*d^6*e^2*f + 6*d^5*
e*f^2 - d^4*f^3)) - f)/(4*d^4*e^2 - 4*d^3*e*f + d^2*f^2))))
```

Sympy [A] time = 19.1665, size = 136, normalized size = 0.18

$$\operatorname{RootSum}\left(t^8\left(1048576d^6e^4 - 2097152d^5e^3f + 1572864d^4e^2f^2 - 524288d^3ef^3 + 65536d^2f^4\right) + t^4\left(-1024d^2e^2f + 1024def^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/(e**2*x**8-f*x**4+d**2),x)
```

```
[Out] RootSum(_t**8*(1048576*d**6*e**4 - 2097152*d**5*e**3*f + 1572864*
d**4*e**2*f**2 - 524288*d**3*e*f**3 + 65536*d**2*f**4) + _t**4*(-
1024*d**2*e**2*f + 1024*d*e*f**2 - 256*f**3) + e**2, Lambda(_t, _
t*log(x + (4096*_t**5*d**4*e**2 - 4096*_t**5*d**3*e*f + 1024*_t**
5*d**2*f**2 + 4*_t*d*e - 4*_t*f)/e)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{e^2x^8 - fx^4 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(e^2*x^8 - f*x^4 + d^2), x)
```

3.9 $\int \frac{1+x^4}{1+bx^4+x^8}\,dx$

Optimal. Leaf size=411

$$-\frac{\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}+\frac{\log\left(\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*S
qrt[2 + Sqrt[2 - b]]) - ArcTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt
[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) + ArcTan[(Sqrt[2 - S
qrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]
]) + ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]]/
(4*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2
]/(8*Sqrt[2 - Sqrt[2 - b]]) + Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x
^2]/(8*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x +
 x^2]/(8*Sqrt[2 + Sqrt[2 - b]]) + Log[1 + Sqrt[2 + Sqrt[2 - b]]*x
 + x^2]/(8*Sqrt[2 + Sqrt[2 - b]])
```

Rubi [A] time = 0.605884, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}+\frac{\log\left(\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-\sqrt{2-b}}}-\frac{\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}+\frac{\log\left(\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{\sqrt{2-b}+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{\sqrt{2-b}+2}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{2-\sqrt{2-b}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*S
qrt[2 + Sqrt[2 - b]]) - ArcTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt
[2 - Sqrt[2 - b]]]/(4*Sqrt[2 - Sqrt[2 - b]]) + ArcTan[(Sqrt[2 - S
qrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]]/(4*Sqrt[2 + Sqrt[2 - b]
]) + ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*x)/Sqrt[2 - Sqrt[2 - b]]]/
(4*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2
]/(8*Sqrt[2 - Sqrt[2 - b]]) + Log[1 + Sqrt[2 - Sqrt[2 - b]]*x + x
^2]/(8*Sqrt[2 - Sqrt[2 - b]]) - Log[1 - Sqrt[2 + Sqrt[2 - b]]*x +
 x^2]/(8*Sqrt[2 + Sqrt[2 - b]]) + Log[1 + Sqrt[2 + Sqrt[2 - b]]*x
 + x^2]/(8*Sqrt[2 + Sqrt[2 - b]])
```

Rubi in Sympy [A] time = 82.8809, size = 304, normalized size = 0.74

$$-\frac{\log\left(x^2 - x\sqrt{\sqrt{-b+2}+2}+1\right)}{8\sqrt{\sqrt{-b+2}+2}} + \frac{\log\left(x^2 + x\sqrt{\sqrt{-b+2}+2}+1\right)}{8\sqrt{\sqrt{-b+2}+2}} + \frac{\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{-b+2}+2}}{\sqrt{\sqrt{-b+2}+2}}\right)}{4\sqrt{\sqrt{-b+2}+2}} + \frac{\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{-b+2}+2}}{\sqrt{\sqrt{-b+2}+2}}\right)}{4\sqrt{\sqrt{-b+2}+2}} - \frac{\log\left(x^2 - x\sqrt{-\sqrt{-b+2}+2}+1\right)}{8\sqrt{-\sqrt{-b+2}+2}} + \frac{\log\left(x^2 + x\sqrt{-\sqrt{-b+2}+2}+1\right)}{8\sqrt{-\sqrt{-b+2}+2}} + \frac{\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{-b+2}+2}}{\sqrt{-\sqrt{-b+2}+2}}\right)}{4\sqrt{-\sqrt{-b+2}+2}} + \frac{\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{-b+2}+2}}{\sqrt{-\sqrt{-b+2}+2}}\right)}{4\sqrt{-\sqrt{-b+2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] -log(x**2 - x*sqrt(sqrt(-b + 2) + 2) + 1)/(8*sqrt(sqrt(-b + 2) +
2)) + log(x**2 + x*sqrt(sqrt(-b + 2) + 2) + 1)/(8*sqrt(sqrt(-b +
2) + 2)) + atan((2*x - sqrt(-sqrt(-b + 2) + 2))/sqrt(sqrt(-b + 2)
 + 2))/(4*sqrt(sqrt(-b + 2) + 2)) + atan((2*x + sqrt(-sqrt(-b + 2
) + 2))/sqrt(sqrt(-b + 2) + 2))/(4*sqrt(sqrt(-b + 2) + 2)) - log(
x**2 - x*sqrt(-sqrt(-b + 2) + 2) + 1)/(8*sqrt(-sqrt(-b + 2) + 2))
 + log(x**2 + x*sqrt(-sqrt(-b + 2) + 2) + 1)/(8*sqrt(-sqrt(-b + 2
) + 2)) + atan((2*x - sqrt(sqrt(-b + 2) + 2))/sqrt(-sqrt(-b + 2)
+ 2))/(4*sqrt(-sqrt(-b + 2) + 2)) + atan((2*x + sqrt(sqrt(-b + 2)
 + 2))/sqrt(-sqrt(-b + 2) + 2))/(4*sqrt(-sqrt(-b + 2) + 2))
```

Mathematica [C] time = 0.0357245, size = 55, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^8+\#1^4b+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] RootSum[1 + b*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(b
*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.064, size = 42, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8+b\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8+b*x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+1
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4+1}{x^8+bx^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + b*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

Fricas [A] time = 0.292616, size = 1458, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + b*x^4 + 1),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 +
 12*b + 8)) + b)/(b^2 + 4*b + 4)))*arctan(1/2*((b^2 + 4*b + 4)*sq
rt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2)*sqrt(sqrt(1/2)*sqrt
(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b
^2 + 4*b + 4)))/(x + sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 - (b^3 + 6*b^2
 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + 2*b)*sqrt(-
((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2
 + 4*b + 4))))) - sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b -
2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*arctan(1/2*((
b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b + 2)*sq
rt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12
*b + 8)) - b)/(b^2 + 4*b + 4)))/(x + sqrt(x^2 + 1/2*sqrt(1/2)*(b^
2 + (b^3 + 6*b^2 + 12*b + 8)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8
)) + 2*b)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b
+ 8)) - b)/(b^2 + 4*b + 4))))) - 1/4*sqrt(sqrt(1/2)*sqrt(-((b^2 +
 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b
+ 4)))*log(1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b
+ 8)) - b - 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)
/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4*b + 4))) + x) + 1/4*sqrt
(sqrt(1/2)*sqrt(-((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*
b + 8)) + b)/(b^2 + 4*b + 4)))*log(-1/2*((b^2 + 4*b + 4)*sqrt((b
- 2)/(b^3 + 6*b^2 + 12*b + 8)) - b - 2)*sqrt(sqrt(1/2)*sqrt(-((b^
2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b)/(b^2 + 4
*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b
 - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4)))*log(1/2*((
b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) + b + 2)*sq
rt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12
*b + 8)) - b)/(b^2 + 4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(((
b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 +
 4*b + 4)))*log(-1/2*((b^2 + 4*b + 4)*sqrt((b - 2)/(b^3 + 6*b^2 +
 12*b + 8)) + b + 2)*sqrt(sqrt(1/2)*sqrt(((b^2 + 4*b + 4)*sqrt((b
 - 2)/(b^3 + 6*b^2 + 12*b + 8)) - b)/(b^2 + 4*b + 4))) + x)
```

Sympy [A] time = 7.43506, size = 75, normalized size = 0.18

$$\operatorname{RootSum}\left(t^8\left(65536b^4+524288b^3+1572864b^2+2097152b+1048576\right)+t^4\left(256b^3+1024b^2+1024b\right)+1,\left(t\mapsto t\log\left(102\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] RootSum(_t**8*(65536*b**4 + 524288*b**3 + 1572864*b**2 + 2097152*
b + 1048576) + _t**4*(256*b**3 + 1024*b**2 + 1024*b) + 1, Lambda(
_t, _t*log(1024*_t**5*b**2 + 4096*_t**5*b + 4096*_t**5 + 4*_t*b +
 4*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 + bx^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + b*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 + b*x^4 + 1), x)
```

3.10 $\int \frac{1+x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=469

$$-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*
x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5
]) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1
/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqr
t[5]] - 2^(3/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*
Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] + 2^(3/4)*(
3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 -
Sqrt[5])^(1/4)*Log[Sqrt[3 + Sqrt[5]] - 2^(3/4)*(3 + Sqrt[5])^(1/4
)*x + Sqrt[2]*x^2])/(4*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Lo
g[Sqrt[3 + Sqrt[5]] + 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2
])/(4*2^(3/4)*Sqrt[5])
```

Rubi [A] time = 0.782355, antiderivative size = 451, normalized size of antiderivative = 0.96, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$

$$-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*
x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5
]) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1
/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 -
Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt
[5]) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 -
 Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])
^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x +
2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3
+ Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sq
rt[5])
```

Rubi in Sympy [A] time = 88.8959, size = 590, normalized size = 1.26

$$-\frac{2^{\frac{3}{4}}\sqrt{-2\sqrt{5}+6}\left(-\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{8\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}$$
$$+\frac{2^{\frac{3}{4}}\sqrt{-2\sqrt{5}+6}\left(-\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{8\left(-\sqrt{5}+3\right)^{\frac{5}{4}}}$$
$$-\frac{2^{\frac{3}{4}}\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\sqrt{2\sqrt{5}+6}\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{8\left(\sqrt{5}+3\right)^{\frac{5}{4}}}$$
$$+\frac{2^{\frac{3}{4}}\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\sqrt{2\sqrt{5}+6}\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{8\left(\sqrt{5}+3\right)^{\frac{5}{4}}}$$
$$+\frac{2^{\frac{3}{4}}\left(-\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt{-2\sqrt{5}+6}\sqrt[4]{-\sqrt{5}+3}}+\frac{2^{\frac{3}{4}}\left(-\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt{-2\sqrt{5}+6}\sqrt[4]{-\sqrt{5}+3}}$$
$$+\frac{2^{\frac{3}{4}}\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}\sqrt{2\sqrt{5}+6}}+\frac{2^{\frac{3}{4}}\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}\sqrt{2\sqrt{5}+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] -2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-sqrt(5)/10 + 1/2)*log(2*x**2 - 2
*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(8*(-sq
rt(5) + 3)**(5/4)) + 2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-sqrt(5)/10 +
 1/2)*log(2*x**2 + 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*s
qrt(5) + 6))/(8*(-sqrt(5) + 3)**(5/4)) - 2**(3/4)*(sqrt(5)/10 + 1
/2)*sqrt(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(sqrt(5) + 3)**
(1/4) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5) + 3)**(5/4)) + 2**(3/4)*
(sqrt(5)/10 + 1/2)*sqrt(2*sqrt(5) + 6)*log(2*x**2 + 2*2**(1/4)*x*
(sqrt(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5) + 3)**(5/
4)) + 2**(3/4)*(-sqrt(5)/10 + 1/2)*atan(2**(3/4)*(x - (-2*sqrt(5)
 + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(2*sqrt(-2*sqrt(5) + 6)*(-
sqrt(5) + 3)**(1/4)) + 2**(3/4)*(-sqrt(5)/10 + 1/2)*atan(2**(3/4)
*(x + (-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(2*sqrt(-
2*sqrt(5) + 6)*(-sqrt(5) + 3)**(1/4)) + 2**(3/4)*(sqrt(5)/10 + 1/
2)*atan(2**(3/4)*(x - (2*sqrt(5) + 6)**(1/4)/2)/(sqrt(5) + 3)**(1
/4))/(2*(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6)) + 2**(3/4)*(sqr
t(5)/10 + 1/2)*atan(2**(3/4)*(x + (2*sqrt(5) + 6)**(1/4)/2)/(sqrt
(5) + 3)**(1/4))/(2*(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6))
```

Mathematica [C] time = 0.0227927, size = 55, normalized size = 0.12

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(3
*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.01, size = 42, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}+1}{x^{8}+3x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.301058, size = 1681, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(5)*sqrt(2)*(4*(1/250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) +
5))*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*arctan(5*sqrt(1/10)*(1/250)^(
1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*(sqrt(5) + 1)/(2*sqrt(5)*sqr
t(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*x + 5*sqrt(1/10)*(1
/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*(sqrt(5) + 1) + 2*sqr
t(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*sqrt((sqrt(
5)*x^2 - 3*x^2 + (1/250)^(1/4)*(sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*
(sqrt(5)*(3*sqrt(5) - 5))^(1/4) - 2*sqrt(1/10)*sqrt(sqrt(5)*(3*sq
rt(5) - 5)))/(sqrt(5) - 3)))) + 4*(1/250)^(1/4)*sqrt(sqrt(5)*(3*s
qrt(5) + 5))*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*arctan(5*sqrt(1/10)*
(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*(sqrt(5) + 1)/(2*sq
rt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*x - 5*sqrt
(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*(sqrt(5) + 1
) + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*sq
rt((sqrt(5)*x^2 - 3*x^2 - (1/250)^(1/4)*(sqrt(5)*sqrt(2)*x - 5*sq
rt(2)*x)*(sqrt(5)*(3*sqrt(5) - 5))^(1/4) - 2*sqrt(1/10)*sqrt(sqrt
(5)*(3*sqrt(5) - 5)))/(sqrt(5) - 3)))) + 4*(1/250)^(1/4)*(sqrt(5)
*(3*sqrt(5) + 5))^(3/4)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*arctan(5*sq
```

```
rt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*(sqrt(5) -
 1)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*x
 + 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*(sq
rt(5) - 1) + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5)
 + 5))*sqrt((sqrt(5)*x^2 + 3*x^2 + (1/250)^(1/4)*(sqrt(5)*sqrt(2)
*x + 5*sqrt(2)*x)*(sqrt(5)*(3*sqrt(5) + 5))^(1/4) + 2*sqrt(1/10)*
sqrt(sqrt(5)*(3*sqrt(5) + 5)))/(sqrt(5) + 3)))) + 4*(1/250)^(1/4)
*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*ar
ctan(5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*(
sqrt(5) - 1)/(2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5
) + 5))*x - 5*sqrt(1/10)*(1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^
(3/4)*(sqrt(5) - 1) + 2*sqrt(5)*sqrt(2)*sqrt(1/10)*sqrt(sqrt(5)*(
3*sqrt(5) + 5))*sqrt((sqrt(5)*x^2 + 3*x^2 - (1/250)^(1/4)*(sqrt(5
)*sqrt(2)*x + 5*sqrt(2)*x)*(sqrt(5)*(3*sqrt(5) + 5))^(1/4) + 2*sq
rt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5)))/(sqrt(5) + 3)))) - (1/250
)^(1/4)*(sqrt(5)*(3*sqrt(5) + 5))^(3/4)*sqrt(sqrt(5)*(3*sqrt(5) -
 5))*log(sqrt(5)*x^2 + 3*x^2 + (1/250)^(1/4)*(sqrt(5)*sqrt(2)*x +
 5*sqrt(2)*x)*(sqrt(5)*(3*sqrt(5) + 5))^(1/4) + 2*sqrt(1/10)*sqrt
(sqrt(5)*(3*sqrt(5) + 5))) + (1/250)^(1/4)*(sqrt(5)*(3*sqrt(5) +
5))^(3/4)*sqrt(sqrt(5)*(3*sqrt(5) - 5))*log(sqrt(5)*x^2 + 3*x^2 -
 (1/250)^(1/4)*(sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(sqrt(5)*(3*sqrt
(5) + 5))^(1/4) + 2*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) + 5))) - (
1/250)^(1/4)*sqrt(sqrt(5)*(3*sqrt(5) + 5))*(sqrt(5)*(3*sqrt(5) -
5))^(3/4)*log(sqrt(5)*x^2 - 3*x^2 + (1/250)^(1/4)*(sqrt(5)*sqrt(2
)*x - 5*sqrt(2)*x)*(sqrt(5)*(3*sqrt(5) - 5))^(1/4) - 2*sqrt(1/10)
*sqrt(sqrt(5)*(3*sqrt(5) - 5))) + (1/250)^(1/4)*sqrt(sqrt(5)*(3*s
qrt(5) + 5))*(sqrt(5)*(3*sqrt(5) - 5))^(3/4)*log(sqrt(5)*x^2 - 3*
x^2 - (1/250)^(1/4)*(sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(sqrt(5)*(3
*sqrt(5) - 5))^(1/4) - 2*sqrt(1/10)*sqrt(sqrt(5)*(3*sqrt(5) - 5))
))/(sqrt(3*sqrt(5) + 5)*sqrt(3*sqrt(5) - 5))
```

Sympy [A] time = 3.76062, size = 24, normalized size = 0.05

$$\operatorname{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t\log\left(25600t^5 + 16t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(25600
*_t**5 + 16*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

3.11 $\int \frac{1+x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=85

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) - Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi [A] time = 0.0869912, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) - Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 15.5855, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*
x + 1)/4
```

Mathematica [A] time = 0.0311356, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqr
t[2]*x + x^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A] time = 0.004, size = 58, normalized size = 0.7

$$\frac{\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)+\frac{\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8+2*x^4+1),x)
```

```
[Out] 1/4*arctan(2^(1/2)*x-1)*2^(1/2)+1/8*2^(1/2)*ln((1+x^2+2^(1/2)*x)/
(1+x^2-2^(1/2)*x))+1/4*arctan(1+2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.836387, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*
x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 0.288385, size = 131, normalized size = 1.54

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x +
 1) + 1)) - 1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 -
sqrt(2)*x + 1) - 1)) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8
*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.416257, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*
```

```
x + 1)/4
```

GIAC/XCAS [A] time = 0.271182, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$
$$+\frac{1}{8}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*ln(x^2 + sqrt(2)*x
 + 1) - 1/8*sqrt(2)*ln(x^2 - sqrt(2)*x + 1)
```

3.12 $\int \frac{1+x^4}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4
+ ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4
 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x +
x^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi [A] time = 0.185168, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4
+ ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4
 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x +
x^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 28.9472, size = 128, normalized size = 0.91

$$-\frac{\log\left(x^2-x+1\right)}{8}+\frac{\log\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{24}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{24}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}+\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{4}+\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8+x**4+1),x)
```

```
[Out] -log(x**2 - x + 1)/8 + log(x**2 + x + 1)/8 - sqrt(3)*log(x**2 - s
qrt(3)*x + 1)/24 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/24 + sqrt(3)
*atan(sqrt(3)*(2*x/3 - 1/3))/12 + sqrt(3)*atan(sqrt(3)*(2*x/3 + 1
/3))/12 + atan(2*x - sqrt(3))/4 + atan(2*x + sqrt(3))/4
```

Mathematica [C] time = 0.323501, size = 135, normalized size = 0.96

$$\frac{1}{48}\left(-6\log\left(x^2-x+1\right)+6\log\left(x^2+x+1\right)+4i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-4i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+4\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x^4)/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (
4*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 4*S
qrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] - 6*Log[1 - x + x^2] + 6*Log[1 + x + x^2])/48
```

Maple [A] time = 0.026, size = 109, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8+x^4+1),x)
```

```
[Out] 1/8*ln(x^2+x+1)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/24*ln(
1+x^2-x*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^(1/2))+1/24*ln(1+x^2+x*
3^(1/2))*3^(1/2)+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/12*3^(
1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\int\frac{1}{x^4-x^2+1}\,dx+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/12*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) + 1/
8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 0.291556, size = 212, normalized size = 1.51

$$-\frac{1}{24}\sqrt{3}\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2+\sqrt{3}x+1}+3}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^2-\sqrt{3}x+1}-3}\right)-\sqrt{3}\log\left(x^2+x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(4*sqrt(3)*arctan(sqrt(3)/(2*sqrt(3)*x + 2*sqrt(3)*
sqrt(x^2 + sqrt(3)*x + 1) + 3)) + 4*sqrt(3)*arctan(sqrt(3)/(2*sqr
t(3)*x + 2*sqrt(3)*sqrt(x^2 - sqrt(3)*x + 1) - 3)) - sqrt(3)*log(
x^2 + x + 1) + sqrt(3)*log(x^2 - x + 1) - 2*arctan(1/3*sqrt(3)*(2
*x + 1)) - 2*arctan(1/3*sqrt(3)*(2*x - 1)) - log(x^2 + sqrt(3)*x
```

```
+ 1) + log(x^2 - sqrt(3)*x + 1))
```

Sympy [A] time = 2.87798, size = 190, normalized size = 1.36

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}+9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{5}\right)$$
$$+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1+9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{5}+\frac{\sqrt{3}i}{3}\right)$$
$$+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}+9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^{5}\right)$$
$$+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1+9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^{5}+\frac{\sqrt{3}i}{3}\right)$$
$$+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(9216t^{5}+8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 + 9216*(-1/8 - sqrt
(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x - 1 + 9216*(-1/8 + sq
rt(3)*I/24)**5 + sqrt(3)*I/3) + (1/8 - sqrt(3)*I/24)*log(x + 1 -
sqrt(3)*I/3 + 9216*(1/8 - sqrt(3)*I/24)**5) + (1/8 + sqrt(3)*I/24
)*log(x + 1 + 9216*(1/8 + sqrt(3)*I/24)**5 + sqrt(3)*I/3) + RootS
um(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(9216*_t**5 + 8*_t
 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{4}+1}{x^{8}+x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 + x^4 + 1), x)
```

3.13 $\int \frac{1+x^4}{1+x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)}{8\sqrt{2-\sqrt{2}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)}{8\sqrt{2+\sqrt{2}}}-\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

```
[Out] -(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 +
 Sqrt[2]]])/4 - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]]
- 2*x)/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqr
t[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]])/4 + (Sqrt[(2 + Sqrt[2])
/2]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]])/4 - Log[
1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) + Log[1 + Sq
rt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) - Log[1 - Sqrt[2 +
 Sqrt[2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]]) + Log[1 + Sqrt[2 + Sqrt[
2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]])
```

Rubi [A] time = 0.564652, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{8\sqrt{2-\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)}{8\sqrt{2-\sqrt{2}}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)}{8\sqrt{2+\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)}{8\sqrt{2+\sqrt{2}}}-\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 + x^8),x]
```

```
[Out] -(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 +
 Sqrt[2]]])/4 - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]]
- 2*x)/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqr
t[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]])/4 + (Sqrt[(2 + Sqrt[2])
/2]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]])/4 - Log[
1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) + Log[1 + Sq
rt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2 - Sqrt[2]]) - Log[1 - Sqrt[2 +
 Sqrt[2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]]) + Log[1 + Sqrt[2 + Sqrt[
2]]*x + x^2]/(8*Sqrt[2 + Sqrt[2]])
```

Rubi in Sympy [A] time = 39.2716, size = 270, normalized size = 0.78

$$-\frac{\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)}{8\sqrt{-\sqrt{2}+2}}+\frac{\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)}{8\sqrt{-\sqrt{2}+2}}$$
$$-\frac{\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)}{8\sqrt{\sqrt{2}+2}}+\frac{\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)}{8\sqrt{\sqrt{2}+2}}+\frac{\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{4\sqrt{-\sqrt{2}+2}}$$
$$+\frac{\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{4\sqrt{-\sqrt{2}+2}}+\frac{\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{4\sqrt{\sqrt{2}+2}}+\frac{\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{4\sqrt{\sqrt{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8+1),x)
```

```
[Out] -log(x**2 - x*sqrt(-sqrt(2) + 2) + 1)/(8*sqrt(-sqrt(2) + 2)) + lo
g(x**2 + x*sqrt(-sqrt(2) + 2) + 1)/(8*sqrt(-sqrt(2) + 2)) - log(x
**2 - x*sqrt(sqrt(2) + 2) + 1)/(8*sqrt(sqrt(2) + 2)) + log(x**2 +
 x*sqrt(sqrt(2) + 2) + 1)/(8*sqrt(sqrt(2) + 2)) + atan((2*x - sqr
t(sqrt(2) + 2))/sqrt(-sqrt(2) + 2))/(4*sqrt(-sqrt(2) + 2)) + atan
((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2))/(4*sqrt(-sqrt(2) +
 2)) + atan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))/(4*sqrt
(sqrt(2) + 2)) + atan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
))/(4*sqrt(sqrt(2) + 2))
```

Mathematica [A] time = 0.334667, size = 258, normalized size = 0.74

$$\frac{1}{8}\left(-\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)\right.$$
$$+\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)$$
$$+\left(\sin\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)$$
$$+\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)$$
$$+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x-\cos\left(\frac{\pi}{8}\right)\right)\right)$$
$$+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x+\cos\left(\frac{\pi}{8}\right)\right)\right)$$
$$+2\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)$$
$$\left.+2\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*(Cos[Pi/8] - Sin[Pi/8]) + 2*
ArcTan[x*Sec[Pi/8] - Tan[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 +
 x^2 + 2*x*Cos[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[1 + x^2 - 2*x
*Cos[Pi/8]]*(-Cos[Pi/8] + Sin[Pi/8]) + 2*ArcTan[(x - Cos[Pi/8])*C
sc[Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) + 2*ArcTan[(x + Cos[Pi/8])*Csc[
Pi/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x*Sin[Pi/8]]*(Co
s[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]*(Cos[Pi/8] +
Sin[Pi/8]))/8
```

Maple [C] time = 0.009, size = 27, normalized size = 0.1

$$\frac{1}{8}\sum_{\_R=RootOf\left(\_Z^8+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8+1),x)
```

```
[Out] 1/8*sum((_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 + 1), x)
```

Fricas [A] time = 0.281364, size = 1343, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + sqrt(
-sqrt(2) + 2))/(2*sqrt(2)*x + 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*
sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(
sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-sqrt(2) +
 2)*arctan((sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(2*sqrt(2)*x
+ 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt
(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2)
 + 2))) + 1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2
) - sqrt(-sqrt(2) + 2))/(2*sqrt(2)*x + 2*sqrt(2)*sqrt(x^2 + 1/2*s
qrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1
) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/8*sqrt(2)*sqrt(s
qrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(2*s
qrt(2)*x + 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) -
 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) - sqrt
(-sqrt(2) + 2))) - 1/8*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*a
rctan(sqrt(sqrt(2) + 2)/(2*x + 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2)
+ 1) + sqrt(-sqrt(2) + 2))) - 1/8*(sqrt(sqrt(2) + 2) - sqrt(-sqrt
(2) + 2))*arctan(sqrt(sqrt(2) + 2)/(2*x + 2*sqrt(x^2 - x*sqrt(-sq
rt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))) - 1/8*(sqrt(sqrt(2) + 2) +
 sqrt(-sqrt(2) + 2))*arctan(sqrt(-sqrt(2) + 2)/(2*x + 2*sqrt(x^2
+ x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2))) - 1/8*(sqrt(sqrt
(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(sqrt(-sqrt(2) + 2)/(2*x + 2
*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))) + 1/32
*sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2)
+ 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/32*sqrt(2)*sqrt(
sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt
(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*lo
g(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqr
t(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*log(x^2 - 1/2*sq
rt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1)
 + 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt
(sqrt(2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2)
)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/32*(sqrt(sqrt(2) + 2) +
sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/32*(s
qrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^2 - x*sqrt(-sqrt(2)
+ 2) + 1)
```

Sympy [A] time = 4.44836, size = 19, normalized size = 0.05

$$\operatorname{RootSum}\left(1048576t^8 + 1, \left(t \mapsto t \log\left(4096t^5 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8+1),x)
```

```
[Out] RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 + 4*_t +
x)))
```

GIAC/XCAS [A] time = 0.298775, size = 333, normalized size = 0.96

$$\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{-2\sqrt{2}+4}\ln\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-2\sqrt{2}+4}\ln\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{2\sqrt{2}+4}\ln\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{2\sqrt{2}+4}\ln\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 + 1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(s
qrt(2) + 2)) + 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(
2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x
 + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*sqrt(2*sqrt(2) +
4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/16*sq
rt(-2*sqrt(2) + 4)*ln(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(
-2*sqrt(2) + 4)*ln(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(2*s
qrt(2) + 4)*ln(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(2*sqrt
(2) + 4)*ln(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

3.14 $\int \frac{1+x^4}{1-x^4+x^8} \, dx$

Optimal. Leaf size=331

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{2-\sqrt{3}}} -\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{2+\sqrt{3}}} -\frac{1}{4}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) +\frac{1}{4}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] -(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqr
t[3]]])/4 - (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3
]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 - Log[1 - Sqrt[2 - Sqr
t[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) + Log[1 + Sqrt[2 - Sqrt[3]]*
x + x^2]/(8*Sqrt[2 - Sqrt[3]]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^
2]/(8*Sqrt[2 + Sqrt[3]]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*
Sqrt[2 + Sqrt[3]])
```

Rubi [A] time = 0.479201, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{2-\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{2-\sqrt{3}}} -\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{2+\sqrt{3}}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{2+\sqrt{3}}} -\frac{1}{4}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) +\frac{1}{4}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqr
t[3]]])/4 - (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3
]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 - Log[1 - Sqrt[2 - Sqr
t[3]]*x + x^2]/(8*Sqrt[2 - Sqrt[3]]) + Log[1 + Sqrt[2 - Sqrt[3]]*
x + x^2]/(8*Sqrt[2 - Sqrt[3]]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^
2]/(8*Sqrt[2 + Sqrt[3]]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*
Sqrt[2 + Sqrt[3]])
```

Rubi in Sympy [A] time = 43.0162, size = 270, normalized size = 0.82

$$-\frac{\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{8\sqrt{-\sqrt{3}+2}}+\frac{\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{8\sqrt{-\sqrt{3}+2}}-\frac{\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{8\sqrt{\sqrt{3}+2}}+\frac{\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{8\sqrt{\sqrt{3}+2}}+\frac{\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{4\sqrt{-\sqrt{3}+2}}+\frac{\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{4\sqrt{-\sqrt{3}+2}}+\frac{\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{4\sqrt{\sqrt{3}+2}}+\frac{\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{4\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(8*sqrt(-sqrt(3) + 2)) + lo
g(x**2 + x*sqrt(-sqrt(3) + 2) + 1)/(8*sqrt(-sqrt(3) + 2)) - log(x
**2 - x*sqrt(sqrt(3) + 2) + 1)/(8*sqrt(sqrt(3) + 2)) + log(x**2 +
 x*sqrt(sqrt(3) + 2) + 1)/(8*sqrt(sqrt(3) + 2)) + atan((2*x - sqr
t(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(4*sqrt(-sqrt(3) + 2)) + atan
((2*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(4*sqrt(-sqrt(3) +
 2)) + atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(4*sqrt
(sqrt(3) + 2)) + atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2
))/(4*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0235172, size = 55, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]   Integrate[(1 + x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-#1
^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.011, size = 42, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4+1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - x^4 + 1),x, algorithm="maxima")

[Out] integrate((x^4 + 1)/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.293455, size = 1048, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - x^4 + 1),x, algorithm="fricas")

[Out] -1/8*(4*(4*sqrt(3) + 7)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arcta
n((sqrt(3)*sqrt(2) - 2*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqrt(3)
 - 2)/(4*sqrt(3) - 7)) + 2)*(sqrt(3) - 2)*sqrt((sqrt(3) - 2)/(4*s
qrt(3) - 7)) + 2*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt((sqrt(3)
- 2)/(4*sqrt(3) - 7)) - sqrt(2))) + 4*(4*sqrt(3) + 7)*sqrt((sqrt(
3) + 2)/(4*sqrt(3) + 7))*arctan((sqrt(3)*sqrt(2) - 2*sqrt(2))/(2*
sqrt(2*x^2 - 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2)*(sqrt(3
) - 2)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*sqrt(2)*x
 - 2*sqrt(2)*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(2))) -
 (sqrt(3) + 2)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(2*x^2 + 2*
x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2) + (sqrt(3) + 2)*sqrt((
sqrt(3) - 2)/(4*sqrt(3) - 7))*log(2*x^2 - 2*x*sqrt((sqrt(3) + 2)/
(4*sqrt(3) + 7)) + 2) - (sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(
3) + 7))*log(2*x^2 + 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2)
 + (sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(2*x^2 -
2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2) + 4*sqrt((sqrt(3) -
2)/(4*sqrt(3) - 7))*arctan((sqrt(3)*sqrt(2) + 2*sqrt(2))/(2*sqrt(
2*x^2 + 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2)*(sqrt(3) + 2
)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*sqrt(2)*x + 2*
sqrt(2)*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + sqrt(2))) + 4*sq
rt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan((sqrt(3)*sqrt(2) + 2*sqr
t(2))/(2*sqrt(2*x^2 - 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2
)*(sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*
sqrt(2)*x + 2*sqrt(2)*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) - sq
rt(2))))/((sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt(
(sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.85068, size = 20, normalized size = 0.06

$$\operatorname{RootSum}\left(65536t^8 - 256t^4 + 1, \left(t \mapsto t\log\left(1024t^5 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8-x**4+1),x)

[Out] RootSum(65536*_t**8 - 256*_t**4 + 1, Lambda(_t, _t*log(1024*_t**5
 + x)))
```

GIAC/XCAS [A] time = 0.283116, size = 331, normalized size = 1.

$$\frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{16}\left(\sqrt{6}-\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{16}\left(\sqrt{6}+\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(6) - sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6)
 + sqrt(2))) + 1/8*(sqrt(6) - sqrt(2))*arctan((4*x - sqrt(6) + sq
rt(2))/(sqrt(6) + sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*arctan((4*x
 + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/8*(sqrt(6) + sqrt(
2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/16*
(sqrt(6) - sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/1
6*(sqrt(6) - sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1
/16*(sqrt(6) + sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
 1/16*(sqrt(6) + sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.15 $\int \frac{1+x^4}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=27

$$\frac{x}{2(1-x^4)}+\frac{1}{4}\tan^{-1}(x)+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] x/(2*(1 - x^4)) + ArcTan[x]/4 + ArcTanh[x]/4
```

Rubi [A] time = 0.0208994, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{x}{2(1-x^4)}+\frac{1}{4}\tan^{-1}(x)+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 - x^4)) + ArcTan[x]/4 + ArcTanh[x]/4
```

Rubi in Sympy [A] time = 5.04567, size = 17, normalized size = 0.63

$$\frac{x}{2(-x^4+1)}+\frac{\operatorname{atan}(x)}{4}+\frac{\operatorname{atanh}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8-2*x**4+1),x)
```

```
[Out] x/(2*(-x**4 + 1)) + atan(x)/4 + atanh(x)/4
```

Mathematica [A] time = 0.0213793, size = 31, normalized size = 1.15

$$\frac{1}{8}\left(-\frac{4x}{x^4-1}-\log(1-x)+\log(x+1)+2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-4*x)/(-1 + x^4) + 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/8
```

Maple [A] time = 0.018, size = 42, normalized size = 1.6

$$-\frac{1}{-8+8x}-\frac{\ln(-1+x)}{8}-\frac{1}{8+8x}+\frac{\ln(1+x)}{8}+\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8-2*x^4+1),x)
```

```
[Out] -1/8/(-1+x)-1/8*ln(-1+x)-1/8/(1+x)+1/8*ln(1+x)+1/4*x/(x^2+1)+1/4*
arctan(x)
```

Maxima [A] time = 0.824372, size = 36, normalized size = 1.33

$$-\frac{x}{2\,(x^4-1)}+\frac{1}{4}\,\arctan\left(x\right)+\frac{1}{8}\,\log\left(x+1\right)-\frac{1}{8}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1
)
```

Fricas [A] time = 0.267941, size = 58, normalized size = 2.15

$$\frac{2\left(x^4-1\right)\arctan\left(x\right)+\left(x^4-1\right)\log\left(x+1\right)-\left(x^4-1\right)\log\left(x-1\right)-4\,x}{8\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log
(x - 1) - 4*x)/(x^4 - 1)
```

Sympy [A] time = 0.434841, size = 26, normalized size = 0.96

$$-\frac{x}{2x^4-2}-\frac{\log\left(x-1\right)}{8}+\frac{\log\left(x+1\right)}{8}+\frac{\operatorname{atan}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8-2*x**4+1),x)
```

```
[Out] -x/(2*x**4 - 2) - log(x - 1)/8 + log(x + 1)/8 + atan(x)/4
```

GIAC/XCAS [A] time = 0.268793, size = 39, normalized size = 1.44

$$-\frac{x}{2\,(x^4-1)}+\frac{1}{4}\,\arctan\left(x\right)+\frac{1}{8}\,\ln\left(|x+1|\right)-\frac{1}{8}\,\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^4 - 1) + 1/4*arctan(x) + 1/8*ln(abs(x + 1)) - 1/8*ln(ab
s(x - 1))
```

3.16 $\int \frac{1+x^4}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=131

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[
Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(
-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sq
rt[5])]*x]/Sqrt[2*(1 + Sqrt[5])]
```

Rubi [A] time = 0.181362, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[
Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(
-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sq
rt[5])]*x]/Sqrt[2*(1 + Sqrt[5])]
```

Rubi in Sympy [A] time = 14.183, size = 141, normalized size = 1.08

$$\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{\sqrt{-1+\sqrt{5}}}\right)}{2\sqrt{-1+\sqrt{5}}}-\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{\sqrt{1+\sqrt{5}}}\right)}{2\sqrt{1+\sqrt{5}}}+\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+\sqrt{5}}}\right)}{2\sqrt{-1+\sqrt{5}}}-\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}x}{\sqrt{1+\sqrt{5}}}\right)}{2\sqrt{1+\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x/sqrt(-1 + sqrt(5)))/(2*sqrt(-1 + sqrt(5)))
 - sqrt(2)*atan(sqrt(2)*x/sqrt(1 + sqrt(5)))/(2*sqrt(1 + sqrt(5))
) + sqrt(2)*atanh(sqrt(2)*x/sqrt(-1 + sqrt(5)))/(2*sqrt(-1 + sqrt
(5))) - sqrt(2)*atanh(sqrt(2)*x/sqrt(1 + sqrt(5)))/(2*sqrt(1 + sq
rt(5)))
```

Mathematica [A] time = 0.124403, size = 131, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{2\left(\sqrt{5}-1\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{2\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTan[
Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[2*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(
-1 + Sqrt[5])]*x]/Sqrt[2*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sq
rt[5])]*x]/Sqrt[2*(1 + Sqrt[5])]
```

Maple [A] time = 0.045, size = 96, normalized size = 0.7

$$-\frac{1}{\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{1}{\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)-\frac{1}{\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8-3*x^4+1),x)
```

```
[Out] -1/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+1/(-2+2*5^
(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(
1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2))-1/(2*5^(1/2)+2)^(1/2)*arcta
nh(2*x/(2*5^(1/2)+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 3*x^4 + 1), x)
```

Fricas [A] time = 0.312941, size = 289, normalized size = 2.21

$$\frac{1}{8}\sqrt{2}\left(4\sqrt{\sqrt{5}-1}\arctan\left(\frac{\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-1}}{2\left(\sqrt{2}x+\sqrt{2x^2+\sqrt{5}+1}\right)}\right)-4\sqrt{\sqrt{5}+1}\arctan\left(\frac{\sqrt{\sqrt{5}+1}\left(\sqrt{5}-1\right)}{2\left(\sqrt{2}x+\sqrt{2x^2+\sqrt{5}-1}\right)}\right)-\sqrt{\sqrt{5}-1}\log\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(4*sqrt(sqrt(5) - 1)*arctan(1/2*(sqrt(5) + 1)*sqrt(sq
rt(5) - 1)/(sqrt(2)*x + sqrt(2*x^2 + sqrt(5) + 1))) - 4*sqrt(sqrt
(5) + 1)*arctan(1/2*sqrt(sqrt(5) + 1)*(sqrt(5) - 1)/(sqrt(2)*x +
sqrt(2*x^2 + sqrt(5) - 1))) - sqrt(sqrt(5) - 1)*log(2*sqrt(2)*x +
 (sqrt(5) + 1)*sqrt(sqrt(5) - 1)) + sqrt(sqrt(5) - 1)*log(2*sqrt(
2)*x - (sqrt(5) + 1)*sqrt(sqrt(5) - 1)) + sqrt(sqrt(5) + 1)*log(2
*sqrt(2)*x + sqrt(sqrt(5) + 1)*(sqrt(5) - 1)) - sqrt(sqrt(5) + 1)
*log(2*sqrt(2)*x - sqrt(sqrt(5) + 1)*(sqrt(5) - 1)))
```

Sympy [A] time = 3.19619, size = 49, normalized size = 0.37

$$\operatorname{RootSum}\left(256t^4 - 16t^2 - 1, \left(t \mapsto t\log\left(1024t^5 - 8t + x\right)\right)\right) + \operatorname{RootSum}\left(256t^4 + 16t^2 - 1, \left(t \mapsto t\log\left(1024t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(256*_t**4 - 16*_t**2 - 1, Lambda(_t, _t*log(1024*_t**5 -
8*_t + x))) + RootSum(256*_t**4 + 16*_t**2 - 1, Lambda(_t, _t*log
(1024*_t**5 - 8*_t + x)))
```

GIAC/XCAS [A] time = 0.341627, size = 198, normalized size = 1.51

$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{8}\sqrt{2\sqrt{5}-2}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{8}\sqrt{2\sqrt{5}-2}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{8}\sqrt{2\sqrt{5}+2}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{8}\sqrt{2\sqrt{5}+2}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2*sqrt(5) - 2)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/4*
sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/8*sqrt(
2*sqrt(5) - 2)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*
sqrt(5) - 2)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/8*sqrt(2*sq
rt(5) + 2)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/8*sqrt(2*sqrt
(5) + 2)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

3.17 $\int \frac{1+x^4}{1-4x^4+x^8}\,dx$

Optimal. Leaf size=157

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[
3]]) - ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 +
Sqrt[3]]) + ArcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sq
rt[-1 + Sqrt[3]]) - ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(
1/4)*Sqrt[1 + Sqrt[3]])
```

Rubi [A] time = 0.177235, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{\sqrt{3}-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{1+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[-1 + Sqrt[
3]]) - ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[1 +
Sqrt[3]]) + ArcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sq
rt[-1 + Sqrt[3]]) - ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(
1/4)*Sqrt[1 + Sqrt[3]])
```

Rubi in Sympy [A] time = 17.4474, size = 148, normalized size = 0.94

$$\frac{\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{2}+\sqrt{6}}}\right)}{2\sqrt{-\sqrt{2}+\sqrt{6}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{2}+\sqrt{6}}}\right)}{2\sqrt{\sqrt{2}+\sqrt{6}}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{2}+\sqrt{6}}}\right)}{2\sqrt{-\sqrt{2}+\sqrt{6}}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{2}+\sqrt{6}}}\right)}{2\sqrt{\sqrt{2}+\sqrt{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] atan(sqrt(2)*x/sqrt(-sqrt(2) + sqrt(6)))/(2*sqrt(-sqrt(2) + sqrt(
6))) - atan(sqrt(2)*x/sqrt(sqrt(2) + sqrt(6)))/(2*sqrt(sqrt(2) +
sqrt(6))) + atanh(sqrt(2)*x/sqrt(-sqrt(2) + sqrt(6)))/(2*sqrt(-sq
rt(2) + sqrt(6))) - atanh(sqrt(2)*x/sqrt(sqrt(2) + sqrt(6)))/(2*s
qrt(sqrt(2) + sqrt(6)))
```

Mathematica [C] time = 0.020404, size = 53, normalized size = 0.34

$$\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{\#1^7-2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] RootSum[1 - 4*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-
2*#1^3 + #1^7) & ]/8
```

Maple [C] time = 0.011, size = 40, normalized size = 0.3

$$\frac{1}{8} \sum_{\_R = RootOf\left(\_Z^8 - 4\_Z^4 + 1\right)} \frac{\left(\_R^4 + 1\right) \ln\left(x - \_R\right)}{\_R^7 - 2\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8-4*x^4+1),x)
```

```
[Out] 1/8*sum((_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 4x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 4*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

Fricas [A] time = 0.288496, size = 331, normalized size = 2.11

$$-\frac{1}{8}\sqrt{2}\left(4\left(\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\sqrt{3}+2\right)^{\frac{1}{4}}\left(\sqrt{3}-1\right)}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)}}\right)-4\left(-\sqrt{3}+2\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\sqrt{3}+1\right)\left(-\sqrt{3}+2\right)^{\frac{1}{4}}}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\left(\sqrt{3}+2\right)\sqrt{-\sqrt{}}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 4*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*(4*(sqrt(3) + 2)^(1/4)*arctan((sqrt(3) + 2)^(1/4)*(s
qrt(3) - 1)/(sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(sqrt(3) + 2)*(sq
rt(3) - 2)))) - 4*(-sqrt(3) + 2)^(1/4)*arctan((sqrt(3) + 1)*(-sqr
t(3) + 2)^(1/4)/(sqrt(2)*x + sqrt(2)*sqrt(x^2 + (sqrt(3) + 2)*sqr
t(-sqrt(3) + 2)))) - (sqrt(3) + 2)^(1/4)*log(sqrt(2)*x + (sqrt(3)
 + 2)^(1/4)*(sqrt(3) - 1)) + (sqrt(3) + 2)^(1/4)*log(sqrt(2)*x -
(sqrt(3) + 2)^(1/4)*(sqrt(3) - 1)) + (-sqrt(3) + 2)^(1/4)*log(sqr
t(2)*x + (sqrt(3) + 1)*(-sqrt(3) + 2)^(1/4)) - (-sqrt(3) + 2)^(1/
4)*log(sqrt(2)*x - (sqrt(3) + 1)*(-sqrt(3) + 2)^(1/4)))
```

Sympy [A] time = 0.537586, size = 24, normalized size = 0.15

$$\operatorname{RootSum}\left(1048576t^8 - 4096t^4 + 1, \left(t \mapsto t\log\left(4096t^5 - 12t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] RootSum(1048576*_t**8 - 4096*_t**4 + 1, Lambda(_t, _t*log(4096*_t
**5 - 12*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 4x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 4*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 4*x^4 + 1), x)
```

3.18 $\int \frac{1+x^4}{1-5x^4+x^8}\,dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7]
)] - ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt
[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3]
+ Sqrt[7])] - ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt
[3] + Sqrt[7])]
```

Rubi [A] time = 0.272449, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{6\left(\sqrt{7}-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{6\left(\sqrt{3}+\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3] + Sqrt[7]
)] - ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt[3] + Sqrt
[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(-Sqrt[3]
+ Sqrt[7])] - ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[6*(Sqrt
[3] + Sqrt[7])]
```

Rubi in Sympy [A] time = 17.7604, size = 168, normalized size = 0.98

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{6\sqrt{-\sqrt{3}+\sqrt{7}}}-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{6\sqrt{\sqrt{3}+\sqrt{7}}}+\frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{6\sqrt{-\sqrt{3}+\sqrt{7}}}-\frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{6\sqrt{\sqrt{3}+\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] sqrt(6)*atan(sqrt(2)*x/sqrt(-sqrt(3) + sqrt(7)))/(6*sqrt(-sqrt(3)
 + sqrt(7))) - sqrt(6)*atan(sqrt(2)*x/sqrt(sqrt(3) + sqrt(7)))/(6
*sqrt(sqrt(3) + sqrt(7))) + sqrt(6)*atanh(sqrt(2)*x/sqrt(-sqrt(3)
 + sqrt(7)))/(6*sqrt(-sqrt(3) + sqrt(7))) - sqrt(6)*atanh(sqrt(2)
*x/sqrt(sqrt(3) + sqrt(7)))/(6*sqrt(sqrt(3) + sqrt(7)))
```

Mathematica [C] time = 0.0209534, size = 55, normalized size = 0.32

$$\frac{1}{4}\text{RootSum}\left[\#1^8-5\#1^4+1\&,\frac{\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7-5\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] RootSum[1 - 5*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(-
5*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.012, size = 42, normalized size = 0.3

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8-5*x^4+1),x)
```

```
[Out] 1/4*sum((_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+1
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4+1}{x^8-5\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 5*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

Fricas [A] time = 0.289364, size = 701, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 5*x^4 + 1),x, algorithm="fricas")
```

```
[Out] sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(-sqrt(3)*(3*sqrt(7) - 5*sqrt(3))))*
arctan(3/2*sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(-sqrt(3)*(3*sqrt(7) - 5*
sqrt(3))))*(sqrt(7) + sqrt(3))/(sqrt(3)*x + sqrt(3)*sqrt(1/2*sqrt
(1/6)*sqrt(-sqrt(3)*(3*sqrt(7) - 5*sqrt(3)))*(sqrt(7)*sqrt(3) + 5
) + x^2))) - sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(sqrt(3)*(3*sqrt(7) + 5
*sqrt(3))))*arctan(3/2*sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(sqrt(3)*(3*s
qrt(7) + 5*sqrt(3))))*(sqrt(7) - sqrt(3))/(sqrt(3)*x + sqrt(3)*sq
rt(-1/2*sqrt(1/6)*sqrt(sqrt(3)*(3*sqrt(7) + 5*sqrt(3)))*(sqrt(7)*
sqrt(3) - 5) + x^2))) - 1/4*sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(-sqrt(3
)*(3*sqrt(7) - 5*sqrt(3))))*log(3/2*sqrt(1/3)*sqrt(sqrt(1/6)*sqrt
(-sqrt(3)*(3*sqrt(7) - 5*sqrt(3))))*(sqrt(7) + sqrt(3)) + sqrt(3)
*x) + 1/4*sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(-sqrt(3)*(3*sqrt(7) - 5*s
qrt(3))))*log(-3/2*sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(-sqrt(3)*(3*sqrt
(7) - 5*sqrt(3))))*(sqrt(7) + sqrt(3)) + sqrt(3)*x) + 1/4*sqrt(1/
3)*sqrt(sqrt(1/6)*sqrt(sqrt(3)*(3*sqrt(7) + 5*sqrt(3))))*log(3/2*
sqrt(1/3)*sqrt(sqrt(1/6)*sqrt(sqrt(3)*(3*sqrt(7) + 5*sqrt(3))))*(
sqrt(7) - sqrt(3)) + sqrt(3)*x) - 1/4*sqrt(1/3)*sqrt(sqrt(1/6)*sq
rt(sqrt(3)*(3*sqrt(7) + 5*sqrt(3))))*log(-3/2*sqrt(1/3)*sqrt(sqrt
(1/6)*sqrt(sqrt(3)*(3*sqrt(7) + 5*sqrt(3))))*(sqrt(7) - sqrt(3))
+ sqrt(3)*x)
```

Sympy [A] time = 0.562695, size = 24, normalized size = 0.14

$$\text{RootSum}\left(5308416t^8 - 11520t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 16t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 11520*_t**4 + 1, Lambda(_t, _t*log(9216*_
t**5 - 16*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 + 1}{x^8 - 5\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 5*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 5*x^4 + 1), x)
```

3.19 $\int \frac{1+x^4}{1-6x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sq
rt[1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]]) + ArcTanh[x/Sqrt[-1 + Sqrt
[2]]]/(4*Sqrt[-1 + Sqrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sq
rt[1 + Sqrt[2]])
```

Rubi [A] time = 0.113384, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[-1 + Sqrt[2]]) - ArcTan[x/Sq
rt[1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]]) + ArcTanh[x/Sqrt[-1 + Sqrt
[2]]]/(4*Sqrt[-1 + Sqrt[2]]) - ArcTanh[x/Sqrt[1 + Sqrt[2]]]/(4*Sq
rt[1 + Sqrt[2]])
```

Rubi in Sympy [A] time = 10.3975, size = 100, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\operatorname{atan}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\operatorname{atanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\operatorname{atanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] atan(x/sqrt(-1 + sqrt(2)))/(4*sqrt(-1 + sqrt(2))) - atan(x/sqrt(1
 + sqrt(2)))/(4*sqrt(1 + sqrt(2))) + atanh(x/sqrt(-1 + sqrt(2)))/
(4*sqrt(-1 + sqrt(2))) - atanh(x/sqrt(1 + sqrt(2)))/(4*sqrt(1 + s
qrt(2)))
```

Mathematica [A] time = 0.0732383, size = 111, normalized size = 0.95

$$\frac{1}{4}\left(\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] - Sqrt[-1 + Sqrt[
2]]*ArcTan[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqr
t[-1 + Sqrt[2]]] - Sqrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]
])/4
```

Maple [A] time = 0.07, size = 78, normalized size = 0.7

$$\frac{1}{4\sqrt{\sqrt{2}-1}}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\frac{1}{4\sqrt{\sqrt{2}-1}}Artanh\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\frac{1}{4\sqrt{1+\sqrt{2}}}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)-\frac{1}{4\sqrt{1+\sqrt{2}}}Artanh\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^8-6*x^4+1),x)
```

```
[Out] 1/4*arctan(x/(2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)+1/4*arctanh(x/(
2^(1/2)-1)^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan(x/(1+2^(1/2))^(1/2
))/(1+2^(1/2))^(1/2)-1/4*arctanh(x/(1+2^(1/2))^(1/2))/(1+2^(1/2))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{x^8-6x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 6*x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(x^8 - 6*x^4 + 1), x)
```

Fricas [A] time = 0.285506, size = 390, normalized size = 3.33

$$\begin{aligned}
&\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\left(\sqrt{2}+1\right)}{\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}(x^2+1)+2\right)}+x}\right)\\
&-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\left(\sqrt{2}-1\right)}{\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}(x^2-1)+2\right)}+x}\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\log\left(\sqrt{\frac{1}{2}}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\left(\sqrt{2}+1\right)+x\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\log\left(-\sqrt{\frac{1}{2}}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\left(\sqrt{2}+1\right)+x\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\log\left(\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\left(\sqrt{2}-1\right)+x\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\log\left(-\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\left(\sqrt{2}-1\right)+x\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 6*x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-sqrt(2)*(sqrt(2) - 2))*arctan(sqrt(1/2)*sqrt(
-sqrt(2)*(sqrt(2) - 2))*(sqrt(2) + 1)/(sqrt(1/2)*sqrt(sqrt(2)*(sq
rt(2)*(x^2 + 1) + 2)) + x)) - 1/2*sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2)
 + 2))*arctan(sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2) + 2))*(sqrt(2) - 1)
/(sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2)*(x^2 - 1) + 2)) + x)) - 1/8*sqr
t(1/2)*sqrt(-sqrt(2)*(sqrt(2) - 2))*log(sqrt(1/2)*sqrt(-sqrt(2)*(
sqrt(2) - 2))*(sqrt(2) + 1) + x) + 1/8*sqrt(1/2)*sqrt(-sqrt(2)*(s
qrt(2) - 2))*log(-sqrt(1/2)*sqrt(-sqrt(2)*(sqrt(2) - 2))*(sqrt(2)
 + 1) + x) + 1/8*sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2) + 2))*log(sqrt(1
/2)*sqrt(sqrt(2)*(sqrt(2) + 2))*(sqrt(2) - 1) + x) - 1/8*sqrt(1/2
)*sqrt(sqrt(2)*(sqrt(2) + 2))*log(-sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2
) + 2))*(sqrt(2) - 1) + x)
```

Sympy [A] time = 3.17126, size = 49, normalized size = 0.42

$$\operatorname{RootSum}\left(4096t^4 - 128t^2 - 1, \left(t \mapsto t\log\left(16384t^5 - 20t + x\right)\right)\right) + \operatorname{RootSum}\left(4096t^4 + 128t^2 - 1, \left(t \mapsto t\log\left(16384t^5 - 20t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] RootSum(4096*_t**4 - 128*_t**2 - 1, Lambda(_t, _t*log(16384*_t**5
 - 20*_t + x))) + RootSum(4096*_t**4 + 128*_t**2 - 1, Lambda(_t,
_t*log(16384*_t**5 - 20*_t + x)))
```

GIAC/XCAS [A] time = 0.346108, size = 166, normalized size = 1.42

$$-\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right) + \frac{1}{4}\sqrt{\sqrt{2}+1}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) - \frac{1}{8}\sqrt{\sqrt{2}-1}\ln\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right) + \frac{1}{8}\sqrt{\sqrt{2}-1}\ln\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right) + \frac{1}{8}\sqrt{\sqrt{2}+1}\ln\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right) - \frac{1}{8}\sqrt{\sqrt{2}+1}\ln\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^8 - 6*x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(sqrt(2) - 1)*arctan(x/sqrt(sqrt(2) + 1)) + 1/4*sqrt(sqr
t(2) + 1)*arctan(x/sqrt(sqrt(2) - 1)) - 1/8*sqrt(sqrt(2) - 1)*ln(
abs(x + sqrt(sqrt(2) + 1))) + 1/8*sqrt(sqrt(2) - 1)*ln(abs(x - sq
rt(sqrt(2) + 1))) + 1/8*sqrt(sqrt(2) + 1)*ln(abs(x + sqrt(sqrt(2)
 - 1))) - 1/8*sqrt(sqrt(2) + 1)*ln(abs(x - sqrt(sqrt(2) - 1)))
```

3.20 $\int \frac{1-x^4}{1+bx^4+x^8}\,dx$

Optimal. Leaf size=511

$$\frac{\sqrt{2-\sqrt{2-b}}\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{2-\sqrt{2-b}}\log\left(\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{\sqrt{2-b}+2}\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{2-b}}+\frac{\sqrt{\sqrt{2-b}+2}\log\left(\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}+\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\sqrt{2-b}}+\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}-\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\sqrt{2-b}}$$

```
[Out] -(Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[
2 - b]]])/(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*Ar
cTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqr
t[2 + Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 - S
qrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]])/(4*Sqrt[2 - Sqrt[2 - b
]]*Sqrt[2 - b]) - (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*
x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 - b])
+ (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2])/
(8*Sqrt[2 - b]) - (Sqrt[2 - Sqrt[2 - b]]*Log[1 + Sqrt[2 - Sqrt[2
- b]]*x + x^2])/(8*Sqrt[2 - b]) - (Sqrt[2 + Sqrt[2 - b]]*Log[1 -
Sqrt[2 + Sqrt[2 - b]]*x + x^2])/(8*Sqrt[2 - b]) + (Sqrt[2 + Sqrt[
2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2])/(8*Sqrt[2 - b])
```

Rubi [A] time = 0.875449, antiderivative size = 511, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\sqrt{2-\sqrt{2-b}}\log\left(-\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{2-\sqrt{2-b}}\log\left(\sqrt{2-\sqrt{2-b}}x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{\sqrt{2-b}+2}\log\left(-\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{2-b}}+\frac{\sqrt{\sqrt{2-b}+2}\log\left(\sqrt{\sqrt{2-b}+2}x+x^2+1\right)}{8\sqrt{2-b}}-\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}-2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}+\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}-2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\sqrt{2-b}}+\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2-b}}+2x}{\sqrt{\sqrt{2-b}+2}}\right)}{4\sqrt{2-\sqrt{2-b}}\sqrt{2-b}}-\frac{\sqrt{b+2}\tan^{-1}\left(\frac{\sqrt{\sqrt{2-b}+2}+2x}{\sqrt{2-\sqrt{2-b}}}\right)}{4\sqrt{\sqrt{2-b}+2}\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -(Sqrt[2 + b]*ArcTan[(Sqrt[2 - Sqrt[2 - b]] - 2*x)/Sqrt[2 + Sqrt[
2 - b]]])/(4*Sqrt[2 - Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*Ar
cTan[(Sqrt[2 + Sqrt[2 - b]] - 2*x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqr
t[2 + Sqrt[2 - b]]*Sqrt[2 - b]) + (Sqrt[2 + b]*ArcTan[(Sqrt[2 - S
qrt[2 - b]] + 2*x)/Sqrt[2 + Sqrt[2 - b]]])/(4*Sqrt[2 - Sqrt[2 - b
]]*Sqrt[2 - b]) - (Sqrt[2 + b]*ArcTan[(Sqrt[2 + Sqrt[2 - b]] + 2*
x)/Sqrt[2 - Sqrt[2 - b]]])/(4*Sqrt[2 + Sqrt[2 - b]]*Sqrt[2 - b])
+ (Sqrt[2 - Sqrt[2 - b]]*Log[1 - Sqrt[2 - Sqrt[2 - b]]*x + x^2])/
(8*Sqrt[2 - b]) - (Sqrt[2 - Sqrt[2 - b]]*Log[1 + Sqrt[2 - Sqrt[2
- b]]*x + x^2])/(8*Sqrt[2 - b]) - (Sqrt[2 + Sqrt[2 - b]]*Log[1 -
Sqrt[2 + Sqrt[2 - b]]*x + x^2])/(8*Sqrt[2 - b]) + (Sqrt[2 + Sqrt[
```

```
2 - b]]*Log[1 + Sqrt[2 + Sqrt[2 - b]]*x + x^2])/(8*Sqrt[2 - b])
```

Rubi in Sympy [A] time = 119.994, size = 359, normalized size = 0.7

$$\frac{\sqrt{-\sqrt{-b+2}+2}\log\left(x^2-x\sqrt{-\sqrt{-b+2}+2}+1\right)}{8\sqrt{-b+2}}-\frac{\sqrt{-\sqrt{-b+2}+2}\log\left(x^2+x\sqrt{-\sqrt{-b+2}+2}+1\right)}{8\sqrt{-b+2}}-\frac{\sqrt{-\sqrt{-b+2}+2}\,\mathrm{atan}\left(\frac{2x-\sqrt{\sqrt{-b+2}+2}}{\sqrt{-\sqrt{-b+2}+2}}\right)}{4\sqrt{-b+2}}-\frac{\sqrt{-\sqrt{-b+2}+2}\,\mathrm{atan}\left(\frac{2x+\sqrt{\sqrt{-b+2}+2}}{\sqrt{-\sqrt{-b+2}+2}}\right)}{4\sqrt{-b+2}}-\frac{\sqrt{\sqrt{-b+2}+2}\log\left(x^2-x\sqrt{\sqrt{-b+2}+2}+1\right)}{8\sqrt{-b+2}}+\frac{\sqrt{\sqrt{-b+2}+2}\log\left(x^2+x\sqrt{\sqrt{-b+2}+2}+1\right)}{8\sqrt{-b+2}}+\frac{\sqrt{\sqrt{-b+2}+2}\,\mathrm{atan}\left(\frac{2x-\sqrt{-\sqrt{-b+2}+2}}{\sqrt{\sqrt{-b+2}+2}}\right)}{4\sqrt{-b+2}}+\frac{\sqrt{\sqrt{-b+2}+2}\,\mathrm{atan}\left(\frac{2x+\sqrt{-\sqrt{-b+2}+2}}{\sqrt{\sqrt{-b+2}+2}}\right)}{4\sqrt{-b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((-x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] sqrt(-sqrt(-b + 2) + 2)*log(x**2 - x*sqrt(-sqrt(-b + 2) + 2) + 1)
/(8*sqrt(-b + 2)) - sqrt(-sqrt(-b + 2) + 2)*log(x**2 + x*sqrt(-sq
rt(-b + 2) + 2) + 1)/(8*sqrt(-b + 2)) - sqrt(-sqrt(-b + 2) + 2)*a
tan((2*x - sqrt(sqrt(-b + 2) + 2))/sqrt(-sqrt(-b + 2) + 2))/(4*sq
rt(-b + 2)) - sqrt(-sqrt(-b + 2) + 2)*atan((2*x + sqrt(sqrt(-b +
2) + 2))/sqrt(-sqrt(-b + 2) + 2))/(4*sqrt(-b + 2)) - sqrt(sqrt(-b
 + 2) + 2)*log(x**2 - x*sqrt(sqrt(-b + 2) + 2) + 1)/(8*sqrt(-b +
2)) + sqrt(sqrt(-b + 2) + 2)*log(x**2 + x*sqrt(sqrt(-b + 2) + 2)
+ 1)/(8*sqrt(-b + 2)) + sqrt(sqrt(-b + 2) + 2)*atan((2*x - sqrt(-
sqrt(-b + 2) + 2))/sqrt(sqrt(-b + 2) + 2))/(4*sqrt(-b + 2)) + sqr
t(sqrt(-b + 2) + 2)*atan((2*x + sqrt(-sqrt(-b + 2) + 2))/sqrt(sqr
t(-b + 2) + 2))/(4*sqrt(-b + 2))
```

Mathematica [C] time = 0.0358951, size = 57, normalized size = 0.11

$$-\frac{1}{4}\mathrm{RootSum}\left[\#1^8+\#1^4b+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In]   Integrate[(1 - x^4)/(1 + b*x^4 + x^8),x]
```

```
[Out] -RootSum[1 + b*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(b*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.004, size = 44, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf(\_Z^8+\_Z^4b+1)}\frac{(-\_R^4+1)\ln(x-\_R)}{2\_R^7+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8+b*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8+_Z^4*b+
1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4 - 1}{x^8 + bx^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

Fricas [A] time = 0.291944, size = 1458, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2
+ 12*b - 8)) + b)/(b^2 - 4*b + 4)))*arctan(1/2*((b^2 - 4*b + 4)*s
qrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b + 2)*sqrt(sqrt(1/2)*sqr
t(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(
b^2 - 4*b + 4)))/(x + sqrt(x^2 + 1/2*sqrt(1/2)*(b^2 - (b^3 - 6*b^
2 + 12*b - 8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - 2*b)*sqrt(
-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^
2 - 4*b + 4))))) + sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b +
 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*arctan(1/2*(
(b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b - 2)*s
qrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 1
2*b - 8)) - b)/(b^2 - 4*b + 4)))/(x + sqrt(x^2 + 1/2*sqrt(1/2)*(b
^2 + (b^3 - 6*b^2 + 12*b - 8)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b -
8)) - 2*b)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b
 - 8)) - b)/(b^2 - 4*b + 4))))) + 1/4*sqrt(sqrt(1/2)*sqrt(-((b^2
- 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b
 + 4)))*log(1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b
 - 8)) - b + 2)*sqrt(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2
)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 - 4*b + 4))) + x) - 1/4*sqr
t(sqrt(1/2)*sqrt(-((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12
*b - 8)) + b)/(b^2 - 4*b + 4)))*log(-1/2*((b^2 - 4*b + 4)*sqrt((b
 + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b + 2)*sqrt(sqrt(1/2)*sqrt(-((b
^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b)/(b^2 -
4*b + 4))) + x) - 1/4*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((
b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4)))*log(1/2*(
(b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) + b - 2)*s
qrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 1
2*b - 8)) - b)/(b^2 - 4*b + 4))) + x) + 1/4*sqrt(sqrt(1/2)*sqrt((
(b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2
- 4*b + 4)))*log(-1/2*((b^2 - 4*b + 4)*sqrt((b + 2)/(b^3 - 6*b^2
+ 12*b - 8)) + b - 2)*sqrt(sqrt(1/2)*sqrt(((b^2 - 4*b + 4)*sqrt((
b + 2)/(b^3 - 6*b^2 + 12*b - 8)) - b)/(b^2 - 4*b + 4))) + x)
```

Sympy [A] time = 7.47082, size = 76, normalized size = 0.15

$$-\operatorname{RootSum}\left(t^8\left(65536b^4 - 524288b^3 + 1572864b^2 - 2097152b + 1048576\right) + t^4\left(256b^3 - 1024b^2 + 1024b\right) + 1, \left(t \mapsto t\log\left(1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8+b*x**4+1),x)
```

```
[Out] -RootSum(_t**8*(65536*b**4 - 524288*b**3 + 1572864*b**2 - 2097152
*b + 1048576) + _t**4*(256*b**3 - 1024*b**2 + 1024*b) + 1, Lambda
(_t, _t*log(1024*_t**5*b**2 - 4096*_t**5*b + 4096*_t**5 + 4*_t*b
- 4*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4 - 1}{x^8 + bx^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 + b*x^4 + 1), x)
```

3.21 $\int \frac{1-x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=429

$$-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3-\sqrt{5}}x+\sqrt{3-\sqrt{5}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2}x^2-2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2}x^2+2^{3/4}\sqrt[4]{3+\sqrt{5}}x+\sqrt{3+\sqrt{5}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}}$$

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 -
Sqrt[5])^(1/4)])/(2*2^(3/4)) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2
^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)) - ((3 - Sqrt[5])^(1/4
)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)) - ((3
+ Sqrt[5])^(1/4)*Log[Sqrt[3 - Sqrt[5]] - 2^(3/4)*(3 - Sqrt[5])^(1
/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt
[3 - Sqrt[5]] + 2^(3/4)*(3 - Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*
2^(3/4)) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[3 + Sqrt[5]] - 2^(3/4)*(
3 + Sqrt[5])^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)) - ((3 - Sqrt[5])
^(1/4)*Log[Sqrt[3 + Sqrt[5]] + 2^(3/4)*(3 + Sqrt[5])^(1/4)*x + Sq
rt[2]*x^2])/(4*2^(3/4))
```

Rubi [A] time = 0.704994, antiderivative size = 411, normalized size of antiderivative = 0.96, number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 -
Sqrt[5])^(1/4)])/(2*2^(3/4)) + ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2
^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)) - ((3 - Sqrt[5])^(1/4
)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)) - ((3
+ Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^
(1/4)*x + 2*x^2])/(4*2^(3/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(
3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4))
 + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqr
t[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)) - ((3 - Sqrt[5])^(1/4)*Log[S
qrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2
^(3/4))
```

Rubi in Sympy [A] time = 85.7917, size = 590, normalized size = 1.38

$$\frac{2^{\frac{3}{4}}\sqrt{-2\sqrt{5}+6}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{8\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{2^{\frac{3}{4}}\sqrt{-2\sqrt{5}+6}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{-\sqrt{5}+3}+\sqrt{-2\sqrt{5}+6}\right)}{8\left(-\sqrt{5}+3\right)^{\frac{5}{4}}} + \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{2\sqrt{5}+6}\log\left(2x^2-2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{8\left(\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{2\sqrt{5}+6}\log\left(2x^2+2\sqrt[4]{2}x\sqrt[4]{\sqrt{5}+3}+\sqrt{2\sqrt{5}+6}\right)}{8\left(\sqrt{5}+3\right)^{\frac{5}{4}}} - \frac{2^{\frac{3}{4}}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt{-2\sqrt{5}+6}\sqrt[4]{-\sqrt{5}+3}} - \frac{2^{\frac{3}{4}}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{-2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{-\sqrt{5}+3}}\right)}{2\sqrt{-2\sqrt{5}+6}\sqrt[4]{-\sqrt{5}+3}} - \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x-\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}\sqrt{2\sqrt{5}+6}} - \frac{2^{\frac{3}{4}}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\left(x+\frac{\sqrt[4]{2\sqrt{5}+6}}{2}\right)}{\sqrt[4]{\sqrt{5}+3}}\right)}{2\sqrt[4]{\sqrt{5}+3}\sqrt{2\sqrt{5}+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] 2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-sqrt(5)/2 + 1/2)*log(2*x**2 - 2*2
**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt(5) + 6))/(8*(-sqrt
(5) + 3)**(5/4)) - 2**(3/4)*sqrt(-2*sqrt(5) + 6)*(-sqrt(5)/2 + 1/
2)*log(2*x**2 + 2*2**(1/4)*x*(-sqrt(5) + 3)**(1/4) + sqrt(-2*sqrt
(5) + 6))/(8*(-sqrt(5) + 3)**(5/4)) + 2**(3/4)*(1/2 + sqrt(5)/2)*
sqrt(2*sqrt(5) + 6)*log(2*x**2 - 2*2**(1/4)*x*(sqrt(5) + 3)**(1/4
) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5) + 3)**(5/4)) - 2**(3/4)*(1/2
 + sqrt(5)/2)*sqrt(2*sqrt(5) + 6)*log(2*x**2 + 2*2**(1/4)*x*(sqrt
(5) + 3)**(1/4) + sqrt(2*sqrt(5) + 6))/(8*(sqrt(5) + 3)**(5/4)) -
 2**(3/4)*(-sqrt(5)/2 + 1/2)*atan(2**(3/4)*(x - (-2*sqrt(5) + 6)*
*(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(2*sqrt(-2*sqrt(5) + 6)*(-sqrt(5
) + 3)**(1/4)) - 2**(3/4)*(-sqrt(5)/2 + 1/2)*atan(2**(3/4)*(x + (
-2*sqrt(5) + 6)**(1/4)/2)/(-sqrt(5) + 3)**(1/4))/(2*sqrt(-2*sqrt(
5) + 6)*(-sqrt(5) + 3)**(1/4)) - 2**(3/4)*(1/2 + sqrt(5)/2)*atan(
2**(3/4)*(x - (2*sqrt(5) + 6)**(1/4)/2)/(sqrt(5) + 3)**(1/4))/(2*
(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6)) - 2**(3/4)*(1/2 + sqrt(
5)/2)*atan(2**(3/4)*(x + (2*sqrt(5) + 6)**(1/4)/2)/(sqrt(5) + 3)*
*(1/4))/(2*(sqrt(5) + 3)**(1/4)*sqrt(2*sqrt(5) + 6))
```

Mathematica [C] time = 0.0214814, size = 57, normalized size = 0.13

$$-\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 + 3*x^4 + x^8),x]
```

```
[Out] -RootSum[1 + 3*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.009, size = 44, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8+3*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+
1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4-1}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

Fricas [A] time = 0.295184, size = 1007, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/16*2^(3/4)*(4*sqrt(2)*(sqrt(5) + 3)^(1/4)*arctan((sqrt(5)*sqrt
(2) - sqrt(2))*(sqrt(5) + 3)^(1/4)/(4*2^(1/4)*x + (sqrt(5)*sqrt(2
) - sqrt(2))*(sqrt(5) + 3)^(1/4) + 2*2^(1/4)*sqrt(sqrt(2)*(2*sqrt
(2)*x^2 - sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + (sqrt(5)*2^(3/4)*x -
2^(3/4)*x)*(sqrt(5) + 3)^(1/4)))) + 4*sqrt(2)*(sqrt(5) + 3)^(1/4
)*arctan((sqrt(5)*sqrt(2) - sqrt(2))*(sqrt(5) + 3)^(1/4)/(4*2^(1/
4)*x - (sqrt(5)*sqrt(2) - sqrt(2))*(sqrt(5) + 3)^(1/4) + 2*2^(1/4
)*sqrt(sqrt(2)*(2*sqrt(2)*x^2 - sqrt(sqrt(5) + 3)*(sqrt(5) - 3) -
 (sqrt(5)*2^(3/4)*x - 2^(3/4)*x)*(sqrt(5) + 3)^(1/4)))) - 4*sqrt
(2)*(-sqrt(5) + 3)^(1/4)*arctan((sqrt(5)*sqrt(2) + sqrt(2))*(-sqr
t(5) + 3)^(1/4)/(4*2^(1/4)*x + (sqrt(5)*sqrt(2) + sqrt(2))*(-sqrt
(5) + 3)^(1/4) + 2*2^(1/4)*sqrt(sqrt(2)*(2*sqrt(2)*x^2 + (sqrt(5)
 + 3)*sqrt(-sqrt(5) + 3) + (sqrt(5)*2^(3/4)*x + 2^(3/4)*x)*(-sqrt
(5) + 3)^(1/4)))) - 4*sqrt(2)*(-sqrt(5) + 3)^(1/4)*arctan((sqrt(
5)*sqrt(2) + sqrt(2))*(-sqrt(5) + 3)^(1/4)/(4*2^(1/4)*x - (sqrt(5
)*sqrt(2) + sqrt(2))*(-sqrt(5) + 3)^(1/4) + 2*2^(1/4)*sqrt(sqrt(2
)*(2*sqrt(2)*x^2 + (sqrt(5) + 3)*sqrt(-sqrt(5) + 3) - (sqrt(5)*2^
(3/4)*x + 2^(3/4)*x)*(-sqrt(5) + 3)^(1/4)))) - sqrt(2)*(sqrt(5)
```

```
+ 3)^(1/4)*log(2*sqrt(2)*x^2 - sqrt(sqrt(5) + 3)*(sqrt(5) - 3) +
(sqrt(5)*2^(3/4)*x - 2^(3/4)*x)*(sqrt(5) + 3)^(1/4)) + sqrt(2)*(s
qrt(5) + 3)^(1/4)*log(2*sqrt(2)*x^2 - sqrt(sqrt(5) + 3)*(sqrt(5)
- 3) - (sqrt(5)*2^(3/4)*x - 2^(3/4)*x)*(sqrt(5) + 3)^(1/4)) + sqr
t(2)*(-sqrt(5) + 3)^(1/4)*log(2*sqrt(2)*x^2 + (sqrt(5) + 3)*sqrt(
-sqrt(5) + 3) + (sqrt(5)*2^(3/4)*x + 2^(3/4)*x)*(-sqrt(5) + 3)^(1
/4)) - sqrt(2)*(-sqrt(5) + 3)^(1/4)*log(2*sqrt(2)*x^2 + (sqrt(5)
+ 3)*sqrt(-sqrt(5) + 3) - (sqrt(5)*2^(3/4)*x + 2^(3/4)*x)*(-sqrt(
5) + 3)^(1/4)))
```

Sympy [A] time = 3.80062, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(65536t^8 + 768t^4 + 1, \left(t \mapsto t \log\left(1024t^5 + 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8+3*x**4+1),x)
```

```
[Out] -RootSum(65536*_t**8 + 768*_t**4 + 1, Lambda(_t, _t*log(1024*_t**
5 + 8*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4 - 1}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

3.22 $\int \frac{1-x^4}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=97

$$\frac{x}{2(x^4+1)} - \frac{\log\left(x^2-\sqrt{2}x+1\right)}{8\sqrt{2}} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{8\sqrt{2}} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

```
[Out] x/(2*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 +
Sqrt[2]*x]/(4*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + L
og[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rubi [A] time = 0.107823, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{x}{2(x^4+1)} - \frac{\log\left(x^2-\sqrt{2}x+1\right)}{8\sqrt{2}} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{8\sqrt{2}} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] x/(2*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 +
Sqrt[2]*x]/(4*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + L
og[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rubi in Sympy [A] time = 17.1031, size = 82, normalized size = 0.85

$$\frac{x}{2(x^4+1)} - \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{16} + \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{16} + \frac{\sqrt{2}\,\text{atan}\left(\sqrt{2}x-1\right)}{8} + \frac{\sqrt{2}\,\text{atan}\left(\sqrt{2}x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] x/(2*(x**4 + 1)) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 + sqrt(2)
*log(x**2 + sqrt(2)*x + 1)/16 + sqrt(2)*atan(sqrt(2)*x - 1)/8 + s
qrt(2)*atan(sqrt(2)*x + 1)/8
```

Mathematica [A] time = 0.110991, size = 90, normalized size = 0.93

$$\frac{1}{16}\left(\frac{8x}{x^4+1} - \sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right) + 2\sqrt{2}\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*Ar
cTan[1 + Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*
Log[1 + Sqrt[2]*x + x^2])/16
```

Maple [A] time = 0.011, size = 68, normalized size = 0.7

$$\frac{x}{2x^4+2}+\frac{\arctan\left(\sqrt{2}x-1\right)\sqrt{2}}{8}+\frac{\sqrt{2}}{16}\ln\left(\frac{1+x^2+\sqrt{2}x}{1+x^2-\sqrt{2}x}\right)+\frac{\arctan\left(1+\sqrt{2}x\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8+2*x^4+1),x)
```

```
[Out] 1/2*x/(x^4+1)+1/8*arctan(2^(1/2)*x-1)*2^(1/2)+1/16*2^(1/2)*ln((1+
x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+1/8*arctan(1+2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.817646, size = 111, normalized size = 1.14

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 2*x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)
*x + 1) - 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

Fricas [A] time = 0.266288, size = 173, normalized size = 1.78

$$-\frac{4\sqrt{2}(x^4+1)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)+4\sqrt{2}(x^4+1)\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)-\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{2}x+1\right)+}{16(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 2*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2
 + sqrt(2)*x + 1) + 1)) + 4*sqrt(2)*(x^4 + 1)*arctan(1/(sqrt(2)*x
 + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) - 1)) - sqrt(2)*(x^4 + 1)*lo
g(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x +
1) - 8*x)/(x^4 + 1)
```

Sympy [A] time = 0.511541, size = 82, normalized size = 0.85

$$\frac{x}{2x^4+2}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{16}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8+2*x**4+1),x)
```

```
[Out] x/(2*x**4 + 2) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/16 + sqrt(2)*l
og(x**2 + sqrt(2)*x + 1)/16 + sqrt(2)*atan(sqrt(2)*x - 1)/8 + sqr
t(2)*atan(sqrt(2)*x + 1)/8
```

GIAC/XCAS [A] time = 0.284509, size = 111, normalized size = 1.14

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)+\frac{x}{2\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 2*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*ln(x^2 + sqrt(2)*
x + 1) - 1/16*sqrt(2)*ln(x^2 - sqrt(2)*x + 1) + 1/2*x/(x^4 + 1)
```

3.23 $\int \frac{1-x^4}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]])/4 + ArcTan[Sqrt[3] - 2*x]/4
+ (Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]])/4 - ArcTan[Sqrt[3] + 2*x]/4
 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 - (Sqrt[3]*Log[1 - Sqr
t[3]*x + x^2])/8 + (Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/8
```

Rubi [A] time = 0.204099, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{8}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{4}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]])/4 + ArcTan[Sqrt[3] - 2*x]/4
+ (Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]])/4 - ArcTan[Sqrt[3] + 2*x]/4
 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 - (Sqrt[3]*Log[1 - Sqr
t[3]*x + x^2])/8 + (Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/8
```

Rubi in Sympy [A] time = 40.9273, size = 128, normalized size = 0.91

$$\frac{\log\left(x^2-x+1\right)}{8}-\frac{\log\left(x^2+x+1\right)}{8}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{8}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{4}-\frac{\mathrm{atan}\left(2x-\sqrt{3}\right)}{4}-\frac{\mathrm{atan}\left(2x+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8+x**4+1),x)
```

```
[Out] log(x**2 - x + 1)/8 - log(x**2 + x + 1)/8 - sqrt(3)*log(x**2 - sq
rt(3)*x + 1)/8 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/8 + sqrt(3)*at
an(sqrt(3)*(2*x/3 - 1/3))/4 + sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))
/4 - atan(2*x - sqrt(3))/4 - atan(2*x + sqrt(3))/4
```

Mathematica [C] time = 0.300273, size = 129, normalized size = 0.92

$$\frac{1}{8}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)-2\sqrt{-2-2i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2\sqrt{-2+2i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]   Integrate[(1 - x^4)/(1 + x^4 + x^8),x]
```

```
[Out] (-2*Sqrt[-2 - (2*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - 2*Sq
rt[-2 + (2*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] + 2*Sqrt[3]*
ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
+ Log[1 - x + x^2] - Log[1 + x + x^2])/8
```

Maple [A] time = 0.019, size = 109, normalized size = 0.8

$$-\frac{\ln\left(x^{2}+x+1\right)}{8}+\frac{\sqrt{3}}{4}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(1+x^{2}-x\sqrt{3}\right)\sqrt{3}}{8}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^{2}+x\sqrt{3}\right)\sqrt{3}}{8}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\ln\left(x^{2}-x+1\right)}{8}+\frac{\sqrt{3}}{4}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^4+1)/(x^8+x^4+1),x)
```

```
[Out] -1/8*ln(x^2+x+1)+1/4*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/8*ln(1
+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))+1/8*ln(1+x^2+x*3^
(1/2))*3^(1/2)-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)+1/4*3^(1/2
)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{2}\int\frac{2x^{2}-1}{x^{4}-x^{2}+1}\,dx-\frac{1}{8}\log\left(x^{2}+x+1\right)+\frac{1}{8}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^4 - 1)/(x^8 + x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) - 1/2*integrate((2*x^2 - 1)/(x^4 - x^2 + 1),
 x) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Fricas [A] time = 0.28571, size = 190, normalized size = 1.36

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\sqrt{3}\log\left(x^{2}+\sqrt{3}x+1\right)-\frac{1}{8}\sqrt{3}\log\left(x^{2}-\sqrt{3}x+1\right)+\frac{1}{2}\arctan\left(\frac{1}{2x+\sqrt{3}+2\sqrt{x^{2}+\sqrt{3}x+1}}\right)+\frac{1}{2}\arctan\left(\frac{1}{2x-\sqrt{3}+2\sqrt{x^{2}-\sqrt{3}x+1}}\right)-\frac{1}{8}\log\left(x^{2}+x+1\right)+\frac{1}{8}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^4 - 1)/(x^8 + x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/8*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/8
```

```
*sqrt(3)*log(x^2 - sqrt(3)*x + 1) + 1/2*arctan(1/(2*x + sqrt(3) +
 2*sqrt(x^2 + sqrt(3)*x + 1))) + 1/2*arctan(1/(2*x - sqrt(3) + 2*
sqrt(x^2 - sqrt(3)*x + 1))) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2
- x + 1)
```

Sympy [A] time = 2.68794, size = 148, normalized size = 1.06

$$-\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right)-\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(-\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right)$$
$$-\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}-\frac{\sqrt{3}i}{8}\right)^5\right)-\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)\log\left(x+1024\left(\frac{1}{8}+\frac{\sqrt{3}i}{8}\right)^5\right)$$
$$-\operatorname{RootSum}\left(256t^4-16t^2+1,\left(t\mapsto t\log\left(1024t^5+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8+x**4+1),x)
```

```
[Out] -(-1/8 - sqrt(3)*I/8)*log(x + 1024*(-1/8 - sqrt(3)*I/8)**5) - (-1
/8 + sqrt(3)*I/8)*log(x + 1024*(-1/8 + sqrt(3)*I/8)**5) - (1/8 -
sqrt(3)*I/8)*log(x + 1024*(1/8 - sqrt(3)*I/8)**5) - (1/8 + sqrt(3
)*I/8)*log(x + 1024*(1/8 + sqrt(3)*I/8)**5) - RootSum(256*_t**4 -
 16*_t**2 + 1, Lambda(_t, _t*log(1024*_t**5 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4-1}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 + x^4 + 1), x)
```

3.24 $\int \frac{1-x^4}{1+x^8}\,dx$

Optimal. Leaf size=347

$$\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 -
Sqrt[2]]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(
4*Sqrt[2 + Sqrt[2]]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 +
Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x
)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]]) + (Sqrt[(2 - Sqrt[2])/
2]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[2])/2]
*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[2])/2]*L
og[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[2])/2]*Log
[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/8
```

Rubi [A] time = 0.669772, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2 -
Sqrt[2]]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(
4*Sqrt[2 + Sqrt[2]]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 +
Sqrt[2]]]/(4*Sqrt[2 - Sqrt[2]]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x
)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2 + Sqrt[2]]) + (Sqrt[(2 - Sqrt[2])/
2]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[2])/2]
*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[2])/2]*L
og[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[2])/2]*Log
[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/8
```

Rubi in Sympy [A] time = 68.0091, size = 468, normalized size = 1.35

$$-\frac{\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)}{4\sqrt{-\sqrt{2}+2}}+\frac{\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)}{4\sqrt{-\sqrt{2}+2}}$$
$$-\frac{\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)}{4\sqrt{\sqrt{2}+2}}+\frac{\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)}{4\sqrt{\sqrt{2}+2}}$$
$$-\frac{\left(-\sqrt{\sqrt{2}+2}+\frac{(1+\sqrt{2})\sqrt{\sqrt{2}+2}}{2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{2\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}-\frac{\left(-\sqrt{\sqrt{2}+2}+\frac{(1+\sqrt{2})\sqrt{\sqrt{2}+2}}{2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)}{2\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}$$
$$+\frac{\left(-\frac{(-\sqrt{2}+1)\sqrt{-\sqrt{2}+2}}{2}+\sqrt{-\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{2\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}$$
$$+\frac{\left(-\frac{(-\sqrt{2}+1)\sqrt{-\sqrt{2}+2}}{2}+\sqrt{-\sqrt{2}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)}{2\sqrt{-\sqrt{2}+2}\sqrt{\sqrt{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8+1),x)
```

```
[Out] -(-sqrt(2)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(2) + 2) + 1)/(4*sqrt(
-sqrt(2) + 2)) + (-sqrt(2)/2 + 1/2)*log(x**2 + x*sqrt(-sqrt(2) +
2) + 1)/(4*sqrt(-sqrt(2) + 2)) - (1/2 + sqrt(2)/2)*log(x**2 - x*s
qrt(sqrt(2) + 2) + 1)/(4*sqrt(sqrt(2) + 2)) + (1/2 + sqrt(2)/2)*l
og(x**2 + x*sqrt(sqrt(2) + 2) + 1)/(4*sqrt(sqrt(2) + 2)) - (-sqrt
(sqrt(2) + 2) + (1 + sqrt(2))*sqrt(sqrt(2) + 2)/2)*atan((2*x - sq
rt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2))/(2*sqrt(-sqrt(2) + 2)*sqrt(s
qrt(2) + 2)) - (-sqrt(sqrt(2) + 2) + (1 + sqrt(2))*sqrt(sqrt(2) +
 2)/2)*atan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2))/(2*sqrt
(-sqrt(2) + 2)*sqrt(sqrt(2) + 2)) + (-(-sqrt(2) + 1)*sqrt(-sqrt(2
) + 2)/2 + sqrt(-sqrt(2) + 2))*atan((2*x - sqrt(-sqrt(2) + 2))/sq
rt(sqrt(2) + 2))/(2*sqrt(-sqrt(2) + 2)*sqrt(sqrt(2) + 2)) + (-(-s
qrt(2) + 1)*sqrt(-sqrt(2) + 2)/2 + sqrt(-sqrt(2) + 2))*atan((2*x
+ sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))/(2*sqrt(-sqrt(2) + 2)*sq
rt(sqrt(2) + 2))
```

Mathematica [A] time = 0.289129, size = 257, normalized size = 0.74

$$\frac{1}{8}\left(-\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)+\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)\right.$$
$$+\left(\sin\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)+\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)$$
$$+2\left(\sin\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\csc\left(\frac{\pi}{8}\right)\left(x+\cos\left(\frac{\pi}{8}\right)\right)\right)$$
$$+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)$$
$$+2\left(\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(x\sec\left(\frac{\pi}{8}\right)-\tan\left(\frac{\pi}{8}\right)\right)$$
$$\left.+2\left(\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right)\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right)-x\csc\left(\frac{\pi}{8}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 + x^8),x]
```

```
[Out] (2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + Log[
1 + x^2 - 2*x*Sin[Pi/8]]*(Cos[Pi/8] - Sin[Pi/8]) + 2*ArcTan[(x +
Cos[Pi/8])*Csc[Pi/8]]*(-Cos[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*
x*Sin[Pi/8]]*(-Cos[Pi/8] + Sin[Pi/8]) + 2*ArcTan[Sec[Pi/8]*(x + S
```

```
in[Pi/8])]*(Cos[Pi/8] + Sin[Pi/8]) + 2*ArcTan[x*Sec[Pi/8] - Tan[P
i/8]]*(Cos[Pi/8] + Sin[Pi/8]) - Log[1 + x^2 - 2*x*Cos[Pi/8]]*(Cos
[Pi/8] + Sin[Pi/8]) + Log[1 + x^2 + 2*x*Cos[Pi/8]]*(Cos[Pi/8] + S
in[Pi/8]))/8
```

Maple [C] time = 0.01, size = 29, normalized size = 0.1

$$\frac{1}{8}\sum_{\_R=RootOf(\_Z^8+1)}\frac{(-\_R^4+1)\ln(x-\_R)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8+1),x)
```

```
[Out] 1/8*sum((-_R^4+1)/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^4-1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 + 1), x)
```

Fricas [A] time = 0.283889, size = 1343, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + sqrt(-
sqrt(2) + 2))/(2*sqrt(2)*x + 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*s
qrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(s
qrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(sqrt(2) + 2
)*arctan((sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(2*sqrt(2)*x +
2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2
)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) +
 2))) - 1/8*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2)
 - sqrt(-sqrt(2) + 2))/(2*sqrt(2)*x + 2*sqrt(2)*sqrt(x^2 + 1/2*sq
rt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1)
 + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/8*sqrt(2)*sqrt(-s
qrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(2*s
qrt(2)*x + 2*sqrt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) -
 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2) - sqrt
(-sqrt(2) + 2))) - 1/8*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*a
rctan(sqrt(sqrt(2) + 2)/(2*x + 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2)
+ 1) + sqrt(-sqrt(2) + 2))) - 1/8*(sqrt(sqrt(2) + 2) + sqrt(-sqrt
(2) + 2))*arctan(sqrt(sqrt(2) + 2)/(2*x + 2*sqrt(x^2 - x*sqrt(-sq
rt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(sqrt(2) + 2) -
 sqrt(-sqrt(2) + 2))*arctan(sqrt(-sqrt(2) + 2)/(2*x + 2*sqrt(x^2
+ x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2))) + 1/8*(sqrt(sqrt
(2) + 2) - sqrt(-sqrt(2) + 2))*arctan(sqrt(-sqrt(2) + 2)/(2*x + 2
```

```
*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))) + 1/32
*sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) +
 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(-
sqrt(2) + 2)*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt
(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/32*sqrt(2)*sqrt(-sqrt(2) + 2)*l
og(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sq
rt(2) + 2) + 1) - 1/32*sqrt(2)*sqrt(sqrt(2) + 2)*log(x^2 - 1/2*sq
rt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1)
 + 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt
(sqrt(2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2)
)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/32*(sqrt(sqrt(2) + 2) -
sqrt(-sqrt(2) + 2))*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/32*(s
qrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*log(x^2 - x*sqrt(-sqrt(2)
+ 2) + 1)
```

Sympy [A] time = 4.30534, size = 20, normalized size = 0.06

$$-\operatorname{RootSum}\left(1048576t^8+1,\left(t\mapsto t\log\left(4096t^5-4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8+1),x)
```

```
[Out] -RootSum(1048576*_t**8 + 1, Lambda(_t, _t*log(4096*_t**5 - 4*_t +
 x)))
```

GIAC/XCAS [A] time = 0.313585, size = 333, normalized size = 0.96

$$\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{2\sqrt{2}+4}\ln\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{2\sqrt{2}+4}\ln\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-2\sqrt{2}+4}\ln\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{-2\sqrt{2}+4}\ln\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 + 1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sq
rt(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2)
 + 2))/sqrt(sqrt(2) + 2)) - 1/8*sqrt(-2*sqrt(2) + 4)*arctan((2*x
+ sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(-2*sqrt(2) +
4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/16*sq
rt(2*sqrt(2) + 4)*ln(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(2
*sqrt(2) + 4)*ln(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(-2*sq
rt(2) + 4)*ln(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-2*sqrt
(2) + 4)*ln(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

3.25 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]
]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/S
qrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sq
rt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqr
t[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[
(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2
 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 +
 Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.658743, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]
]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/S
qrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sq
rt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqr
t[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[
(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2
 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 +
 Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi in Sympy [A] time = 72.8165, size = 495, normalized size = 1.39

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(12
*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 + x*sqrt
(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(sqrt(3)/2
+ 1)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) + 2)) +
 sqrt(3)*(sqrt(3)/2 + 1)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(12*
sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3) + 2)**(3/2)/2 + sqrt(3)*s
qrt(sqrt(3) + 2))*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) +
2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3)
 + 2)**(3/2)/2 + sqrt(3)*sqrt(sqrt(3) + 2))*atan((2*x + sqrt(sqrt
(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3)
+ 2)) + sqrt(3)*((-sqrt(3) + 2)**(3/2)/2 + sqrt(3)*sqrt(-sqrt(3)
+ 2))*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(
-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*((-sqrt(3) + 2)**(3/2)
/2 + sqrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x + sqrt(-sqrt(3) + 2))/
sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0237335, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-
#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.01, size = 44, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.289335, size = 1223, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*(4*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arct
an(1/(2*(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) + (209
*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56
*sqrt(3) - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*
x - 2*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - sqrt(3) + 2)) + 4*
(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2*
(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3)
*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3)
 - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x)
*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(3) - 2)) + (2*sqrt(3)
 + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(194*x^2 + 112*sqrt(
3)*(x^2 + 1) + 2*(209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sq
rt(3) + 7)) + 194) - (2*sqrt(3) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3
) - 7))*log(194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x +
362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 194) - (2*sqrt(3) +
3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(-194*x^2 + 112*sqrt(3)
*(x^2 + 1) + 2*(209*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt
(3) - 7)) - 194) + (2*sqrt(3) + 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3)
+ 7))*log(-194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x - 3
62*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - 194) + 4*sqrt(3)*sqrt
((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/(2*(sqrt(3) + 2)*sqrt((9
7*x^2 + 56*sqrt(3)*(x^2 + 1) + (209*sqrt(3)*x + 362*x)*sqrt((sqrt
(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt((sqrt(3)
+ 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(3) + 2)/(4
*sqrt(3) + 7)) + sqrt(3) + 2)) + 4*sqrt(3)*sqrt((sqrt(3) - 2)/(4*
sqrt(3) - 7))*arctan(1/(2*(sqrt(3) + 2)*sqrt((97*x^2 + 56*sqrt(3)
*(x^2 + 1) - (209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3
) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt((sqrt(3) + 2)/(4*sqrt(3) +
7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) - s
qrt(3) - 2)))/((sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*
sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.89696, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(9216t^5-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_
t**5 - 8*_t + x)))
```

GIAC/XCAS [A] time = 0.28578, size = 342, normalized size = 0.96

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*ar
ctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(
6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(
2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + s
qrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6)
 - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqr
t(6) - sqrt(2)) + 1)
```

3.26 $\int \frac{1-x^4}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=13

$$\frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + ArcTanh[x]/2
```

Rubi [A] time = 0.0109466, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] ArcTan[x]/2 + ArcTanh[x]/2
```

Rubi in Sympy [A] time = 3.97275, size = 8, normalized size = 0.62

$$\frac{\operatorname{atan}(x)}{2} + \frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8-2*x**4+1),x)
```

```
[Out] atan(x)/2 + atanh(x)/2
```

Mathematica [A] time = 0.00645278, size = 25, normalized size = 1.92

$$-\frac{1}{4}\log(1-x) + \frac{1}{4}\log(x+1) + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 - 2*x^4 + x^8),x]
```

```
[Out] ArcTan[x]/2 - Log[1 - x]/4 + Log[1 + x]/4
```

Maple [A] time = 0.002, size = 10, normalized size = 0.8

$$\frac{\arctan(x)}{2} + \frac{\mathit{Artanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8-2*x^4+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)
```

Maxima [A] time = 0.817391, size = 23, normalized size = 1.77

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 2*x^4 + 1),x, algorithm="maxima")

[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 0.267531, size = 23, normalized size = 1.77

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 2*x^4 + 1),x, algorithm="fricas")

[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Sympy [A] time = 0.353536, size = 17, normalized size = 1.31

$$-\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8-2*x**4+1),x)

[Out] -log(x - 1)/4 + log(x + 1)/4 + atan(x)/2
```

GIAC/XCAS [A] time = 0.266856, size = 26, normalized size = 2.

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\ln(|x+1|) - \frac{1}{4}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 2*x^4 + 1),x, algorithm="giac")

[Out] 1/2*arctan(x) + 1/4*ln(abs(x + 1)) - 1/4*ln(abs(x - 1))
```

3.27 $\int \frac{1-x^4}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=129

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan
[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2
/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 +
 Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])]
```

Rubi [A] time = 0.243565, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan
[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2
/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 +
 Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])]
```

Rubi in Sympy [A] time = 13.3094, size = 141, normalized size = 1.09

$$\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-1+\sqrt{5}}}\right)}{10\sqrt{-1+\sqrt{5}}}+\frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{1+\sqrt{5}}}\right)}{10\sqrt{1+\sqrt{5}}}+\frac{\sqrt{10}\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+\sqrt{5}}}\right)}{10\sqrt{-1+\sqrt{5}}}+\frac{\sqrt{10}\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{1+\sqrt{5}}}\right)}{10\sqrt{1+\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(10)*atan(sqrt(2)*x/sqrt(-1 + sqrt(5)))/(10*sqrt(-1 + sqrt(5)
)) + sqrt(10)*atan(sqrt(2)*x/sqrt(1 + sqrt(5)))/(10*sqrt(1 + sqrt
(5))) + sqrt(10)*atanh(sqrt(2)*x/sqrt(-1 + sqrt(5)))/(10*sqrt(-1
+ sqrt(5))) + sqrt(10)*atanh(sqrt(2)*x/sqrt(1 + sqrt(5)))/(10*sqr
t(1 + sqrt(5)))
```

Mathematica [A] time = 0.122379, size = 129, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10\left(\sqrt{5}-1\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(1 - x^4)/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTan
[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2
/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] + ArcTanh[Sqrt[2/(1 +
 Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])]
```

Maple [A] time = 0.034, size = 110, normalized size = 0.9

$$\frac{\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)+\frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}Artanh\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{\sqrt{5}}{5\sqrt{-2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{\sqrt{5}}{5\sqrt{2\sqrt{5}+2}}Artanh\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^4+1)/(x^8-3*x^4+1),x)
```

```
[Out] 1/5*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))+1
/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))
+1/5*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*x/(-2+2*5^(1/2))^(1/2)
)+1/5*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4-1}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-(x^4 - 1)/(x^8 - 3*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 3*x^4 + 1), x)
```

Fricas [A] time = 0.280235, size = 401, normalized size = 3.11

$$-\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\left(\sqrt{5}+5\right)}{10\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2+1)+5\right)}+x\right)}\right)+\frac{1}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\left(\sqrt{5}-5\right)}{10\left(\sqrt{\frac{1}{10}}\sqrt{\sqrt{5}\left(\sqrt{5}(2x^2-1)+5\right)}+x\right)}\right)+\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\log\left(\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\left(\sqrt{5}+5\right)+x\right)-\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\log\left(-\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{-\sqrt{5}\left(\sqrt{5}-5\right)}\left(\sqrt{5}+5\right)+x\right)-\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\log\left(\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\left(\sqrt{5}-5\right)+x\right)+\frac{1}{20}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\log\left(-\frac{1}{10}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}\left(\sqrt{5}+5\right)}\left(\sqrt{5}-5\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 3*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*arctan(1/10*sqrt(1/2)
*sqrt(-sqrt(5)*(sqrt(5) - 5))*(sqrt(5) + 5)/(sqrt(1/10)*sqrt(sqrt
(5)*(sqrt(5)*(2*x^2 + 1) + 5)) + x)) + 1/5*sqrt(1/2)*sqrt(sqrt(5)
*(sqrt(5) + 5))*arctan(1/10*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))
*(sqrt(5) - 5)/(sqrt(1/10)*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 - 1) + 5)
) + x)) + 1/20*sqrt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*log(1/10*sq
rt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*(sqrt(5) + 5) + x) - 1/20*sq
rt(1/2)*sqrt(-sqrt(5)*(sqrt(5) - 5))*log(-1/10*sqrt(1/2)*sqrt(-sq
rt(5)*(sqrt(5) - 5))*(sqrt(5) + 5) + x) - 1/20*sqrt(1/2)*sqrt(sqr
t(5)*(sqrt(5) + 5))*log(1/10*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5)
)*(sqrt(5) - 5) + x) + 1/20*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))
*log(-1/10*sqrt(1/2)*sqrt(sqrt(5)*(sqrt(5) + 5))*(sqrt(5) - 5) +
x)
```

Sympy [A] time = 3.18749, size = 51, normalized size = 0.4

$$-\operatorname{RootSum}\left(6400t^4 - 80t^2 - 1, \left(t \mapsto t\log\left(25600t^5 - 16t + x\right)\right)\right) - \operatorname{RootSum}\left(6400t^4 + 80t^2 - 1, \left(t \mapsto t\log\left(25600t^5 - 16t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8-3*x**4+1),x)
```

```
[Out] -RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(25600*_t**5
 - 16*_t + x))) - RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _
t*log(25600*_t**5 - 16*_t + x)))
```

GIAC/XCAS [A] time = 0.343793, size = 198, normalized size = 1.53

$$\frac{1}{20}\sqrt{10\sqrt{5} - 10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{20}\sqrt{10\sqrt{5} + 10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}}\right) + \frac{1}{40}\sqrt{10\sqrt{5} - 10}\ln\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5} - 10}\ln\left(\left|x - \sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5} + 10}\ln\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5} + 10}\ln\left(\left|x - \sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 3*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/
20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) + 1/40
*sqrt(10*sqrt(5) - 10)*ln(abs(x + sqrt(1/2*sqrt(5) + 1/2))) - 1/4
0*sqrt(10*sqrt(5) - 10)*ln(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/
40*sqrt(10*sqrt(5) + 10)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1
/40*sqrt(10*sqrt(5) + 10)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2)))
```

3.28 $\int \frac{1-x^4}{1-4x^4+x^8}\,dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sq
rt[3])]) + ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[
3*(1 + Sqrt[3])]) + ArcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^
(1/4)*Sqrt[3*(-1 + Sqrt[3])]) + ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt
[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])])
```

Rubi [A] time = 0.211679, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{\sqrt{3}-1}}\right)}{2\sqrt[4]{2}\sqrt{3\left(\sqrt{3}-1\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+\sqrt{3}}}\right)}{2\sqrt[4]{2}\sqrt{3\left(1+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[3*(-1 + Sq
rt[3])]) + ArcTan[(2^(1/4)*x)/Sqrt[1 + Sqrt[3]]]/(2*2^(1/4)*Sqrt[
3*(1 + Sqrt[3])]) + ArcTanh[(2^(1/4)*x)/Sqrt[-1 + Sqrt[3]]]/(2*2^
(1/4)*Sqrt[3*(-1 + Sqrt[3])]) + ArcTanh[(2^(1/4)*x)/Sqrt[1 + Sqrt
[3]]]/(2*2^(1/4)*Sqrt[3*(1 + Sqrt[3])])
```

Rubi in Sympy [A] time = 17.078, size = 168, normalized size = 1.02

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{2}+\sqrt{6}}}\right)}{6\sqrt{-\sqrt{2}+\sqrt{6}}}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{2}+\sqrt{6}}}\right)}{6\sqrt{\sqrt{2}+\sqrt{6}}}+\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{2}+\sqrt{6}}}\right)}{6\sqrt{-\sqrt{2}+\sqrt{6}}}+\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{2}+\sqrt{6}}}\right)}{6\sqrt{\sqrt{2}+\sqrt{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] sqrt(3)*atan(sqrt(2)*x/sqrt(-sqrt(2) + sqrt(6)))/(6*sqrt(-sqrt(2)
 + sqrt(6))) + sqrt(3)*atan(sqrt(2)*x/sqrt(sqrt(2) + sqrt(6)))/(6
*sqrt(sqrt(2) + sqrt(6))) + sqrt(3)*atanh(sqrt(2)*x/sqrt(-sqrt(2)
 + sqrt(6)))/(6*sqrt(-sqrt(2) + sqrt(6))) + sqrt(3)*atanh(sqrt(2)
*x/sqrt(sqrt(2) + sqrt(6)))/(6*sqrt(sqrt(2) + sqrt(6)))
```

Mathematica [C] time = 0.0206923, size = 55, normalized size = 0.33

$$-\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{\#1^7-2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 - 4*x^4 + x^8),x]
```

```
[Out] -RootSum[1 - 4*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(-2*#1^3 + #1^7) & ]/8
```

Maple [C] time = 0.01, size = 42, normalized size = 0.3

$$\frac{1}{8}\sum_{\_R=RootOf(\_Z^8-4\_Z^4+1)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{\_R^7-2\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8-4*x^4+1),x)
```

```
[Out] 1/8*sum((-_R^4+1)/(_R^7-2*_R^3)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^4+1)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^4-1}{x^8-4x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 4*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

Fricas [A] time = 0.300408, size = 581, normalized size = 3.52

$$\begin{aligned}
&-\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}\arctan\left(\frac{3\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}\left(\sqrt{3}+1\right)}{\sqrt{3}x+\sqrt{3}\sqrt{x^2+\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}\left(\sqrt{3}+2\right)}}\right)\\
&-\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}}\arctan\left(\frac{3\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}}\left(\sqrt{3}-1\right)}{\sqrt{3}x+\sqrt{3}\sqrt{x^2-\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}\left(\sqrt{3}-2\right)}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}\log\left(3\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}\left(\sqrt{3}+1\right)+\sqrt{3}x\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}\log\left(-3\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}\left(\sqrt{3}+1\right)+\sqrt{3}x\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}}\log\left(3\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}}\left(\sqrt{3}-1\right)+\sqrt{3}x\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}}\log\left(-3\sqrt{\frac{1}{6}}\sqrt{\sqrt{\frac{1}{3}}\sqrt{\sqrt{3}\left(2\sqrt{3}+3\right)}}\left(\sqrt{3}-1\right)+\sqrt{3}x\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 4*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*sqrt(3) - 3)))*arctan(3
*sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*sqrt(3) - 3)))*(sqrt(3)
 + 1)/(sqrt(3)*x + sqrt(3)*sqrt(x^2 + sqrt(1/3)*sqrt(sqrt(3)*(2*s
qrt(3) - 3))*(sqrt(3) + 2)))) - sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqr
t(3)*(2*sqrt(3) + 3)))*arctan(3*sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqr
t(3)*(2*sqrt(3) + 3)))*(sqrt(3) - 1)/(sqrt(3)*x + sqrt(3)*sqrt(x^
2 - sqrt(1/3)*sqrt(sqrt(3)*(2*sqrt(3) + 3))*(sqrt(3) - 2)))) + 1/
4*sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*sqrt(3) - 3)))*log(3*s
qrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*sqrt(3) - 3)))*(sqrt(3) +
 1) + sqrt(3)*x) - 1/4*sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*s
qrt(3) - 3)))*log(-3*sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*sqr
t(3) - 3)))*(sqrt(3) + 1) + sqrt(3)*x) + 1/4*sqrt(1/6)*sqrt(sqrt(
1/3)*sqrt(sqrt(3)*(2*sqrt(3) + 3)))*log(3*sqrt(1/6)*sqrt(sqrt(1/3
)*sqrt(sqrt(3)*(2*sqrt(3) + 3)))*(sqrt(3) - 1) + sqrt(3)*x) - 1/4
*sqrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*sqrt(3) + 3)))*log(-3*s
qrt(1/6)*sqrt(sqrt(1/3)*sqrt(sqrt(3)*(2*sqrt(3) + 3)))*(sqrt(3) -
 1) + sqrt(3)*x)
```

Sympy [A] time = 0.583771, size = 26, normalized size = 0.16

$$-\operatorname{RootSum}\left(84934656t^{8}-36864t^{4}+1,\left(t\mapsto t\log\left(36864t^{5}-20t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8-4*x**4+1),x)
```

```
[Out] -RootSum(84934656*_t**8 - 36864*_t**4 + 1, Lambda(_t, _t*log(3686
4*_t**5 - 20*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^{4}-1}{x^{8}-4\,x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 4*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 - 4*x^4 + 1), x)
```

3.29 $\int \frac{1-x^4}{1-5x^4+x^8}\,dx$

Optimal. Leaf size=169

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7
])] + ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sq
rt[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[
3] + Sqrt[7])] + ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(
Sqrt[3] + Sqrt[7])]
```

Rubi [A] time = 0.292635, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-\sqrt{3}}}x\right)}{\sqrt{14\left(\sqrt{7}-\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{3}+\sqrt{7}}}x\right)}{\sqrt{14\left(\sqrt{3}+\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[3] + Sqrt[7
])] + ArcTan[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(Sqrt[3] + Sq
rt[7])] + ArcTanh[Sqrt[2/(-Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(-Sqrt[
3] + Sqrt[7])] + ArcTanh[Sqrt[2/(Sqrt[3] + Sqrt[7])]*x]/Sqrt[14*(
Sqrt[3] + Sqrt[7])]
```

Rubi in Sympy [A] time = 18.8649, size = 168, normalized size = 0.99

$$\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{14\sqrt{-\sqrt{3}+\sqrt{7}}}+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{14\sqrt{\sqrt{3}+\sqrt{7}}}+\frac{\sqrt{14}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-\sqrt{3}+\sqrt{7}}}\right)}{14\sqrt{-\sqrt{3}+\sqrt{7}}}+\frac{\sqrt{14}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{3}+\sqrt{7}}}\right)}{14\sqrt{\sqrt{3}+\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] sqrt(14)*atan(sqrt(2)*x/sqrt(-sqrt(3) + sqrt(7)))/(14*sqrt(-sqrt(
3) + sqrt(7))) + sqrt(14)*atan(sqrt(2)*x/sqrt(sqrt(3) + sqrt(7)))
/(14*sqrt(sqrt(3) + sqrt(7))) + sqrt(14)*atanh(sqrt(2)*x/sqrt(-sq
rt(3) + sqrt(7)))/(14*sqrt(-sqrt(3) + sqrt(7))) + sqrt(14)*atanh(
sqrt(2)*x/sqrt(sqrt(3) + sqrt(7)))/(14*sqrt(sqrt(3) + sqrt(7)))
```

Mathematica [C] time = 0.0207839, size = 57, normalized size = 0.34

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-5\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-5\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 - 5*x^4 + x^8),x]
```

```
[Out] -RootSum[1 - 5*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(-5*#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.01, size = 44, normalized size = 0.3

$$\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^8-5\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-5\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8-5*x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-5*_R^3)*ln(x-_R),_R=RootOf(_Z^8-5*_Z^4+
1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4-1}{x^8-5\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 5*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

Fricas [A] time = 0.310394, size = 693, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 5*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) - 7*sqrt(3))))
*arctan(7/2*sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) - 7
*sqrt(3))))*(sqrt(7) + sqrt(3))/(sqrt(7)*x + sqrt(7)*sqrt(1/2*sqr
t(1/14)*sqrt(sqrt(7)*(5*sqrt(7) - 7*sqrt(3)))*(sqrt(7)*sqrt(3) +
5) + x^2))) - sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) +
 7*sqrt(3))))*arctan(7/2*sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(
5*sqrt(7) + 7*sqrt(3))))*(sqrt(7) - sqrt(3))/(sqrt(7)*x + sqrt(7)
*sqrt(-1/2*sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) + 7*sqrt(3)))*(sqrt
(7)*sqrt(3) - 5) + x^2))) + 1/4*sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sq
rt(7)*(5*sqrt(7) - 7*sqrt(3))))*log(7/2*sqrt(1/7)*sqrt(sqrt(1/14)
*sqrt(sqrt(7)*(5*sqrt(7) - 7*sqrt(3))))*(sqrt(7) + sqrt(3)) + sqr
t(7)*x) - 1/4*sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) -
 7*sqrt(3))))*log(-7/2*sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(5*
sqrt(7) - 7*sqrt(3))))*(sqrt(7) + sqrt(3)) + sqrt(7)*x) + 1/4*sqr
t(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) + 7*sqrt(3))))*log
(7/2*sqrt(1/7)*sqrt(sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) + 7*sqrt(3
))))*(sqrt(7) - sqrt(3)) + sqrt(7)*x) - 1/4*sqrt(1/7)*sqrt(sqrt(1
/14)*sqrt(sqrt(7)*(5*sqrt(7) + 7*sqrt(3))))*log(-7/2*sqrt(1/7)*sq
rt(sqrt(1/14)*sqrt(sqrt(7)*(5*sqrt(7) + 7*sqrt(3))))*(sqrt(7) - s
qrt(3)) + sqrt(7)*x)
```

Sympy [A] time = 0.57599, size = 26, normalized size = 0.15

$$-\operatorname{RootSum}\left(157351936t^8 - 62720t^4 + 1, \left(t \mapsto t \log\left(50176t^5 - 24t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8-5*x**4+1),x)
```

```
[Out] -RootSum(157351936*_t**8 - 62720*_t**4 + 1, Lambda(_t, _t*log(501
76*_t**5 - 24*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4 - 1}{x^8 - 5\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 5*x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(x^8 - 5*x^4 + 1), x)
```

3.30 $\int \frac{1-x^4}{1-6x^4+x^8} \, dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[2*(-1 + Sqrt[2])]) + ArcTan[
x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-
1 + Sqrt[2]]]/(4*Sqrt[2*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqr
t[2]]]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rubi [A] time = 0.146701, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[2*(-1 + Sqrt[2])]) + ArcTan[
x/Sqrt[1 + Sqrt[2]]]/(4*Sqrt[2*(1 + Sqrt[2])]) + ArcTanh[x/Sqrt[-
1 + Sqrt[2]]]/(4*Sqrt[2*(-1 + Sqrt[2])]) + ArcTanh[x/Sqrt[1 + Sqr
t[2]]]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rubi in Sympy [A] time = 10.2948, size = 121, normalized size = 0.97

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{8\sqrt{-1+\sqrt{2}}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\operatorname{atanh}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)}{8\sqrt{-1+\sqrt{2}}}+\frac{\sqrt{2}\operatorname{atanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{8\sqrt{1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] sqrt(2)*atan(x/sqrt(-1 + sqrt(2)))/(8*sqrt(-1 + sqrt(2))) + sqrt(
2)*atan(x/sqrt(1 + sqrt(2)))/(8*sqrt(1 + sqrt(2))) + sqrt(2)*atan
h(x/sqrt(-1 + sqrt(2)))/(8*sqrt(-1 + sqrt(2))) + sqrt(2)*atanh(x/
sqrt(1 + sqrt(2)))/(8*sqrt(1 + sqrt(2)))
```

Mathematica [A] time = 0.0849075, size = 114, normalized size = 0.91

$$\frac{\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]] + Sqrt[-1 + Sqrt[
2]]*ArcTan[x/Sqrt[1 + Sqrt[2]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[x/Sqr
t[-1 + Sqrt[2]]] + Sqrt[-1 + Sqrt[2]]*ArcTanh[x/Sqrt[1 + Sqrt[2]]
])/(4*Sqrt[2])
```

Maple [A] time = 0.034, size = 90, normalized size = 0.7

$$\frac{\sqrt{2}}{8\sqrt{\sqrt{2}-1}}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)+\frac{\sqrt{2}}{8\sqrt{1+\sqrt{2}}}Artanh\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\frac{\sqrt{2}}{8\sqrt{1+\sqrt{2}}}\arctan\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)+\frac{\sqrt{2}}{8\sqrt{\sqrt{2}-1}}Artanh\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8-6*x^4+1),x)
```

```
[Out] 1/8*2^(1/2)/(2^(1/2)-1)^(1/2)*arctan(x/(2^(1/2)-1)^(1/2))+1/8*2^(
1/2)/(1+2^(1/2))^(1/2)*arctanh(x/(1+2^(1/2))^(1/2))+1/8*2^(1/2)/(
1+2^(1/2))^(1/2)*arctan(x/(1+2^(1/2))^(1/2))+1/8*2^(1/2)/(2^(1/2)
-1)^(1/2)*arctanh(x/(2^(1/2)-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4-1}{x^8-6\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 6*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - 6*x^4 + 1), x)
```

Fricas [A] time = 0.289652, size = 347, normalized size = 2.78

$$-\frac{1}{4}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\arctan\left(\frac{\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\left(\sqrt{2}+2\right)}{2\left(\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}(x^2+1)+2\right)}+x\right)}\right)+\frac{1}{4}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\arctan\left(\frac{\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\left(\sqrt{2}-2\right)}{2\left(\sqrt{\frac{1}{2}}\sqrt{\sqrt{2}\left(\sqrt{2}(x^2-1)+2\right)}+x\right)}\right)+\frac{1}{16}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\log\left(\frac{1}{2}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\left(\sqrt{2}+2\right)+x\right)-\frac{1}{16}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\log\left(-\frac{1}{2}\sqrt{-\sqrt{2}\left(\sqrt{2}-2\right)}\left(\sqrt{2}+2\right)+x\right)-\frac{1}{16}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\log\left(\frac{1}{2}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\left(\sqrt{2}-2\right)+x\right)+\frac{1}{16}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\log\left(-\frac{1}{2}\sqrt{\sqrt{2}\left(\sqrt{2}+2\right)}\left(\sqrt{2}-2\right)+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 6*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-sqrt(2)*(sqrt(2) - 2))*arctan(1/2*sqrt(-sqrt(2)*(sqrt(
2) - 2))*(sqrt(2) + 2)/(sqrt(1/2)*sqrt(sqrt(2)*(sqrt(2)*(x^2 + 1)
 + 2)) + x)) + 1/4*sqrt(sqrt(2)*(sqrt(2) + 2))*arctan(1/2*sqrt(sq
rt(2)*(sqrt(2) + 2))*(sqrt(2) - 2)/(sqrt(1/2)*sqrt(sqrt(2)*(sqrt(
2)*(x^2 - 1) + 2)) + x)) + 1/16*sqrt(-sqrt(2)*(sqrt(2) - 2))*log(
1/2*sqrt(-sqrt(2)*(sqrt(2) - 2))*(sqrt(2) + 2) + x) - 1/16*sqrt(-
sqrt(2)*(sqrt(2) - 2))*log(-1/2*sqrt(-sqrt(2)*(sqrt(2) - 2))*(sqr
t(2) + 2) + x) - 1/16*sqrt(sqrt(2)*(sqrt(2) + 2))*log(1/2*sqrt(sq
rt(2)*(sqrt(2) + 2))*(sqrt(2) - 2) + x) + 1/16*sqrt(sqrt(2)*(sqrt
(2) + 2))*log(-1/2*sqrt(sqrt(2)*(sqrt(2) + 2))*(sqrt(2) - 2) + x)
```

Sympy [A] time = 3.15438, size = 51, normalized size = 0.41

$$-\operatorname{RootSum}\left(16384t^4 - 256t^2 - 1, \left(t \mapsto t\log\left(65536t^5 - 28t + x\right)\right)\right) - \operatorname{RootSum}\left(16384t^4 + 256t^2 - 1, \left(t \mapsto t\log\left(65536t^5 - 28t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8-6*x**4+1),x)
```

```
[Out] -RootSum(16384*_t**4 - 256*_t**2 - 1, Lambda(_t, _t*log(65536*_t*
*5 - 28*_t + x))) - RootSum(16384*_t**4 + 256*_t**2 - 1, Lambda(_
t, _t*log(65536*_t**5 - 28*_t + x)))
```

GIAC/XCAS [A] time = 0.346548, size = 182, normalized size = 1.46

$$\frac{1}{8}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}+1}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \frac{1}{16}\sqrt{2\sqrt{2}-2}\ln\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right)-\frac{1}{16}\sqrt{2\sqrt{2}-2}\ln\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right) + \frac{1}{16}\sqrt{2\sqrt{2}+2}\ln\left(\left|x+\sqrt{\sqrt{2}-1}\right|\right)-\frac{1}{16}\sqrt{2\sqrt{2}+2}\ln\left(\left|x-\sqrt{\sqrt{2}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - 6*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2*sqrt(2) - 2)*arctan(x/sqrt(sqrt(2) + 1)) + 1/8*sqrt(2*
sqrt(2) + 2)*arctan(x/sqrt(sqrt(2) - 1)) + 1/16*sqrt(2*sqrt(2) -
2)*ln(abs(x + sqrt(sqrt(2) + 1))) - 1/16*sqrt(2*sqrt(2) - 2)*ln(a
bs(x - sqrt(sqrt(2) + 1))) + 1/16*sqrt(2*sqrt(2) + 2)*ln(abs(x +
sqrt(sqrt(2) - 1))) - 1/16*sqrt(2*sqrt(2) + 2)*ln(abs(x - sqrt(sq
rt(2) - 1)))
```

3.31 $\int \frac{-1+\sqrt{3}+2x^4}{1-x^4+x^8} \, dx$

Optimal. Leaf size=135

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2]) +
ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - Log
[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2 - Sq
rt[3]]*x + x^2]/(2*Sqrt[2])
```

Rubi [A] time = 0.250058, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2]) +
ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - Log
[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2 - Sq
rt[3]]*x + x^2]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 51.7622, size = 202, normalized size = 1.5

$$\frac{\left(-\sqrt{3}+1\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{4\sqrt{-\sqrt{3}+2}}-\frac{\left(-\sqrt{3}+1\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{4\sqrt{-\sqrt{3}+2}}+\frac{\sqrt{2}\left(-\sqrt{3}+3\right)^2\operatorname{atan}\left(\frac{\sqrt{6}\left(x\left(-\frac{\sqrt{3}}{3}+1\right)-\frac{\left(-3+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{6}\right)}{-\sqrt{3}+2}\right)}{12\left(-\sqrt{3}+2\right)}+\frac{\sqrt{2}\left(-\sqrt{3}+3\right)^2\operatorname{atan}\left(\frac{\sqrt{6}\left(x\left(-\frac{\sqrt{3}}{3}+1\right)+\frac{\left(-3+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{6}\right)}{-\sqrt{3}+2}\right)}{12\left(-\sqrt{3}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+2*x**4+3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] (-sqrt(3) + 1)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(4*sqrt(-sqrt
(3) + 2)) - (-sqrt(3) + 1)*log(x**2 + x*sqrt(-sqrt(3) + 2) + 1)/(
4*sqrt(-sqrt(3) + 2)) + sqrt(2)*(-sqrt(3) + 3)**2*atan(sqrt(6)*(x
*(-sqrt(3)/3 + 1) - (-3 + sqrt(3))*sqrt(sqrt(3) + 2)/6)/(-sqrt(3)
 + 2))/(12*(-sqrt(3) + 2)) + sqrt(2)*(-sqrt(3) + 3)**2*atan(sqrt(
6)*(x*(-sqrt(3)/3 + 1) + (-3 + sqrt(3))*sqrt(sqrt(3) + 2)/6)/(-sq
rt(3) + 2))/(12*(-sqrt(3) + 2))
```

Mathematica [C] time = 0.0522052, size = 71, normalized size = 0.53

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{2\#1^4\log(x-\#1)+\sqrt{3}\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + Sqrt[3] + 2*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Sqrt[3]*Log[x - #1] +
 2*Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.076, size = 47, normalized size = 0.4

$$\frac{1}{4}\sum_{\_R=RootOf(\_Z^8-\_Z^4+1)}\frac{\left(-1+2\_R^4+\sqrt{3}\right)\ln(x-\_R)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+2*x^4+3^(1/2))/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(1/(2*_R^7-_R^3)*(-1+2*_R^4+3^(1/2))*ln(x-_R),_R=RootOf(_Z
^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^4+\sqrt{3}-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.84787, size = 136, normalized size = 1.01

$$\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{x\left(\sqrt{6}+2\sqrt{2}\right)}{1+\sqrt{3}}\right)+2\operatorname{atan}\left(\frac{x^3\left(\sqrt{6}+2\sqrt{2}\right)}{1+\sqrt{3}}-\sqrt{2}x\right)\right)}{4}-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x\left(2+2\sqrt{3}\right)}{4\left(\sqrt{3}+2\right)}+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x\left(2+2\sqrt{3}\right)}{4\left(\sqrt{3}+2\right)}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+2*x**4+3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(x*(sqrt(6) + 2*sqrt(2))/(1 + sqrt(3))) + 2*atan(x
**3*(sqrt(6) + 2*sqrt(2))/(1 + sqrt(3)) - sqrt(2)*x))/4 - sqrt(2)
*log(x**2 - sqrt(2)*x*(2 + 2*sqrt(3))/(4*(sqrt(3) + 2)) + 1)/4 +
sqrt(2)*log(x**2 + sqrt(2)*x*(2 + 2*sqrt(3))/(4*(sqrt(3) + 2)) +
1)/4
```

GIAC/XCAS [A] time = 0.291177, size = 144, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$
$$+\frac{1}{4}\sqrt{2}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{4}\sqrt{2}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^4 + sqrt(3) - 1)/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2)))
 + 1/2*sqrt(2)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2
))) + 1/4*sqrt(2)*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/4*s
qrt(2)*ln(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.32 $\int \frac{1+\left(1+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=164

$$-\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] -(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqr
t[3]]])/2 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/S
qrt[2 - Sqrt[3]]])/2 - (Sqrt[2 + Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3
]]*x + x^2])/4 + (Sqrt[2 + Sqrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x +
 x^2])/4
```

Rubi [A] time = 0.210683, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{4}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqr
t[3]]])/2 + (Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/S
qrt[2 - Sqrt[3]]])/2 - (Sqrt[2 + Sqrt[3]]*Log[1 - Sqrt[2 - Sqrt[3
]]*x + x^2])/4 + (Sqrt[2 + Sqrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x +
 x^2])/4
```

Rubi in Sympy [A] time = 44.5854, size = 165, normalized size = 1.01

$$-\frac{\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{4\sqrt{-\sqrt{3}+2}}+\frac{\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{4\sqrt{-\sqrt{3}+2}}+\frac{\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3\sqrt{3}+6}}{3}\right)}{\sqrt{-\sqrt{3}+2}}\right)}{2\sqrt{-\sqrt{3}+2}}+\frac{\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3\sqrt{3}+6}}{3}\right)}{\sqrt{-\sqrt{3}+2}}\right)}{2\sqrt{-\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**4*(1+3**(1/2)))/(x**8-x**4+1),x)
```

```
[Out] -log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(4*sqrt(-sqrt(3) + 2)) + lo
g(x**2 + x*sqrt(-sqrt(3) + 2) + 1)/(4*sqrt(-sqrt(3) + 2)) + atan(
sqrt(3)*(2*sqrt(3)*x/3 - sqrt(3*sqrt(3) + 6)/3)/sqrt(-sqrt(3) + 2
))/(2*sqrt(-sqrt(3) + 2)) + atan(sqrt(3)*(2*sqrt(3)*x/3 + sqrt(3*
sqrt(3) + 6)/3)/sqrt(-sqrt(3) + 2))/(2*sqrt(-sqrt(3) + 2))
```

Mathematica [C] time = 0.0534861, size = 72, normalized size = 0.44

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\sqrt{3}\#1^4\log(x-\#1)+\#1^4\log(x-\#1)+\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (1 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4 + Sqr
t[3]*Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.054, size = 62, normalized size = 0.4

$$\frac{1}{8}\sum_{\_R=RootOf(\_Z^8-\_Z^4+1)}\frac{\left(2\_R^4+2\sqrt{3}\_R^4+\left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^4*(1+3^(1/2)))/(x^8-x^4+1),x)
```

```
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(2*_R^4+2*3^(1/2)*_R^4+(1+3^(1/2))*(3^(1/
2)-1))*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\left(\sqrt{3}+1\right)+1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**4*(1+3**(1/2)))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError
```

GIAC/XCAS [A] time = 0.297529, size = 166, normalized size = 1.01

$$\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\ln\left(x^{2}+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{8}\left(\sqrt{6}+\sqrt{2}\right)\ln\left(x^{2}-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(sqrt(3) + 1) + 1)/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(6) + sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6)
 - sqrt(2))) + 1/4*(sqrt(6) + sqrt(2))*arctan((4*x - sqrt(6) - sq
rt(2))/(sqrt(6) - sqrt(2))) + 1/8*(sqrt(6) + sqrt(2))*ln(x^2 + 1/
2*x*(sqrt(6) - sqrt(2)) + 1) - 1/8*(sqrt(6) + sqrt(2))*ln(x^2 - 1
/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.33 $\int \frac{3-2\sqrt{3}+\left(-3+\sqrt{3}\right)x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=180

$$\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 -
Sqrt[3]]])/2 - (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] +
 2*x)/Sqrt[2 - Sqrt[3]]])/2 + (Sqrt[3*(2 - Sqrt[3])]*Log[1 - Sqrt
[2 - Sqrt[3]]*x + x^2])/4 - (Sqrt[3*(2 - Sqrt[3])]*Log[1 + Sqrt[2
 - Sqrt[3]]*x + x^2])/4
```

Rubi [A] time = 0.27579, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\sqrt{3\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 -
Sqrt[3]]])/2 - (Sqrt[3*(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] +
 2*x)/Sqrt[2 - Sqrt[3]]])/2 + (Sqrt[3*(2 - Sqrt[3])]*Log[1 - Sqrt
[2 - Sqrt[3]]*x + x^2])/4 - (Sqrt[3*(2 - Sqrt[3])]*Log[1 + Sqrt[2
 - Sqrt[3]]*x + x^2])/4
```

Rubi in Sympy [A] time = 54.4499, size = 216, normalized size = 1.2

$$-\frac{\left(-2\sqrt{3}+3\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{4\sqrt{-\sqrt{3}+2}}+\frac{\left(-2\sqrt{3}+3\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{4\sqrt{-\sqrt{3}+2}}+\frac{\sqrt{3}\left(-3\sqrt{3}+6\right)^2\operatorname{atan}\left(\frac{x\left(-4+2\sqrt{3}\right)-\left(-2+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{\sqrt{-15\sqrt{3}+26}}\right)}{18\sqrt{-15\sqrt{3}+26}}+\frac{\sqrt{3}\left(-3\sqrt{3}+6\right)^2\operatorname{atan}\left(\frac{x\left(-4+2\sqrt{3}\right)+\left(-2+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{\sqrt{-15\sqrt{3}+26}}\right)}{18\sqrt{-15\sqrt{3}+26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+x**4*(-3+3**(1/2))-2*3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] -(-2*sqrt(3) + 3)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(4*sqrt(-s
qrt(3) + 2)) + (-2*sqrt(3) + 3)*log(x**2 + x*sqrt(-sqrt(3) + 2) +
 1)/(4*sqrt(-sqrt(3) + 2)) + sqrt(3)*(-3*sqrt(3) + 6)**2*atan((x*
(-4 + 2*sqrt(3)) - (-2 + sqrt(3))*sqrt(sqrt(3) + 2))/sqrt(-15*sqr
t(3) + 26))/(18*sqrt(-15*sqrt(3) + 26)) + sqrt(3)*(-3*sqrt(3) + 6
)**2*atan((x*(-4 + 2*sqrt(3)) + (-2 + sqrt(3))*sqrt(sqrt(3) + 2))
/sqrt(-15*sqrt(3) + 26))/(18*sqrt(-15*sqrt(3) + 26))
```

Mathematica [C] time = 0.0687416, size = 89, normalized size = 0.49

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\sqrt{3}\#1^4\log(x-\#1)-3\#1^4\log(x-\#1)-2\sqrt{3}\log(x-\#1)+3\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 2*Sqrt[3] + (-3 + Sqrt[3])*x^4)/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (3*Log[x - #1] - 2*Sqrt[3]*Log[x - #1
] - 3*Log[x - #1]*#1^4 + Sqrt[3]*Log[x - #1]*#1^4)/(-#1^3 + 2*#1^
7) & ]/4
```

Maple [C] time = 0.013, size = 62, normalized size = 0.3

$$\frac{1}{8}\sum_{\_R=RootOf(\_Z^8-\_Z^4+1)}\frac{\left(-6\_R^4+2\sqrt{3}\_R^4+\left(-3+\sqrt{3}\right)\left(\sqrt{3}-1\right)\right)\ln(x-\_R)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+x^4*(-3+3^(1/2))-2*3^(1/2))/(x^8-x^4+1),x)
```

```
[Out] 1/8*sum(1/(2*_R^7-_R^3)*(-6*_R^4+2*3^(1/2)*_R^4+(-3+3^(1/2))*(3^(
1/2)-1))*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\left(\sqrt{3}-3\right)-2\sqrt{3}+3}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1),x, algorithm="maxima
```

```
[Out] integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1),x, algorithm="fricas
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+x**4*(-3+3**(1/2))-2*3**(1/2))/(x**8-x**4+1),x)
```

```
[Out] Exception raised: PolynomialError
```

GIAC/XCAS [A] time = 0.290372, size = 177, normalized size = 0.98

$$\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{8}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4*(sqrt(3) - 3) - 2*sqrt(3) + 3)/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(
6) - sqrt(2))) + 1/4*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6)
- sqrt(2))/(sqrt(6) - sqrt(2))) + 1/8*(sqrt(6) - 3*sqrt(2))*ln(x^
2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/8*(sqrt(6) - 3*sqrt(2))*ln
(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.34 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}+\frac{dx}{c}$$

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Lo
g[a + c*x^2])/(2*c)
```

Rubi [A] time = 0.0844758, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x)/(c + a/x^2),x]
```

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Lo
g[a + c*x^2])/(2*c)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{a}\,d\,\text{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}+\frac{e\log\left(a+cx^2\right)}{2c}+\frac{\int d\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x)/(c+a/x**2),x)
```

```
[Out] -sqrt(a)*d*atan(sqrt(c)*x/sqrt(a))/c**(3/2) + e*log(a + c*x**2)/(
2*c) + Integral(d, x)/c
```

Mathematica [A] time = 0.0388683, size = 49, normalized size = 1.

$$-\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\log\left(a+cx^2\right)}{2c}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x)/(c + a/x^2),x]
```

```
[Out] (d*x)/c - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/c^(3/2) + (e*Lo
g[a + c*x^2])/(2*c)
```

Maple [A] time = 0.006, size = 43, normalized size = 0.9

$$\frac{dx}{c}+\frac{e\ln\left(cx^2+a\right)}{2\,c}-\frac{ad}{c}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x)/(c+a/x^2),x)
```

```
[Out] 1/c*d*x+1/2*e*ln(c*x^2+a)/c-1/c*a*d/(a*c)^(1/2)*arctan(c*x/(a*c)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x)/(c + a/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298733, size = 1, normalized size = 0.02

$$\left[\frac{d\sqrt{-\frac{a}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{a}{c}}-a}{cx^2+a}\right)+2\,dx+e\log\left(cx^2+a\right)}{2\,c},-\frac{2\,d\sqrt{\frac{a}{c}}\arctan\left(\frac{x}{\sqrt{\frac{a}{c}}}\right)-2\,dx-e\log\left(cx^2+a\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x)/(c + a/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-a/c)*log((c*x^2 - 2*c*x*sqrt(-a/c) - a)/(c*x^2 + a)
) + 2*d*x + e*log(c*x^2 + a))/c, -1/2*(2*d*sqrt(a/c)*arctan(x/sqr
t(a/c)) - 2*d*x - e*log(c*x^2 + a))/c]
```

Sympy [A] time = 1.68075, size = 112, normalized size = 2.29

$$\left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}-\frac{d\sqrt{-ac^3}}{2c^3}\right)+e}{d}\right)+\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2c^3}\right)\log\left(x+\frac{-2c\left(\frac{e}{2c}+\frac{d\sqrt{-ac^3}}{2c^3}\right)+e}{d}\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x)/(c+a/x**2),x)
```

```
[Out] (e/(2*c) - d*sqrt(-a*c**3)/(2*c**3))*log(x + (-2*c*(e/(2*c) - d*s
qrt(-a*c**3)/(2*c**3)) + e)/d) + (e/(2*c) + d*sqrt(-a*c**3)/(2*c*
*3))*log(x + (-2*c*(e/(2*c) + d*sqrt(-a*c**3)/(2*c**3)) + e)/d) +
 d*x/c
```

GIAC/XCAS [A] time = 0.267971, size = 58, normalized size = 1.18

$$-\frac{ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}+\frac{dx}{c}+\frac{e\ln\left(cx^2+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x)/(c + a/x^2),x, algorithm="giac")
```

```
[Out] -a*d*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + d*x/c + 1/2*e*ln(c*x^2
 + a)/c
```

3.35 $\int \frac{d+\frac{e}{x}}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=86

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{dx}{c}$$

```
[Out] (d*x)/c - ((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b*d - c*e)*Log[a + b*x +
c*x^2])/(2*c^2)
```

Rubi [A] time = 0.183937, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] (d*x)/c - ((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b*d - c*e)*Log[a + b*x +
c*x^2])/(2*c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int d\,dx}{c}-\frac{(bd-ce)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(-2acd+b^2d-bce\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x)/(c+a/x**2+b/x),x)
```

```
[Out] Integral(d, x)/c - (b*d - c*e)*log(a + b*x + c*x**2)/(2*c**2) - (
-2*a*c*d + b**2*d - b*c*e)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))
/(c**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.145556, size = 86, normalized size = 1.

$$\frac{\frac{2\left(-2acd+b^2d-bce\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(ce-bd)\log(a+x(b+cx))+2cdx}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x)/(c + a/x^2 + b/x),x]
```

```
[Out] (2*c*d*x + (2*(b^2*d - 2*a*c*d - b*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-
b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*d) + c*e)*Log[a + x*(b +
 c*x)])/(2*c^2)
```

Maple [A] time = 0.005, size = 161, normalized size = 1.9

$$\frac{dx}{c}-\frac{\ln\left(cx^2+bx+a\right)bd}{2\,c^2}+\frac{\ln\left(cx^2+bx+a\right)e}{2\,c}-2\,\frac{ad}{c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^2d}{c^2}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{be}{c}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x)/(c+a/x^2+b/x),x)
```

```
[Out] 1/c*d*x-1/2/c^2*ln(c*x^2+b*x+a)*b*d+1/2/c*ln(c*x^2+b*x+a)*e-2/c/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d+1/c^2/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d-1/c/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x)/(c + b/x + a/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267912, size = 1, normalized size = 0.01

$$\left[\frac{\left(bce-\left(b^2-2\,ac\right)d\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\left(2\,cdx-(bd-ce)\log\left(cx^2+bx+a\right)\right)\sqrt{b^2-4\,ac}}{2\,\sqrt{b^2-4\,ac}c^2},\ -\frac{2\left(bce-\left(b^2-2\,ac\right)d\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-\left(2\,cdx-(bd-ce)\log\left(cx^2+bx+a\right)\right)\sqrt{-b^2+4\,ac}}{2\,\sqrt{-b^2+4\,ac}c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x)/(c + b/x + a/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c*e - (b^2 - 2*a*c)*d)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4
*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)
)/(c*x^2 + b*x + a)) + (2*c*d*x - (b*d - c*e)*log(c*x^2 + b*x + a
))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), -1/2*(2*(b*c*e - (
b^2 - 2*a*c)*d)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - (2*c*d*x - (b*d - c*e)*log(c*x^2 + b*x + a))*sqrt(-b^2 + 4
*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 4.56724, size = 423, normalized size = 4.92

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(-\frac{\sqrt{-4ac+b^2}(2acd-b^2d+bce)}{2c^2(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(-\frac{\sqrt{-4ac+b^2}(2acd-b^2d+bce)}{2c^2(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)}{2acd-b^2d+bce}\right) + \left(\frac{\sqrt{-4ac+b^2}\left(2acd-b^2d+bce\right)}{2c^2\left(4ac-b^2\right)} - \frac{bd-ce}{2c^2}\right)\log\left(x+\frac{-abd-4ac^2\left(\frac{\sqrt{-4ac+b^2}(2acd-b^2d+bce)}{2c^2(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)+2ace+b^2c\left(\frac{\sqrt{-4ac+b^2}(2acd-b^2d+bce)}{2c^2(4ac-b^2)}-\frac{bd-ce}{2c^2}\right)}{2acd-b^2d+bce}\right) + \frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x)/(c+a/x**2+b/x),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c -
 b**2)) - (b*d - c*e)/(2*c**2))*log(x + (-a*b*d - 4*a*c**2*(-sqrt
(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)
) - (b*d - c*e)/(2*c**2)) + 2*a*c*e + b**2*c*(-sqrt(-4*a*c + b**2
)*(2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e
)/(2*c**2)))/(2*a*c*d - b**2*d + b*c*e)) + (sqrt(-4*a*c + b**2)*(
2*a*c*d - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(
2*c**2))*log(x + (-a*b*d - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*d
 - b**2*d + b*c*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)
) + 2*a*c*e + b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*d - b**2*d + b*c
*e)/(2*c**2*(4*a*c - b**2)) - (b*d - c*e)/(2*c**2)))/(2*a*c*d - b
**2*d + b*c*e)) + d*x/c
```

GIAC/XCAS [A] time = 0.267372, size = 115, normalized size = 1.34

$$\frac{dx}{c} - \frac{(bd-ce)\ln\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x)/(c + b/x + a/x^2),x, algorithm="giac")
```

```
[Out] d*x/c - 1/2*(b*d - c*e)*ln(c*x^2 + b*x + a)/c^2 + (b^2*d - 2*a*c*
d - b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*
a*c)*c^2)
```

3.36 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}}\,dx$

Optimal. Leaf size=253

$$\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}-\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}+\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}}-\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}}+\frac{dx}{c}$$

```
[Out] (d*x)/c + ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d - Sqrt[c]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5
/4)) + ((Sqrt[a]*d + Sqrt[c]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d
+ Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(1/4)*c^(5/4))
```

Rubi [A] time = 0.420356, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}-\frac{\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{5/4}}+\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}}-\frac{\left(\sqrt{a}d-\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}c^{5/4}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((Sqrt[a]*d - Sqrt[c]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d - Sqrt[c]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(5
/4)) + ((Sqrt[a]*d + Sqrt[c]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(5/4)) - ((Sqrt[a]*d
+ Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(1/4)*c^(5/4))
```

Rubi in Sympy [A] time = 72.4602, size = 235, normalized size = 0.93

$$\frac{dx}{c}+\frac{\sqrt{2}\left(\sqrt{a}d-\sqrt{c}e\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}d-\sqrt{c}e\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8\sqrt[4]{a}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8\sqrt[4]{a}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x**2)/(c+a/x**4),x)
```

```
[Out] d*x/c + sqrt(2)*(sqrt(a)*d - sqrt(c)*e)*atan(1 - sqrt(2)*c**(1/4)
*x/a**(1/4))/(4*a**(1/4)*c**(5/4)) - sqrt(2)*(sqrt(a)*d - sqrt(c)
*e)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(1/4)*c**(5/4)) +
```

```
sqrt(2)*(sqrt(a)*d + sqrt(c)*e)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x
 + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(1/4)*c**(5/4)) - sqrt(2)*(sqr
t(a)*d + sqrt(c)*e)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqr
t(c) + c*x**2)/(8*a**(1/4)*c**(5/4))
```

Mathematica [A] time = 0.170371, size = 293, normalized size = 1.16

$$\frac{\left(a^{5/4}\sqrt{c}d+a^{3/4}ce\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}ac^{7/4}}-\frac{\left(a^{5/4}\sqrt{c}d+a^{3/4}ce\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}ac^{7/4}}+\frac{\left(a^{3/4}ce-a^{5/4}\sqrt{c}d\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}x-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}ac^{7/4}}+\frac{\left(a^{3/4}ce-a^{5/4}\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}ac^{7/4}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x^2)/(c + a/x^4),x]
```

```
[Out] (d*x)/c + ((-(a^(5/4)*Sqrt[c]*d) + a^(3/4)*c*e)*ArcTan[(-(Sqrt[2]
*a^(1/4)) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*a*c^(7/4)
) + ((-(a^(5/4)*Sqrt[c]*d) + a^(3/4)*c*e)*ArcTan[(Sqrt[2]*a^(1/4)
 + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*a*c^(7/4)) + ((a^(
5/4)*Sqrt[c]*d + a^(3/4)*c*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/
4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a*c^(7/4)) - ((a^(5/4)*Sqrt[c]*d
+ a^(3/4)*c*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a*c^(7/4))
```

Maple [A] time = 0.007, size = 266, normalized size = 1.1

$$\frac{dx}{c}-\frac{d\sqrt{2}}{4c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)-\frac{d\sqrt{2}}{4c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)-\frac{d\sqrt{2}}{8c}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{e\sqrt{2}}{8c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e\sqrt{2}}{4c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e\sqrt{2}}{4c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x^2)/(c+a/x^4),x)
```

```
[Out] 1/c*d*x-1/4/c*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4
)*x+1)-1/4/c*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)
*x-1)-1/8/c*d*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/
8/c*e/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4/c*e/(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4/c*e/(1/
```

```
c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^2)/(c + a/x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270875, size = 1018, normalized size = 4.02

$$c\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2\,acd^2e^2+c^2e^4}{ac^5}}+2\,de}{c^2}}\log\left(-\left(a^2d^4-c^2e^4\right)x+\left(ac^4e\sqrt{-\frac{a^2d^4-2\,acd^2e^2+c^2e^4}{ac^5}}+a^2cd^3-ac^2de^2\right)\sqrt{\frac{c^2\sqrt{-\frac{a^2d^4-2\,acd^2e^2+c^2e^4}{ac^5}}+2\,de}{c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^2)/(c + a/x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(c*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5
)) + 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x + (a*c^4*e*sqrt(-(a^2
*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + a^2*c*d^3 - a*c^2*d*e^
2)*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) +
 2*d*e)/c^2)) - c*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*
e^4)/(a*c^5)) + 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*x - (a*c^4*e
*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) + a^2*c*d^3 -
 a*c^2*d*e^2)*sqrt((c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)
/(a*c^5)) + 2*d*e)/c^2)) - c*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^
2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)*log(-(a^2*d^4 - c^2*e^4)*
x + (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) -
 a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^2*sqrt(-(a^2*d^4 - 2*a*c*d^2*e
^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + c*sqrt(-(c^2*sqrt(-(a^2*d
^4 - 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)*log(-(a^2*d^
4 - c^2*e^4)*x - (a*c^4*e*sqrt(-(a^2*d^4 - 2*a*c*d^2*e^2 + c^2*e^
4)/(a*c^5)) - a^2*c*d^3 + a*c^2*d*e^2)*sqrt(-(c^2*sqrt(-(a^2*d^4
- 2*a*c*d^2*e^2 + c^2*e^4)/(a*c^5)) - 2*d*e)/c^2)) + 4*d*x)/c
```

Sympy [A] time = 3.11912, size = 109, normalized size = 0.43

$$\operatorname{RootSum}\left(256t^4ac^5-64t^2ac^3de+a^2d^4+2acd^2e^2+c^2e^4,\left(t\mapsto t\log\left(x+\frac{-64t^3ac^4e-4ta^2cd^3+12tac^2de^2}{a^2d^4-c^2e^4}\right)\right)\right)+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x**2)/(c+a/x**4),x)
```

```
[Out] RootSum(256*_t**4*a*c**5 - 64*_t**2*a*c**3*d*e + a**2*d**4 + 2*a*
c*d**2*e**2 + c**2*e**4, Lambda(_t, _t*log(x + (-64*_t**3*a*c**4*
e - 4*_t*a**2*c*d**3 + 12*_t*a*c**2*d*e**2)/(a**2*d**4 - c**2*e**
4)))) + d*x/c
```

GIAC/XCAS [A] time = 0.272603, size = 347, normalized size = 1.37

$$\frac{dx}{c} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}} acd - \left(ac^3\right)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}} acd + \left(ac^3\right)^{\frac{3}{4}} e\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}} ac^3 d - \left(ac^3\right)^{\frac{3}{4}} c^2 e\right) \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^5} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}} ac^3 d + \left(ac^3\right)^{\frac{3}{4}} c^2 e\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^2)/(c + a/x^4),x, algorithm="giac")
```

```
[Out] d*x/c - 1/4*sqrt(2)*((a*c^3)^(1/4)*a*c*d - (a*c^3)^(3/4)*e)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) +
1/8*sqrt(2)*((a*c^3)^(1/4)*a*c*d + (a*c^3)^(3/4)*e)*ln(x^2 - sqrt
(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/4*sqrt(2)*((a*c^3)^(1/
4)*a*c^3*d - (a*c^3)^(3/4)*c^2*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^5) - 1/8*sqrt(2)*((a*c^3)^(1/4)
*a*c^3*d + (a*c^3)^(3/4)*c^2*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c^5)
```

3.37 $\int \frac{d+\frac{e}{x^2}}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=208

$$-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}$$

```
[Out] (d*x)/c - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b*d - c*e + (b^2*d
- 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]])
```

Rubi [A] time = 1.10437, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b*d - c*e + (b^2*d
- 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]])
```

Rubi in Sympy [A] time = 76.2677, size = 214, normalized size = 1.03

$$\frac{dx}{c}-\frac{\sqrt{2}\left(-2acd+b\left(bd-ce\right)+\sqrt{-4ac+b^2}\left(bd-ce\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2acd+b\left(bd-ce\right)-\sqrt{-4ac+b^2}\left(bd-ce\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x**2)/(c+a/x**4+b/x**2),x)
```

```
[Out] d*x/c - sqrt(2)*(-2*a*c*d + b*(b*d - c*e) + sqrt(-4*a*c + b**2)*(
b*d - c*e))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))
/(2*c**(3/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) +
 sqrt(2)*(-2*a*c*d + b*(b*d - c*e) - sqrt(-4*a*c + b**2)*(b*d - c
*e))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**
(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.324647, size = 251, normalized size = 1.21

$$-\frac{\left(bd\sqrt{b^2-4ac}-ce\sqrt{b^2-4ac}+2acd+b^2(-d)+bce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bd\sqrt{b^2-4ac}-ce\sqrt{b^2-4ac}-2acd+b^2d-bce\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x^2)/(c + a/x^4 + b/x^2),x]
```

```
[Out] (d*x)/c - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + b*c*e -
c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - b*c*e
 - c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqr
t[b^2 - 4*a*c]])
```

Maple [B] time = 0.031, size = 560, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x^2)/(c+a/x^4+b/x^2),x)
```

```
[Out] 1/c*d*x-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/2*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a*d-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d
+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/2/c*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d-1
/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+1/2
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{dx}{c}+\frac{-\int\frac{(bd-ce)x^2+ad}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^2)/(c + b/x^2 + a/x^4),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate(-((b*d - c*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x
)/c
```

Fricas [A] time = 0.332601, size = 3429, normalized size = 16.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^2)/(c + b/x^2 + a/x^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*
c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4
*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*
e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (
a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x + sqrt(1/2)*((b^4 - 5
*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^
2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*
c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c
^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e
^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2
 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*
e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*
c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))
/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*
a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^4)*sqrt(-
(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b
^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2
*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x -
 sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*
c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d
 - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4
 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^
2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 +
(b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^2*c^3 - 4*a*c^
4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(
b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b
*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e - (b^2*c
^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)
*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(3*b^2
*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2 - a^2*c)*d^4 - (b^3
 + a*b*c)*d^3*e)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3
 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2 + ((b^
3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*e)*sqrt(-(4*b*c^3*d*
e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*
c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))
*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2*a*c^2)*d*e
 - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2
 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(b^2*c - 2
*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2
*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4))*log(2*(3*b^2*c*d^2*e^2 - 3*b*c^2*d*e^3 + c^3*e^4 + (a*b^2
 - a^2*c)*d^4 - (b^3 + a*b*c)*d^3*e)*x - sqrt(1/2)*((b^4 - 5*a*b^
2*c + 4*a^2*c^2)*d^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4
*a*c^3)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d - 2*(b^2*c^4 - 4*a*c^5)*
e)*sqrt(-(4*b*c^3*d*e^3 - c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^4 + 4*(b^3*c - a*b*c^2)*d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(
b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(
b^2*c - 2*a*c^2)*d*e - (b^2*c^3 - 4*a*c^4)*sqrt(-(4*b*c^3*d*e^3 -
 c^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(b^3*c - a*b*c^2)*
d^3*e - 2*(3*b^2*c^2 - a*c^3)*d^2*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2
*c^3 - 4*a*c^4))) + 2*d*x)/c
```

Sympy [A] time = 34.7032, size = 428, normalized size = 2.06

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5 - 128ab^2c^4 + 16b^4c^3\right) + t^2\left(48a^2bc^2d^2 - 64a^2c^3de - 28ab^3cd^2 + 48ab^2c^2de - 16abc^3e^2 + 4b^5d^2 - 8b^4\right.\right.$$
$$+\frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x**2)/(c+a/x**4+b/x**2),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) +
_t**2*(48*a**2*b*c**2*d**2 - 64*a**2*c**3*d*e - 28*a*b**3*c*d**2
+ 48*a*b**2*c**2*d*e - 16*a*b*c**3*e**2 + 4*b**5*d**2 - 8*b**4*c*
d*e + 4*b**3*c**2*e**2) + a**3*d**4 - 2*a**2*b*d**3*e + 2*a**2*c*
d**2*e**2 + a*b**2*d**2*e**2 - 2*a*b*c*d*e**3 + a*c**2*e**4, Lamb
da(_t, _t*log(x + (32*_t**3*a*b*c**4*d - 64*_t**3*a*c**5*e - 8*_t
**3*b**3*c**3*d + 16*_t**3*b**2*c**4*e - 4*_t*a**2*c**2*d**3 + 8*
_t*a*b**2*c*d**3 - 18*_t*a*b*c**2*d**2*e + 12*_t*a*c**3*d*e**2 -
2*_t*b**4*d**3 + 6*_t*b**3*c*d**2*e - 6*_t*b**2*c**2*d*e**2 + 2*_
t*b*c**3*e**3)/(a**2*c*d**4 - a*b**2*d**4 + a*b*c*d**3*e + b**3*d
**3*e - 3*b**2*c*d**2*e**2 + 3*b*c**2*d*e**3 - c**3*e**4)))) + d*
x/c
```

GIAC/XCAS [A] time = 1.28784, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^2)/(c + b/x^2 + a/x^4),x, algorithm="giac")
```

```
[Out] Done
```

3.38 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}} \, dx$

Optimal. Leaf size=311

$$\frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}+\frac{\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{a}d+\sqrt{3}\sqrt{c}e\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+\sqrt{3}\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{\sqrt[6]{a}d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}+\frac{dx}{c}$$

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + (
(Sqrt[a]*d - Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^
(1/6)])/(6*a^(1/3)*c^(7/6)) - ((Sqrt[a]*d + Sqrt[3]*Sqrt[c]*e)*Ar
cTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(6*a^(1/3)*c^(7/6)) - (e*L
og[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) + ((Sqrt[3]*Sqrt[a
]*d + Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3
)*x^2])/(12*a^(1/3)*c^(7/6)) - ((Sqrt[3]*Sqrt[a]*d - Sqrt[c]*e)*L
og[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/3
)*c^(7/6))
```

Rubi [A] time = 0.639059, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$\frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{3}\sqrt{a}d-\sqrt{c}e\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}+\frac{\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{a}d+\sqrt{3}\sqrt{c}e\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+\sqrt{3}\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{\sqrt[6]{a}d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x^3)/(c + a/x^6),x]
```


```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + (
(Sqrt[a]*d - Sqrt[3]*Sqrt[c]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^
(1/6)])/(6*a^(1/3)*c^(7/6)) - ((Sqrt[a]*d + Sqrt[3]*Sqrt[c]*e)*Ar
cTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(6*a^(1/3)*c^(7/6)) - (e*L
og[a^(1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) + ((Sqrt[3]*Sqrt[a
]*d + Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3
)*x^2])/(12*a^(1/3)*c^(7/6)) - ((Sqrt[3]*Sqrt[a]*d - Sqrt[c]*e)*L
og[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/3
)*c^(7/6))
```

Rubi in Sympy [A] time = 124.579, size = 314, normalized size = 1.01

$$-\frac{\sqrt[6]{a}d\,\mathrm{atan}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{\frac{7}{6}}}+\frac{dx}{c}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{\frac{2}{3}}}+\frac{\left(\sqrt{a}d-\sqrt{3}\sqrt{c}e\right)\mathrm{atan}\left(\frac{\sqrt{3}\left(\sqrt[6]{a}-\frac{2\sqrt{3}\sqrt[6]{c}x}{3}\right)}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{\frac{7}{6}}}-\frac{\left(\sqrt{a}d+\sqrt{3}\sqrt{c}e\right)\mathrm{atan}\left(\frac{\sqrt{3}\left(\sqrt[6]{a}+\frac{2\sqrt{3}\sqrt[6]{c}x}{3}\right)}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{\frac{7}{6}}}+\frac{\left(-\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{12\sqrt[3]{a}c^{\frac{7}{6}}}+\frac{\left(\sqrt{3}\sqrt{a}d+\sqrt{c}e\right)\log\left(1+\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{12\sqrt[3]{a}c^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x**3)/(c+a/x**6),x)
```

```
[Out] -a**(1/6)*d*atan(c**(1/6)*x/a**(1/6))/(3*c**(7/6)) + d*x/c - e*lo
g(a**(1/3) + c**(1/3)*x**2)/(6*a**(1/3)*c**(2/3)) + (sqrt(a)*d -
sqrt(3)*sqrt(c)*e)*atan(sqrt(3)*(a**(1/6) - 2*sqrt(3)*c**(1/6)*x/
3)/a**(1/6))/(6*a**(1/3)*c**(7/6)) - (sqrt(a)*d + sqrt(3)*sqrt(c)
*e)*atan(sqrt(3)*(a**(1/6) + 2*sqrt(3)*c**(1/6)*x/3)/a**(1/6))/(6
*a**(1/3)*c**(7/6)) + (-sqrt(3)*sqrt(a)*d + sqrt(c)*e)*log(1 + c*
*(1/3)*x**2/a**(1/3) + sqrt(3)*c**(1/6)*x/a**(1/6))/(12*a**(1/3)*
c**(7/6)) + (sqrt(3)*sqrt(a)*d + sqrt(c)*e)*log(1 + c**(1/3)*x**2
/a**(1/3) - sqrt(3)*c**(1/6)*x/a**(1/6))/(12*a**(1/3)*c**(7/6))
```

Mathematica [A] time = 0.210588, size = 346, normalized size = 1.11

$$-\frac{\left(-\sqrt{3}a^{7/6}\sqrt{c}d-a^{2/3}ce\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12ac^{5/3}}-\frac{\left(\sqrt{3}a^{7/6}\sqrt{c}d-a^{2/3}ce\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{12ac^{5/3}}+\frac{\left(\sqrt{3}a^{2/3}ce-a^{7/6}\sqrt{c}d\right)\tan^{-1}\left(\frac{2\sqrt[6]{c}x-\sqrt{3}\sqrt[6]{a}}{\sqrt[6]{a}}\right)}{6ac^{5/3}}+\frac{\left(a^{7/6}\left(-\sqrt{c}\right)d-\sqrt{3}a^{2/3}ce\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/3}}-\frac{\sqrt[6]{a}d\tan^{-1}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x^3)/(c + a/x^6),x]
```

```
[Out] (d*x)/c - (a^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + (
(-(a^(7/6)*Sqrt[c]*d) + Sqrt[3]*a^(2/3)*c*e)*ArcTan[(-(Sqrt[3]*a^
(1/6)) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(5/3)) + ((-(a^(7/6)*Sqrt[
c]*d) - Sqrt[3]*a^(2/3)*c*e)*ArcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*
x)/a^(1/6)])/(6*a*c^(5/3)) - (e*Log[a^(1/3) + c^(1/3)*x^2])/(6*a^
(1/3)*c^(2/3)) - ((-(Sqrt[3]*a^(7/6)*Sqrt[c]*d) - a^(2/3)*c*e)*Lo
g[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/
3)) - ((Sqrt[3]*a^(7/6)*Sqrt[c]*d - a^(2/3)*c*e)*Log[a^(1/3) + Sq
rt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/3))
```

Maple [A] time = 0.07, size = 334, normalized size = 1.1

$$\frac{dx}{c}-\frac{\sqrt{3}d}{12\,a}\left(\frac{a}{c}\right)^{\frac{7}{6}}\ln\left(x^2+\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)+\frac{e}{12\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2+\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)$$
$$-\frac{d}{6\,c}\sqrt[6]{\frac{a}{c}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}+\sqrt{3}\right)-\frac{\sqrt{3}e}{6\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}+\sqrt{3}\right)$$
$$-\frac{e}{6\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2+\sqrt[3]{\frac{a}{c}}\right)-\frac{d}{3\,c}\sqrt[6]{\frac{a}{c}}\arctan\left(x\frac{1}{\sqrt[6]{\frac{a}{c}}}\right)$$
$$+\frac{e}{12\,a}\ln\left(x^2-\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}+\frac{\sqrt{3}d}{12\,c}\ln\left(x^2-\sqrt{3}\sqrt[6]{\frac{a}{c}}x+\sqrt[3]{\frac{a}{c}}\right)\sqrt[6]{\frac{a}{c}}$$
$$+\frac{\sqrt{3}e}{6\,a}\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}-\sqrt{3}\right)-\frac{d}{6\,c}\sqrt[6]{\frac{a}{c}}\arctan\left(2\,x\frac{1}{\sqrt[6]{\frac{a}{c}}}-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x^3)/(c+a/x^6),x)
```

```
[Out] 1/c*d*x-1/12*(1/c*a)^(7/6)/a*ln(x^2+3^(1/2)*(1/c*a)^(1/6)*x+(1/c*
a)^(1/3))*3^(1/2)*d+1/12*(1/c*a)^(2/3)/a*ln(x^2+3^(1/2)*(1/c*a)^(
1/6)*x+(1/c*a)^(1/3))*e-1/6/c*(1/c*a)^(1/6)*arctan(2*x/(1/c*a)^(1
/6)+3^(1/2))*d-1/6*(1/c*a)^(2/3)/a*arctan(2*x/(1/c*a)^(1/6)+3^(1/
2))*3^(1/2)*e-1/6*(1/c*a)^(2/3)/a*e*ln(x^2+(1/c*a)^(1/3))-1/3/c*(
1/c*a)^(1/6)*d*arctan(x/(1/c*a)^(1/6))+1/12/a*ln(x^2-3^(1/2)*(1/c
*a)^(1/6)*x+(1/c*a)^(1/3))*(1/c*a)^(2/3)*e+1/12/c*ln(x^2-3^(1/2)*
(1/c*a)^(1/6)*x+(1/c*a)^(1/3))*3^(1/2)*(1/c*a)^(1/6)*d+1/6/a*(1/c
*a)^(2/3)*arctan(2*x/(1/c*a)^(1/6)-3^(1/2))*3^(1/2)*e-1/6/c*(1/c*
a)^(1/6)*arctan(2*x/(1/c*a)^(1/6)-3^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^3)/(c + a/x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.349835, size = 4070, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^3)/(c + a/x^6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2
*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*arctan(-(s
qrt(3)*a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a
*c^7)) + sqrt(3)*(a^2*c*d^4 - 3*a*c^2*d^2*e^2))*((a*c^3*sqrt(-(a^
```

```
2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e
^3)/(a*c^3))^(1/3)/(2*(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x +
 2*(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*sqrt(((a^3*d^7 - a^2*c
*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 + (2*a^2*c^6*d*e*sq
rt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + a^3*c^2*
d^5 - 4*a^2*c^3*d^3*e^2 + 3*a*c^4*d*e^4)*((a*c^3*sqrt(-(a^2*d^6 -
 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*
c^3))^(2/3) + ((a^2*c^5*d^2*e + a*c^6*e^3)*x*sqrt(-(a^2*d^6 - 6*a
*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + (a^3*c*d^6 - 2*a^2*c^2*d^4
*e^2 - 3*a*c^3*d^2*e^4)*x)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3))/(
a^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)) + (a*c^
5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + a^
2*c*d^4 - 3*a*c^2*d^2*e^2)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)))
- 4*sqrt(3)*c*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2
*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)*arctan(-(sqrt(
3)*a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7
)) - sqrt(3)*(a^2*c*d^4 - 3*a*c^2*d^2*e^2))*(-(a*c^3*sqrt(-(a^2*d
^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)
/(a*c^3))^(1/3)/(2*(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x + 2*
(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*sqrt(((a^3*d^7 - a^2*c*d^
5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 - (2*a^2*c^6*d*e*sqrt(
-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^3*c^2*d^5
 + 4*a^2*c^3*d^3*e^2 - 3*a*c^4*d*e^4)*(-(a*c^3*sqrt(-(a^2*d^6 - 6
*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^
3))^(2/3) - ((a^2*c^5*d^2*e + a*c^6*e^3)*x*sqrt(-(a^2*d^6 - 6*a*c
*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - (a^3*c*d^6 - 2*a^2*c^2*d^4*e
^2 - 3*a*c^3*d^2*e^4)*x)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
+ 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3))/(a
^3*d^7 - a^2*c*d^5*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)) - (a*c^5
*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^2
*c*d^4 + 3*a*c^2*d^2*e^2)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2
 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)))
+ c*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^
7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^3*d^7 - a^2*c*d^5
*e^2 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 - (2*a^2*c^6*d*e*sqrt(-
(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + a^3*c^2*d^5
- 4*a^2*c^3*d^3*e^2 + 3*a*c^4*d*e^4)*((a*c^3*sqrt(-(a^2*d^6 - 6*a
*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3)
)^(2/3) - ((a^2*c^5*d^2*e + a*c^6*e^3)*x*sqrt(-(a^2*d^6 - 6*a*c*d
^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + (a^3*c*d^6 - 2*a^2*c^2*d^4*e^2
 - 3*a*c^3*d^2*e^4)*x)*((a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9
*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)) + c*(
-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7))
- 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)*log(-(a^3*d^7 - a^2*c*d^5*e^2
 - 5*a*c^2*d^3*e^4 - 3*c^3*d*e^6)*x^2 + (2*a^2*c^6*d*e*sqrt(-(a^2
*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^3*c^2*d^5 + 4*
a^2*c^3*d^3*e^2 - 3*a*c^4*d*e^4)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*
d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(
2/3) + ((a^2*c^5*d^2*e + a*c^6*e^3)*x*sqrt(-(a^2*d^6 - 6*a*c*d^4*
e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - (a^3*c*d^6 - 2*a^2*c^2*d^4*e^2 -
3*a*c^3*d^2*e^4)*x)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c
^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)) - 2*c*(
(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) +
 3*a*d^2*e - c*e^3)/(a*c^3))^(1/3)*log(-(a^2*d^5 - 2*a*c*d^3*e^2
- 3*c^2*d*e^4)*x + (a*c^5*e*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^
2*d^2*e^4)/(a*c^7)) + a^2*c*d^4 - 3*a*c^2*d^2*e^2)*((a*c^3*sqrt(-
(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) + 3*a*d^2*e -
c*e^3)/(a*c^3))^(1/3)) - 2*c*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*
e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)
*log(-(a^2*d^5 - 2*a*c*d^3*e^2 - 3*c^2*d*e^4)*x - (a*c^5*e*sqrt(-
(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*d^2*e^4)/(a*c^7)) - a^2*c*d^4 +
3*a*c^2*d^2*e^2)*(-(a*c^3*sqrt(-(a^2*d^6 - 6*a*c*d^4*e^2 + 9*c^2*
d^2*e^4)/(a*c^7)) - 3*a*d^2*e + c*e^3)/(a*c^3))^(1/3)) - 12*d*x)/
c
```

Sympy [A] time = 8.90937, size = 167, normalized size = 0.54

$$\text{RootSum}\left(46656t^6a^2c^7 + t^3\left(-1296a^2c^4d^2e + 432ac^5e^3\right) + a^3d^6 + 3a^2cd^4e^2 + 3ac^2d^2e^4 + c^3e^6, \left(t \mapsto t\log\left(x + \frac{-1296t^4ac^5e}{a^2}\right.\right.\right.$$
$$+ \frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x**3)/(c+a/x**6),x)
```

```
[Out] RootSum(46656*_t**6*a**2*c**7 + _t**3*(-1296*a**2*c**4*d**2*e + 4
32*a*c**5*e**3) + a**3*d**6 + 3*a**2*c*d**4*e**2 + 3*a*c**2*d**2*
e**4 + c**3*e**6, Lambda(_t, _t*log(x + (-1296*_t**4*a*c**5*e - 6
*_t*a**2*c*d**4 + 36*_t*a*c**2*d**2*e**2 - 6*_t*c**3*e**4)/(a**2*
d**5 - 2*a*c*d**3*e**2 - 3*c**2*d*e**4)))) + d*x/c
```

GIAC/XCAS [A] time = 0.324781, size = 406, normalized size = 1.31

$$\frac{dx}{c} - \frac{\left(ac^5\right)^{\frac{1}{6}} d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,c^2} - \frac{\left(ac^5\right)^{\frac{2}{3}} |c| e\ln\left(x^2 + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,ac^5} - \frac{\left(\left(ac^5\right)^{\frac{1}{6}} ac^2d + \sqrt{3}\left(ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} - \frac{\left(\left(ac^5\right)^{\frac{1}{6}} ac^2d - \sqrt{3}\left(ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} - \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}} ac^2d - \left(ac^5\right)^{\frac{2}{3}} e\right)\ln\left(x^2 + \sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}} + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} + \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}} ac^2d + \left(ac^5\right)^{\frac{2}{3}} e\right)\ln\left(x^2 - \sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}} + \left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^3)/(c + a/x^6),x, algorithm="giac")
```

```
[Out] d*x/c - 1/3*(a*c^5)^(1/6)*d*arctan(x/(a/c)^(1/6))/c^2 - 1/6*(a*c^
5)^(2/3)*abs(c)*e*ln(x^2 + (a/c)^(1/3))/(a*c^5) - 1/6*((a*c^5)^(1
/6)*a*c^2*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c
)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/6*((a*c^5)^(1/6)*a*c^2*d - sqrt
(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/
6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d - (a*c^5)^(2/3)
*e)*ln(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) + 1/12*
(sqrt(3)*(a*c^5)^(1/6)*a*c^2*d + (a*c^5)^(2/3)*e)*ln(x^2 - sqrt(3
)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4)
```

3.39 $\int \frac{d+\frac{e}{x^3}}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=716

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{dx}{c}$$

```
[Out] (d*x)/c + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/
3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2
/3)) + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
) - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b*d - c*e + (b^2*d - 2
*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*
c])^(2/3)) + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4
*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - S
qrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4
/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d - 2*a*c
*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/
3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 3.49776, antiderivative size = 716, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/
3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2
/3)) + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
) - ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b*d - c*e + (b^2*d - 2
*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*
c])^(2/3)) + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4
*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - S
qrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4
/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b*d - c*e + (b^2*d - 2*a*c
*d - b*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/
3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x**3)/(c+a/x**6+b/x**3),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0891591, size = 88, normalized size = 0.12

$$\frac{dx}{c}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3bd\log(x-\#1)-\#1^3ce\log(x-\#1)+ad\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x^3)/(c + a/x^6 + b/x^3),x]
```

```
[Out] (d*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*d*Log[x - #1] + b*d*
Log[x - #1]*#1^3 - c*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/
(3*c)
```

Maple [C] time = 0.034, size = 67, normalized size = 0.1

$$\frac{dx}{c}+\frac{1}{3\,c}\sum_{\_R=RootOf\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left((-bd+ce)\_R^3-ad\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x^3)/(c+a/x^6+b/x^3),x)
```

```
[Out] 1/c*d*x+1/3/c*sum(((-b*d+c*e)*_R^3-a*d)/(2*_R^5*c+_R^2*b)*ln(x-_R
),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{dx}{c}+\frac{-\int\frac{(bd-ce)x^3+ad}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^3)/(c + b/x^3 + a/x^6),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate(-((b*d - c*e)*x^3 + a*d)/(c*x^6 + b*x^3 + a), x
)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^3)/(c + b/x^3 + a/x^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x**3)/(c+a/x**6+b/x**3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + \frac{e}{x^3}}{c + \frac{b}{x^3} + \frac{a}{x^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^3)/(c + b/x^3 + a/x^6),x, algorithm="giac")
```

```
[Out] integrate((d + e/x^3)/(c + b/x^3 + a/x^6), x)
```

3.40 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}}\,dx$

Optimal. Leaf size=753

$$-\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}+\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}+\frac{dx}{c}$$

```
[Out] (d*x)/c + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e
)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt
[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 + Sqrt[2]]*(Sqrt[a]
*(d - Sqrt[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) -
 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) -
 (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[
(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1
/8))])/(8*a^(3/8)*c^(9/8)) + (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqr
t[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8
)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - ((Sqrt[a
]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^
(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*
c^(9/8)) + ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) + S
qrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 -
 Sqrt[2])]*a^(3/8)*c^(9/8)) + (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]
*e)*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x
^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8)) - (((1 + Sqrt[2])*
Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^
(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8))
```

Rubi [A] time = 3.00955, antiderivative size = 753, normalized size of antiderivative = 1., number of

steps used = 21, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$

$$-\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\log\left(\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}-\frac{\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\log\left(\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{c}x+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}a^{3/8}c^{9/8}}+\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}-2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}-\frac{\sqrt{2-\sqrt{2}}\left(\left(1+\sqrt{2}\right)\sqrt{a}d+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2+\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}+\frac{\sqrt{2+\sqrt{2}}\left(\sqrt{a}\left(d-\sqrt{2}d\right)+\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}+2\sqrt[8]{c}x}{\sqrt{2-\sqrt{2}}\sqrt[8]{a}}\right)}{8a^{3/8}c^{9/8}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (d*x)/c + (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e
)*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8) - 2*c^(1/8)*x)/(Sqrt[2 + Sqrt
[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - (Sqrt[2 + Sqrt[2]]*(Sqrt[a]
*(d - Sqrt[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) -
 2*c^(1/8)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) -
 (Sqrt[2 - Sqrt[2]]*((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]*e)*ArcTan[
(Sqrt[2 - Sqrt[2]]*a^(1/8) + 2*c^(1/8)*x)/(Sqrt[2 + Sqrt[2]]*a^(1
/8))])/(8*a^(3/8)*c^(9/8)) + (Sqrt[2 + Sqrt[2]]*(Sqrt[a]*(d - Sqr
t[2]*d) + Sqrt[c]*e)*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8) + 2*c^(1/8
)*x)/(Sqrt[2 - Sqrt[2]]*a^(1/8))])/(8*a^(3/8)*c^(9/8)) - ((Sqrt[a
]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) - Sqrt[2 - Sqrt[2]]*a^
(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 - Sqrt[2])]*a^(3/8)*
c^(9/8)) + ((Sqrt[a]*(d - Sqrt[2]*d) + Sqrt[c]*e)*Log[a^(1/4) + S
qrt[2 - Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 -
 Sqrt[2])]*a^(3/8)*c^(9/8)) + (((1 + Sqrt[2])*Sqrt[a]*d + Sqrt[c]
*e)*Log[a^(1/4) - Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*x + c^(1/4)*x
^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8)) - (((1 + Sqrt[2])*
Sqrt[a]*d + Sqrt[c]*e)*Log[a^(1/4) + Sqrt[2 + Sqrt[2]]*a^(1/8)*c^
(1/8)*x + c^(1/4)*x^2])/(8*Sqrt[2*(2 + Sqrt[2])]*a^(3/8)*c^(9/8))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x**4)/(c+a/x**8),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.9331, size = 551, normalized size = 0.73

$$\log\left(2\sqrt[8]{a}\sqrt[8]{c}x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)\left(a^{5/8}ce\cos\left(\frac{\pi}{8}\right)-a^{9/8}\sqrt{c}d\sin\left(\frac{\pi}{8}\right)\right)+\log\left(-2\sqrt[8]{a}\sqrt[8]{c}x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{c}x^2\right)\left(a^{9/8}\sqrt{c}d\,\mathrm{s}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x^4)/(c + a/x^8),x]
```

```
[Out] (8*a*c^(5/8)*d*x + 2*ArcTan[Cot[Pi/8] + (c^(1/8)*x*Csc[Pi/8])/a^(
1/8)]*(a^(5/8)*c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log
[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^(1/8)*x*Sin[Pi/8]]*(a^(5/8)*
c*e*Cos[Pi/8] - a^(9/8)*Sqrt[c]*d*Sin[Pi/8]) + 2*ArcTan[Cot[Pi/8]
 - (c^(1/8)*x*Csc[Pi/8])/a^(1/8)]*(-(a^(5/8)*c*e*Cos[Pi/8]) + a^(
9/8)*Sqrt[c]*d*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 - 2*a^(1/8)
*c^(1/8)*x*Sin[Pi/8]]*(-(a^(5/8)*c*e*Cos[Pi/8]) + a^(9/8)*Sqrt[c]
*d*Sin[Pi/8]) - 2*ArcTan[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8
]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) - 2*ArcT
an[(c^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*(a^(9/8)*Sqrt[c]*d*
Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi/8]) + Log[a^(1/4) + c^(1/4)*x^2 -
2*a^(1/8)*c^(1/8)*x*Cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(
5/8)*c*e*Sin[Pi/8]) - Log[a^(1/4) + c^(1/4)*x^2 + 2*a^(1/8)*c^(1/
8)*x*Cos[Pi/8]]*(a^(9/8)*Sqrt[c]*d*Cos[Pi/8] + a^(5/8)*c*e*Sin[Pi
/8]))/(8*a*c^(13/8))
```

Maple [C] time = 0.007, size = 45, normalized size = 0.1

$$\frac{dx}{c}+\frac{1}{8\,c^2}\sum_{\_R=RootOf\left(c\_Z^8+a\right)}\frac{\left(\_R^4ce-ad\right)\ln\left(x-\_R\right)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x^4)/(c+a/x^8),x)
```

```
[Out] 1/c*d*x+1/8/c^2*sum((_R^4*c*e-a*d)/_R^7*ln(x-_R),_R=RootOf(_Z^8*c
+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{dx}{c}+\frac{\int\frac{cex^4-ad}{cx^8+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^4)/(c + a/x^8),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate((c*e*x^4 - a*d)/(c*x^8 + a), x)/c
```

Fricas [A] time = 0.515055, size = 3742, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^4)/(c + a/x^8),x, algorithm="fricas")
```

```
[Out] 1/8*(4*c*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d
^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c
*d*e^3)/(a*c^4))^(1/4)*arctan((a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*
c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3
*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*e^2 + a*c^3*d*e^4)*((a*c^4*sqr
t(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^
2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/
4)/((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x + (
a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*sqrt(((a^4
*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c
^4*e^8)*x^2 + (2*a^3*c^7*d*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 +
38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^4
*c^2*d^6 - 7*a^3*c^3*d^4*e^2 + 7*a^2*c^4*d^2*e^4 - a*c^5*e^6)*sqr
t((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/
(a*c^4)))/(a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a*c
^3*d^2*e^6 + c^4*e^8)))) - 4*c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c
*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*
c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)*arctan((a^2*c^6*e*s
qrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*
d^2*e^6 + c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a
*c^3*d*e^4)*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c
^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e +
 4*c*d*e^3)/(a*c^4))^(1/4)/((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*
d^2*e^4 + c^3*e^6)*x + (a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e
^4 + c^3*e^6)*sqrt(((a^4*d^8 - 4*a^3*c*d^6*e^2 - 10*a^2*c^2*d^4*e
^4 - 4*a*c^3*d^2*e^6 + c^4*e^8)*x^2 - (2*a^3*c^7*d*e*sqrt(-(a^4*d
^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c
^4*e^8)/(a^3*c^9)) - a^4*c^2*d^6 + 7*a^3*c^3*d^4*e^2 - 7*a^2*c^4*
d^2*e^4 + a*c^5*e^6)*sqrt(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e
^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9))
- 4*a*d^3*e + 4*c*d*e^3)/(a*c^4)))/(a^4*d^8 - 4*a^3*c*d^6*e^2 - 1
0*a^2*c^2*d^4*e^4 - 4*a*c^3*d^2*e^6 + c^4*e^8)))) - c*((a*c^4*sqr
t(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^
2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/
4)*log((a^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x
+ (a^2*c^6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e
^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c
^2*d^3*e^2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e
^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9))
+ 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)) + c*((a*c^4*sqrt(-(a^4*d
^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c
^4*e^8)/(a^3*c^9)) + 4*a*d^3*e - 4*c*d*e^3)/(a*c^4))^(1/4)*log((a
^3*d^6 - 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x - (a^2*c^
6*e*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + a^3*c*d^5 - 6*a^2*c^2*d^3*e^
2 + a*c^3*d*e^4)*((a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) + 4*a*d^3
*e - 4*c*d*e^3)/(a*c^4))^(1/4)) + c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*
a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/
(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d^6 -
 5*a^2*c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x + (a^2*c^6*e*sqrt
(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2
*e^6 + c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a*c^
3*d*e^4)*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*
d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*
c*d*e^3)/(a*c^4))^(1/4)) - c*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d
^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^
9)) - 4*a*d^3*e + 4*c*d*e^3)/(a*c^4))^(1/4)*log((a^3*d^6 - 5*a^2*
c*d^4*e^2 - 5*a*c^2*d^2*e^4 + c^3*e^6)*x - (a^2*c^6*e*sqrt(-(a^4*
d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a*c^3*d^2*e^6 +
c^4*e^8)/(a^3*c^9)) - a^3*c*d^5 + 6*a^2*c^2*d^3*e^2 - a*c^3*d*e^4
)*(-(a*c^4*sqrt(-(a^4*d^8 - 12*a^3*c*d^6*e^2 + 38*a^2*c^2*d^4*e^4
 - 12*a*c^3*d^2*e^6 + c^4*e^8)/(a^3*c^9)) - 4*a*d^3*e + 4*c*d*e^3
)/(a*c^4))^(1/4)) + 8*d*x)/c
```

Sympy [A] time = 47.5118, size = 204, normalized size = 0.27

$$\text{RootSum}\left(16777216t^8a^3c^9 + t^4\left(-32768a^3c^5d^3e + 32768a^2c^6de^3\right) + a^4d^8 + 4a^3cd^6e^2 + 6a^2c^2d^4e^4 + 4ac^3d^2e^6 + c^4e^8, \left(t \mapsto t\right.\right.$$

$$+ \frac{dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x**4)/(c+a/x**8),x)
```

```
[Out] RootSum(16777216*_t**8*a**3*c**9 + _t**4*(-32768*a**3*c**5*d**3*e
 + 32768*a**2*c**6*d*e**3) + a**4*d**8 + 4*a**3*c*d**6*e**2 + 6*a
**2*c**2*d**4*e**4 + 4*a*c**3*d**2*e**6 + c**4*e**8, Lambda(_t, _
t*log(x + (-32768*_t**5*a**2*c**6*e - 8*_t*a**3*c*d**5 + 80*_t*a*
*2*c**2*d**3*e**2 - 40*_t*a*c**3*d*e**4)/(a**3*d**6 - 5*a**2*c*d*
*4*e**2 - 5*a*c**2*d**2*e**4 + c**3*e**6)))) + d*x/c
```

GIAC/XCAS [A] time = 0.311153, size = 873, normalized size = 1.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^4)/(c + a/x^8),x, algorithm="giac")
```

```
[Out] d*x/c - 1/8*(c*sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2
) + 2)*(a/c)^(1/8))*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/c)^(1/8))
/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) - 1/8*(c*sqrt(-sqrt(2) +
2)*(a/c)^(5/8)*e + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*arctan((2*x
 - sqrt(-sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)
))/(a*c) + 1/8*(c*sqrt(sqrt(2) + 2)*(a/c)^(5/8)*e - a*d*sqrt(-sqr
t(2) + 2)*(a/c)^(1/8))*arctan((2*x + sqrt(sqrt(2) + 2)*(a/c)^(1/8
))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c) + 1/8*(c*sqrt(sqrt(2)
+ 2)*(a/c)^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*arctan((
2*x - sqrt(sqrt(2) + 2)*(a/c)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/c)^(1
/8)))/(a*c) - 1/16*(c*sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*e + a*d*sqrt
(sqrt(2) + 2)*(a/c)^(1/8))*ln(x^2 + x*sqrt(sqrt(2) + 2)*(a/c)^(1/
8) + (a/c)^(1/4))/(a*c) + 1/16*(c*sqrt(-sqrt(2) + 2)*(a/c)^(5/8)*
e + a*d*sqrt(sqrt(2) + 2)*(a/c)^(1/8))*ln(x^2 - x*sqrt(sqrt(2) +
2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) + 1/16*(c*sqrt(sqrt(2) + 2)*(
a/c)^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*ln(x^2 + x*sqr
t(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c) - 1/16*(c*sqrt(s
qrt(2) + 2)*(a/c)^(5/8)*e - a*d*sqrt(-sqrt(2) + 2)*(a/c)^(1/8))*l
n(x^2 - x*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + (a/c)^(1/4))/(a*c)
```

3.41 $\int \frac{d+\frac{e}{x^4}}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=433

$$\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{dx}{c}$$

```
[Out] (d*x)/c + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e -
 (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b +
 Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 2.03821, antiderivative size = 433, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-\frac{-2acd+b^2d-bce}{\sqrt{b^2-4ac}}+bd-ce\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{dx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

```
[Out] (d*x)/c + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e -
 (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b +
 Sqrt[b^2 - 4*a*c])^(3/4)) + ((b*d - c*e + (b^2*d - 2*a*c*d - b*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) + ((b*d - c*e - (b^2*d - 2*a*c*d - b*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 171.85, size = 432, normalized size = 1.

$$\frac{dx}{c} - \frac{2^{\frac{3}{4}}\left(-2acd + b(bd-ce) - \sqrt{-4ac+b^2}(bd-ce)\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}} - \frac{2^{\frac{3}{4}}\left(-2acd + b(bd-ce) - \sqrt{-4ac+b^2}(bd-ce)\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}} + \frac{2^{\frac{3}{4}}\left(-2acd + b(bd-ce) + \sqrt{-4ac+b^2}(bd-ce)\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}} + \frac{2^{\frac{3}{4}}\left(-2acd + b(bd-ce) + \sqrt{-4ac+b^2}(bd-ce)\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e/x**4)/(c+a/x**8+b/x**4),x)
```

```
[Out] d*x/c - 2**(3/4)*(-2*a*c*d + b*(b*d - c*e) - sqrt(-4*a*c + b**2)*
(b*d - c*e))*atan(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))*
*(1/4))/(4*c**(5/4)*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c
 + b**2)) - 2**(3/4)*(-2*a*c*d + b*(b*d - c*e) - sqrt(-4*a*c + b*
*2)*(b*d - c*e))*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b*
*2))**(1/4))/(4*c**(5/4)*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-
4*a*c + b**2)) + 2**(3/4)*(-2*a*c*d + b*(b*d - c*e) + sqrt(-4*a*c
 + b**2)*(b*d - c*e))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c
+ b**2))**(1/4))/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*sq
rt(-4*a*c + b**2)) + 2**(3/4)*(-2*a*c*d + b*(b*d - c*e) + sqrt(-4
*a*c + b**2)*(b*d - c*e))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4
*a*c + b**2))**(1/4))/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/
4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.104878, size = 88, normalized size = 0.2

$$\frac{dx}{c} - \frac{\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4bd\log(x-\#1) - \#1^4ce\log(x-\#1) + ad\log(x-\#1)}{2\#1^7c + \#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e/x^4)/(c + a/x^8 + b/x^4),x]
```

```
[Out] (d*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*d*Log[x - #1] + b*d*
Log[x - #1]*#1^4 - c*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/
(4*c)
```

Maple [C] time = 0.008, size = 67, normalized size = 0.2

$$\frac{dx}{c} + \frac{1}{4c}\sum_{\_R = RootOf(c\_Z^8 + \_Z^4 b + a)} \frac{\left((-bd+ce)\_R^4 - ad\right)\ln(x - \_R)}{2\_R^7c + \_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e/x^4)/(c+a/x^8+b/x^4),x)
```

```
[Out] 1/c*d*x+1/4/c*sum(((-b*d+c*e)*_R^4-a*d)/(2*_R^7*c+_R^3*b)*ln(x-_R
),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{dx}{c} + \frac{-\int \frac{(bd-ce)x^4+ad}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^4)/(c + b/x^4 + a/x^8),x, algorithm="maxima")
```

```
[Out] d*x/c + integrate(-((b*d - c*e)*x^4 + a*d)/(c*x^8 + b*x^4 + a), x
)/c
```

Fricas [A] time = 3.28539, size = 17781, normalized size = 41.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^4)/(c + b/x^4 + a/x^8),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c + 5*
a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b
^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 -
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8
+ 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4
*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2
+ 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4
*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*
c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)))*arctan(-1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2
- 4*a^3*c^3)*d^5 - 4*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d^4*e +
6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^3*e^2 - 4*(b^3*c^3 - 4*a*
b*c^4)*d^2*e^3 + (b^2*c^4 - 4*a*c^5)*d*e^4 - ((b^5*c^5 - 8*a*b^3*
c^6 + 16*a^2*b*c^7)*d - 2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*e)
*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^
2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28
*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 +
 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^
6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3
*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6
*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8
 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d
^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e
- 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e
^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*
b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a
*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12
*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2
```

*c^6 + 16*a^2*c^7)))/((5*b*c^4*d*e^5 - c^5*e^6 - (a*b^4 - 3*a^2*b
^2*c + a^3*c^2)*d^6 + (b^5 + a*b^3*c - 7*a^2*b*c^2)*d^5*e - 5*(b^
4*c - a*b^2*c^2 - a^2*c^3)*d^4*e^2 + 10*(b^3*c^2 - a*b*c^3)*d^3*e
^3 - 5*(2*b^2*c^3 - a*c^4)*d^2*e^4)*x + sqrt(1/2)*(5*b*c^4*d*e^5
- c^5*e^6 - (a*b^4 - 3*a^2*b^2*c + a^3*c^2)*d^6 + (b^5 + a*b^3*c
- 7*a^2*b*c^2)*d^5*e - 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^4*e^2 +
10*(b^3*c^2 - a*b*c^3)*d^3*e^3 - 5*(2*b^2*c^3 - a*c^4)*d^2*e^4)*s
qrt((2*(6*b*c^5*d*e^7 - c^6*e^8 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^
2)*d^8 + 2*(a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*d^7*e - (b^6 + 7*a*b
^4*c - 15*a^2*b^2*c^2 - 4*a^3*c^3)*d^6*e^2 + 2*(3*b^5*c + 3*a*b^3
*c^2 - 11*a^2*b*c^3)*d^5*e^3 - 5*(3*b^4*c^2 - a*b^2*c^3 - 2*a^2*c
^4)*d^4*e^4 + 10*(2*b^3*c^3 - a*b*c^4)*d^3*e^5 - (15*b^2*c^4 - 4*
a*c^5)*d^2*e^6)*x^2 - sqrt(1/2)*((b^8 - 9*a*b^6*c + 27*a^2*b^4*c^
2 - 30*a^3*b^2*c^3 + 8*a^4*c^4)*d^6 - 2*(3*b^7*c - 23*a*b^5*c^2 +
 53*a^2*b^3*c^3 - 36*a^3*b*c^4)*d^5*e + 2*(8*b^6*c^2 - 52*a*b^4*c
^3 + 87*a^2*b^2*c^4 - 28*a^3*c^5)*d^4*e^2 - 12*(2*b^5*c^3 - 11*a*
b^3*c^4 + 12*a^2*b*c^5)*d^3*e^3 + 7*(3*b^4*c^4 - 14*a*b^2*c^5 + 8
*a^2*c^6)*d^2*e^4 - 10*(b^3*c^5 - 4*a*b*c^6)*d*e^5 + 2*(b^2*c^6 -
 4*a*c^7)*e^6 - ((b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^
3*b*c^8)*d^2 - 2*(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^
3*c^9)*d*e)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5
*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b
^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*
a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5
+ 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*
b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b^
3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 + (b^4*c
^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 +
 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*
(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 +
 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*
c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c
^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6
 + 16*a^2*c^7)))/(6*b*c^5*d*e^7 - c^6*e^8 - (a^2*b^4 - 3*a^3*b^2*
c + a^4*c^2)*d^8 + 2*(a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*d^7*e - (b
^6 + 7*a*b^4*c - 15*a^2*b^2*c^2 - 4*a^3*c^3)*d^6*e^2 + 2*(3*b^5*c
 + 3*a*b^3*c^2 - 11*a^2*b*c^3)*d^5*e^3 - 5*(3*b^4*c^2 - a*b^2*c^3
 - 2*a^2*c^4)*d^4*e^4 + 10*(2*b^3*c^3 - a*b*c^4)*d^3*e^5 - (15*b^
2*c^4 - 4*a*c^5)*d^2*e^6)))) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^
4 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2
+ 2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3
 - 2*a*c^4)*d*e^3 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8
*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*
c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b
*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^
4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d
^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/2*((b^6 - 7*a
*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 4*(b^5*c - 6*a*b^3*c^2
 + 8*a^2*b*c^3)*d^4*e + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^3
e^2 - 4(b^3*c^3 - 4*a*b*c^4)*d^2*e^3 + (b^2*c^4 - 4*a*c^5)*d*e^
4 + ((b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d - 2*(b^4*c^6 - 8*a*
b^2*c^7 + 16*a^2*c^8)*e)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^
7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6
*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*
b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 -
71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^
3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*
e^4 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^
2 + 2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c
^3 - 2*a*c^4)*d*e^3 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-
(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c

^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^
2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2
*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*
e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)
*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/((5*b*c^4*d*e^5 - c^
5*e^6 - (a*b^4 - 3*a^2*b^2*c + a^3*c^2)*d^6 + (b^5 + a*b^3*c - 7*
a^2*b*c^2)*d^5*e - 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^4*e^2 + 10*(
b^3*c^2 - a*b*c^3)*d^3*e^3 - 5*(2*b^2*c^3 - a*c^4)*d^2*e^4)*x + s
qrt(1/2)*(5*b*c^4*d*e^5 - c^5*e^6 - (a*b^4 - 3*a^2*b^2*c + a^3*c^
2)*d^6 + (b^5 + a*b^3*c - 7*a^2*b*c^2)*d^5*e - 5*(b^4*c - a*b^2*c
^2 - a^2*c^3)*d^4*e^2 + 10*(b^3*c^2 - a*b*c^3)*d^3*e^3 - 5*(2*b^2
*c^3 - a*c^4)*d^2*e^4)*sqrt((2*(6*b*c^5*d*e^7 - c^6*e^8 - (a^2*b^
4 - 3*a^3*b^2*c + a^4*c^2)*d^8 + 2*(a*b^5 - a^2*b^3*c - 3*a^3*b*c
^2)*d^7*e - (b^6 + 7*a*b^4*c - 15*a^2*b^2*c^2 - 4*a^3*c^3)*d^6*e^
2 + 2*(3*b^5*c + 3*a*b^3*c^2 - 11*a^2*b*c^3)*d^5*e^3 - 5*(3*b^4*c
^2 - a*b^2*c^3 - 2*a^2*c^4)*d^4*e^4 + 10*(2*b^3*c^3 - a*b*c^4)*d^
3*e^5 - (15*b^2*c^4 - 4*a*c^5)*d^2*e^6)*x^2 - sqrt(1/2)*((b^8 - 9
*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4)*d^6 - 2*(
3*b^7*c - 23*a*b^5*c^2 + 53*a^2*b^3*c^3 - 36*a^3*b*c^4)*d^5*e + 2
*(8*b^6*c^2 - 52*a*b^4*c^3 + 87*a^2*b^2*c^4 - 28*a^3*c^5)*d^4*e^2
 - 12*(2*b^5*c^3 - 11*a*b^3*c^4 + 12*a^2*b*c^5)*d^3*e^3 + 7*(3*b^
4*c^4 - 14*a*b^2*c^5 + 8*a^2*c^6)*d^2*e^4 - 10*(b^3*c^5 - 4*a*b*c
^6)*d*e^5 + 2*(b^2*c^6 - 4*a*c^7)*e^6 + ((b^7*c^5 - 12*a*b^5*c^6
+ 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*d^2 - 2*(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)*d*e)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^
8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*
d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e
 - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*
e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35
*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*
a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 1
2*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b*c^4*e^4
+ (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 +
2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 -
 2*a*c^4)*d*e^3 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b
*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^
4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c
^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4
+ 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2
*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(6*b*c^5*d*e^7 - c^6*e^8
 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*d^8 + 2*(a*b^5 - a^2*b^3*c -
 3*a^3*b*c^2)*d^7*e - (b^6 + 7*a*b^4*c - 15*a^2*b^2*c^2 - 4*a^3*c
^3)*d^6*e^2 + 2*(3*b^5*c + 3*a*b^3*c^2 - 11*a^2*b*c^3)*d^5*e^3 -
5*(3*b^4*c^2 - a*b^2*c^3 - 2*a^2*c^4)*d^4*e^4 + 10*(2*b^3*c^3 - a
*b*c^4)*d^3*e^5 - (15*b^2*c^4 - 4*a*c^5)*d^2*e^6)))) - c*sqrt(sqr
t(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4
*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3
)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 + (b^4*c^5 - 8*a*b^2*c^6
+ 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b
^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a
*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 2
1*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^
5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(
7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*
log(-(5*b*c^4*d*e^5 - c^5*e^6 - (a*b^4 - 3*a^2*b^2*c + a^3*c^2)*d
^6 + (b^5 + a*b^3*c - 7*a^2*b*c^2)*d^5*e - 5*(b^4*c - a*b^2*c^2 -
 a^2*c^3)*d^4*e^2 + 10*(b^3*c^2 - a*b*c^3)*d^3*e^3 - 5*(2*b^2*c^3
 - a*c^4)*d^2*e^4)*x + 1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4
*a^3*c^3)*d^5 - 4*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d^4*e + 6*(
b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^3*e^2 - 4*(b^3*c^3 - 4*a*b*c
^4)*d^2*e^3 + (b^2*c^4 - 4*a*c^5)*d*e^4 - ((b^5*c^5 - 8*a*b^3*c^6
 + 16*a^2*b*c^7)*d - 2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*e)*sq
rt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^
2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13

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*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*
d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*
c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c + 5*
a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b
^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 -
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8
+ 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4
*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2
+ 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4
*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*
c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5
*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)
*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*
d*e^3 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7
 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c
^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*
c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^
3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3
*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6
*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(-(5*b*c^4*d*e^5 - c^5*e^6 - (a
*b^4 - 3*a^2*b^2*c + a^3*c^2)*d^6 + (b^5 + a*b^3*c - 7*a^2*b*c^2)
*d^5*e - 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^4*e^2 + 10*(b^3*c^2 -
a*b*c^3)*d^3*e^3 - 5*(2*b^2*c^3 - a*c^4)*d^2*e^4)*x - 1/2*((b^6 -
 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 4*(b^5*c - 6*a*b^3
*c^2 + 8*a^2*b*c^3)*d^4*e + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*d^3*e^2 - 4*(b^3*c^3 - 4*a*b*c^4)*d^2*e^3 + (b^2*c^4 - 4*a*c^5)*
d*e^4 - ((b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d - 2*(b^4*c^6 -
8*a*b^2*c^7 + 16*a^2*c^8)*e)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8
*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7
*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8
*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^
4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6
)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*
c^4*e^4 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^
2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b
^2*c^3 - 2*a*c^4)*d*e^3 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^
2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13
*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*
d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*
c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1
/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b
^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d
^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 - (b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*
c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^
4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a
*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 +
 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b
^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log
(-(5*b*c^4*d*e^5 - c^5*e^6 - (a*b^4 - 3*a^2*b^2*c + a^3*c^2)*d^6
+ (b^5 + a*b^3*c - 7*a^2*b*c^2)*d^5*e - 5*(b^4*c - a*b^2*c^2 - a^
2*c^3)*d^4*e^2 + 10*(b^3*c^2 - a*b*c^3)*d^3*e^3 - 5*(2*b^2*c^3 -
a*c^4)*d^2*e^4)*x + 1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^
3*c^3)*d^5 - 4*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d^4*e + 6*(b^4
*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^3*e^2 - 4*(b^3*c^3 - 4*a*b*c^4)
*d^2*e^3 + (b^2*c^4 - 4*a*c^5)*d*e^4 + ((b^5*c^5 - 8*a*b^3*c^6 +
16*a^2*b*c^7)*d - 2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*e)*sqrt(
-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
```

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*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*
c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b
^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^
2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4
*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7
)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*b^3*c + 5*a^2
*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^3*e + 6*(b^3*
c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e^3 - (b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8
*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7
*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8
*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^
4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6
)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*e^4 + (b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^
3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*c^3 - 2*a*c^4)*d*e
^3 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(-(8*b*c^7*d*e^7 -
c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)
*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5
)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 -
 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^
5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)))*log(-(5*b*c^4*d*e^5 - c^5*e^6 - (a*b^
4 - 3*a^2*b^2*c + a^3*c^2)*d^6 + (b^5 + a*b^3*c - 7*a^2*b*c^2)*d^
5*e - 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^4*e^2 + 10*(b^3*c^2 - a*b
*c^3)*d^3*e^3 - 5*(2*b^2*c^3 - a*c^4)*d^2*e^4)*x - 1/2*((b^6 - 7*
a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 4*(b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d^4*e + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^
3*e^2 - 4*(b^3*c^3 - 4*a*b*c^4)*d^2*e^3 + (b^2*c^4 - 4*a*c^5)*d*e
^4 + ((b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d - 2*(b^4*c^6 - 8*a
*b^2*c^7 + 16*a^2*c^8)*e)*sqrt(-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b
^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^
6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7
*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 -
 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d
^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4
*e^4 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^4 - 4*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^2*
c^3 - 2*a*c^4)*d*e^3 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt(
-(8*b*c^7*d*e^7 - c^8*e^8 - (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
*a^3*b^2*c^3 + a^4*c^4)*d^8 + 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*
c^3 - 2*a^3*b*c^4)*d^7*e - 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b
^2*c^4 - 3*a^3*c^5)*d^6*e^2 + 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^
2*b*c^5)*d^5*e^3 - 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4
*e^4 + 8*(7*b^3*c^5 - 8*a*b*c^6)*d^3*e^5 - 4*(7*b^2*c^6 - 3*a*c^7
)*d^2*e^6)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - 4*d*x)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e/x**4)/(c+a/x**8+b/x**4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + \frac{e}{x^4}}{c + \frac{b}{x^4} + \frac{a}{x^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d + e/x^4)/(c + b/x^4 + a/x^8),x, algorithm="giac")
```

```
[Out] integrate((d + e/x^4)/(c + b/x^4 + a/x^8), x)
```

3.42 $\int \frac{(d+ex^n)^3}{a+cx^{2n}}\,dx$

Optimal. Leaf size=141

$$\frac{ex^{n+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(n+1)}+\frac{dx\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{3de^2x}{c}+\frac{e^3x^{n+1}}{c(n+1)}$$

```
[Out] (3*d*e^2*x)/c + (e^3*x^(1 + n))/(c*(1 + n)) + (d*(c*d^2 - 3*a*e^2
)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a
)])/(a*c) + (e*(3*c*d^2 - a*e^2)*x^(1 + n)*Hypergeometric2F1[1, (
1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c*(1 + n))
```

Rubi [A] time = 0.288941, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{ex^{n+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(n+1)}+\frac{dx\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{3de^2x}{c}+\frac{e^3x^{n+1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^3/(a + c*x^(2*n)),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^(1 + n))/(c*(1 + n)) + (d*(c*d^2 - 3*a*e^2
)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a
)])/(a*c) + (e*(3*c*d^2 - a*e^2)*x^(1 + n)*Hypergeometric2F1[1, (
1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c*(1 + n))
```

Rubi in Sympy [A] time = 27.7449, size = 151, normalized size = 1.07

$$\frac{d^3x\,{}_2F_1\left(\begin{matrix}1,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a}+\frac{3d^2ex^{n+1}\,{}_2F_1\left(\begin{matrix}1,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a(n+1)}+\frac{3de^2x^{2n+1}\,{}_2F_1\left(\begin{matrix}1,\frac{n+\frac{1}{2}}{n}\\ 2+\frac{1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a(2n+1)}+\frac{e^3x^{3n+1}\,{}_2F_1\left(\begin{matrix}1,\frac{3n+1}{2n}\\ \frac{5n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a(3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**3/(a+c*x**(2*n)),x)
```

```
[Out] d**3*x*hyper((1, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/a + 3*d
**2*e*x**(n + 1)*hyper((1, (n + 1)/(2*n)), ((3*n + 1)/(2*n),), -c
*x**(2*n)/a)/(a*(n + 1)) + 3*d*e**2*x**(2*n + 1)*hyper((1, (n + 1
/2)/n), (2 + 1/(2*n),), -c*x**(2*n)/a)/(a*(2*n + 1)) + e**3*x**(3
*n + 1)*hyper((1, (3*n + 1)/(2*n)), ((5*n + 1)/(2*n),), -c*x**(2*
n)/a)/(a*(3*n + 1))
```

Mathematica [A] time = 0.188088, size = 128, normalized size = 0.91

$$\frac{d(n+1)x\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+ex\left(x^n\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+ae\left(3d(n+1)+ex\right.\right.}{ac(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^3/(a + c*x^(2*n)),x]
```

```
[Out] (d*(c*d^2 - 3*a*e^2)*(1 + n)*x*Hypergeometric2F1[1, 1/(2*n), 1 +
1/(2*n), -((c*x^(2*n))/a)] + e*x*(a*e*(3*d*(1 + n) + e*x^n) + (3*
c*d^2 - a*e^2)*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1
))/2, -((c*x^(2*n))/a)]))/(a*c*(1 + n))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int\frac{(d+ex^n)^3}{a+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,de^2(n+1)x+e^3xx^n}{c(n+1)}-\int-\frac{cd^3-3\,ade^2+\left(3\,cd^2e-ae^3\right)x^n}{c^2x^{2\,n}+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a),x, algorithm="maxima")
```

```
[Out] (3*d*e^2*(n + 1)*x + e^3*x*x^n)/(c*(n + 1)) - integrate(-(c*d^3 -
 3*a*d*e^2 + (3*c*d^2*e - a*e^3)*x^n)/(c^2*x^(2*n) + a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3\,n}+3\,de^2x^{2\,n}+3\,d^2ex^n+d^3}{cx^{2\,n}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c*x
^(2*n) + a), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**3/(a+c*x**(2*n)),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{cx^{2n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + a), x)
```

3.43 $\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx$

Optimal. Leaf size=107

$$\frac{x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeom
etric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*
(1 + n))
```

Rubi [A] time = 0.197116, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{2dex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2/(a + c*x^(2*n)),x]
```

```
[Out] (e^2*x)/c + ((c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(a*c) + (2*d*e*x^(1 + n)*Hypergeom
etric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*
(1 + n))
```

Rubi in Sympy [A] time = 19.2901, size = 104, normalized size = 0.97

$$\frac{d^2x\,{}_2F_1\left(\begin{matrix}1,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a}+\frac{2dex^{n+1}\,{}_2F_1\left(\begin{matrix}1,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a\,(n+1)}+\frac{e^2x^{2n+1}\,{}_2F_1\left(\begin{matrix}1,\frac{n+\frac{1}{2}}{n}\\ 2+\frac{1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a\,(2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2/(a+c*x**(2*n)),x)
```

```
[Out] d**2*x*hyper((1, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/a + 2*d
*e*x**(n + 1)*hyper((1, (n + 1)/(2*n)), ((3*n + 1)/(2*n),), -c*x*
*(2*n)/a)/(a*(n + 1)) + e**2*x**(2*n + 1)*hyper((1, (n + 1/2)/n),
 (2 + 1/(2*n),), -c*x**(2*n)/a)/(a*(2*n + 1))
```

Mathematica [A] time = 0.122424, size = 107, normalized size = 1.

$$\frac{x\left((n+1)\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+e\left(2cdx^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+ae(n+1)\right)\right)}{ac(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^2/(a + c*x^(2*n)),x]
```

```
[Out] (x*((c*d^2 - a*e^2)*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(
2*n), -((c*x^(2*n))/a)] + e*(a*e*(1 + n) + 2*c*d*x^n*Hypergeometr
```

```
ic2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])))/(a*c
*(1 + n))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^2}{a+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{c}+\int \frac{2\,cdex^n+cd^2-ae^2}{c^2x^{2\,n}+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a),x, algorithm="maxima")
```

```
[Out] e^2*x/c + integrate((2*c*d*e*x^n + c*d^2 - a*e^2)/(c^2*x^(2*n) +
a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2\,n}+2\,dex^n+d^2}{cx^{2\,n}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c*x^(2*n) + a), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2/(a+c*x**(2*n)),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)^2}{cx^{2\,n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a),x, algorithm="giac")

[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + a), x)
```

3.44 $\int \frac{d+ex^n}{a+cx^{2n}}\,dx$

Optimal. Leaf size=83

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/
a)])/a + (e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^
(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

Rubi [A] time = 0.0658563, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + c*x^(2*n)),x]
```

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/
a)])/a + (e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^
(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n))
```

Rubi in Sympy [A] time = 9.96631, size = 60, normalized size = 0.72

$$\frac{dx{}_2F_1\left(\begin{matrix}1,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}{}_2F_1\left(\begin{matrix}1,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+c*x**(2*n)),x)
```

```
[Out] d*x*hyper((1, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/a + e*x**(
n + 1)*hyper((1, (n + 1)/(2*n)), ((3*n + 1)/(2*n),), -c*x**(2*n)/
a)/(a*(n + 1))
```

Mathematica [A] time = 0.0442213, size = 82, normalized size = 0.99

$$\frac{x\left(d(n+1)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+ex^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)/(a + c*x^(2*n)),x]
```

```
[Out] (x*(d*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(
2*n))/a)] + e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1)
)/2, -((c*x^(2*n))/a)]))/(a*(1 + n))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{d + e x^n}{a + c x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e x^n + d}{c x^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e x^n + d}{c x^{2n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c*x^(2*n) + a), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a+c*x**(2*n)),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e x^n + d}{c x^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a),x, algorithm="giac")

[Out] integrate((e*x^n + d)/(c*x^(2*n) + a), x)
```

3.45 $\int \frac{1}{(d+ex^n)(a+cx^{2n})}\,dx$

Optimal. Leaf size=152

$$-\frac{cex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)}+\frac{cdx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)}$$

```
[Out] (c*d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n)
)/a)])/(a*(c*d^2 + a*e^2)) + (e^2*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)) - (c*e*x^(1 + n)*H
ypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/
a)])/(a*(c*d^2 + a*e^2)*(1 + n))
```

Rubi [A] time = 0.224354, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{cex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)}+\frac{cdx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)*(a + c*x^(2*n))),x]
```

```
[Out] (c*d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n)
)/a)])/(a*(c*d^2 + a*e^2)) + (e^2*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)) - (c*e*x^(1 + n)*H
ypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/
a)])/(a*(c*d^2 + a*e^2)*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^{2n}\right)\left(d+ex^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)/(a+c*x**(2*n)),x)
```

```
[Out] Integral(1/((a + c*x**(2*n))*(d + e*x**n)), x)
```

Mathematica [A] time = 0.153087, size = 131, normalized size = 0.86

$$\frac{x\left(cd^2(n+1)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+e\left(ae(n+1)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)-cdx^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)\right)}{ad(n+1)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)*(a + c*x^(2*n))),x]
```

```
[Out] (x*(c*d^2*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c
*x^(2*n))/a)] + e*(a*e*(1 + n)*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), -((e*x^n)/d)] - c*d*x^n*Hypergeometric2F1[1, (1 + n)/(2*n)
, (3 + n^(-1))/2, -((c*x^(2*n))/a)])))/(a*d*(c*d^2 + a*e^2)*(1 +
```

```
n))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)(a + cx^{2n})}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)/(a+c*x^(2*n)),x)

[Out] int(1/(d+e*x^n)/(a+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)(ex^n + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)*(e*x^n + d)),x, algorithm="maxima")

[Out] integrate(1/((c*x^(2*n) + a)*(e*x^n + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{aex^n + ad + (cex^n + cd)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)*(e*x^n + d)),x, algorithm="fricas")

[Out] integral(1/(a*e*x^n + a*d + (c*e*x^n + c*d)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)/(a+c*x**(2*n)),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)(ex^n + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)*(e*x^n + d)),x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + a)*(e*x^n + d)), x)
```

3.46 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})}\,dx$

Optimal. Leaf size=205

$$-\frac{2c^2dex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2}+\frac{cx\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2}+\frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)}$$

```
[Out] (c*(c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^2) + (2*c*e^2*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^2
- (2*c^2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n
^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^2*(1 + n)) + (e^2
*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(
c*d^2 + a*e^2))
```

Rubi [A] time = 0.354646, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{2c^2dex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2}+\frac{cx\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2}+\frac{2ce^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
```

```
[Out] (c*(c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^2) + (2*c*e^2*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(c*d^2 + a*e^2)^2
- (2*c^2*d*e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n
^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^2*(1 + n)) + (e^2
*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(
c*d^2 + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^{2n}\right)\left(d+ex^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)**2/(a+c*x**(2*n)),x)
```

```
[Out] Integral(1/((a + c*x**(2*n))*(d + e*x**n)**2), x)
```

Mathematica [A] time = 1.13004, size = 188, normalized size = 0.92

$$\frac{x\left(e\left(-\frac{2c^2dx^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)}+\frac{\left(ae^3(n-1)+cd^2e(3n-1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2n}+\frac{ae^3+cd^2e}{d^2n+denx^n}\right)+\frac{c\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{a}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]   Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))),x]
```

```
[Out] (x*((c*(c*d^2 - a*e^2)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n),
 -((c*x^(2*n))/a)])/a + e*((c*d^2*e + a*e^3)/(d^2*n + d*e*n*x^n)
+ ((a*e^3*(-1 + n) + c*d^2*e*(-1 + 3*n))*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*n) - (2*c^2*d*x^n*Hypergeome
tric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(
1 + n)))))/(c*d^2 + a*e^2)^2
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+e x^{n}\right)^{2} \left(a+c x^{2 n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{2} x}{c d^{4} n + a d^{2} e^{2} n + \left(c d^{3} e n + a d e^{3} n\right) x^{n}} + \left(c d^{2} e^{2} (3 n - 1) + a e^{4} (n - 1)\right) \int \frac{1}{c^{2} d^{6} n + 2 \, a c d^{4} e^{2} n + a^{2} d^{2} e^{4} n + \left(c^{2} d^{5} e n + 2 \, a c d^{3} e^{3} n + a^{2} d e^{5} n\right) x^{n}}\, dx - \int \frac{2 \, c^{2} d e x^{n} - c^{2} d^{2} + a c e^{2}}{a c^{2} d^{4} + 2 \, a^{2} c d^{2} e^{2} + a^{3} e^{4} + \left(c^{3} d^{4} + 2 \, a c^{2} d^{2} e^{2} + a^{2} c e^{4}\right) x^{2 n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((c*x^(2*n) + a)*(e*x^n + d)^2),x, algorithm="maxima")
```

```
[Out] e^2*x/(c*d^4*n + a*d^2*e^2*n + (c*d^3*e*n + a*d*e^3*n)*x^n) + (c*
d^2*e^2*(3*n - 1) + a*e^4*(n - 1))*integrate(1/(c^2*d^6*n + 2*a*c
*d^4*e^2*n + a^2*d^2*e^4*n + (c^2*d^5*e*n + 2*a*c*d^3*e^3*n + a^2
*d*e^5*n)*x^n), x) - integrate((2*c^2*d*e*x^n - c^2*d^2 + a*c*e^2
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2
*e^2 + a^2*c*e^4)*x^(2*n)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{c e^{2} x^{4 n} + 2 \, a d e x^{n} + a d^{2} + \left(2 \, c d e x^{n} + c d^{2} + a e^{2}\right) x^{2 n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((c*x^(2*n) + a)*(e*x^n + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(c*e^2*x^(4*n) + 2*a*d*e*x^n + a*d^2 + (2*c*d*e*x^n +
c*d^2 + a*e^2)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)**2/(a+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + a)(ex^n + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)*(e*x^n + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)*(e*x^n + d)^2), x)
```

3.47 $\int \frac{d+ex^n}{a-cx^{2n}} \, dx$

Optimal. Leaf size=81

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (c*x^(2*n))/a]
)/a + (e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1
))/2, (c*x^(2*n))/a])/(a*(1 + n))
```

Rubi [A] time = 0.0703563, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{dx\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a - c*x^(2*n)),x]
```

```
[Out] (d*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (c*x^(2*n))/a]
)/a + (e*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1
))/2, (c*x^(2*n))/a])/(a*(1 + n))
```

Rubi in Sympy [A] time = 10.3714, size = 56, normalized size = 0.69

$$\frac{dx{}_2F_1\left(\begin{matrix}1,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|\frac{cx^{2n}}{a}\right)}{a}+\frac{ex^{n+1}{}_2F_1\left(\begin{matrix}1,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|\frac{cx^{2n}}{a}\right)}{a\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a-c*x**(2*n)),x)
```

```
[Out] d*x*hyper((1, 1/(2*n)), ((n + 1/2)/n,), c*x**(2*n)/a)/a + e*x**(n
 + 1)*hyper((1, (n + 1)/(2*n)), ((3*n + 1)/(2*n),), c*x**(2*n)/a)
/(a*(n + 1))
```

Mathematica [A] time = 0.0687256, size = 80, normalized size = 0.99

$$\frac{x\left(d(n+1)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};\frac{cx^{2n}}{a}\right)+ex^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\frac{cx^{2n}}{a}\right)\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)/(a - c*x^(2*n)),x]
```

```
[Out] (x*(d*(1 + n)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), (c*x^(2*
n))/a] + e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2
, (c*x^(2*n))/a]))/(a*(1 + n))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{a - cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a-c*x^(2*n)),x)

[Out] int((d+e*x^n)/(a-c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^n + d}{cx^{2n} - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^n + d)/(c*x^(2*n) - a),x, algorithm="maxima")

[Out] -integrate((e*x^n + d)/(c*x^(2*n) - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{ex^n + d}{cx^{2n} - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^n + d)/(c*x^(2*n) - a),x, algorithm="fricas")

[Out] integral(-(e*x^n + d)/(c*x^(2*n) - a), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a-c*x**(2*n)),x)

[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{ex^n + d}{cx^{2n} - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e*x^n + d)/(c*x^(2*n) - a),x, algorithm="giac")

[Out] integrate(-(e*x^n + d)/(c*x^(2*n) - a), x)
```

3.48 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^2}\,dx$

Optimal. Leaf size=288

$$-\frac{e(1-n)x^{n+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}-\frac{d(1-2n)x\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}+\frac{x\left(ex^n\left(3cd^2-ae^2\right)+d\left(cd^2-3ae^2\right)\right)}{2acn\left(a+cx^{2n}\right)}+\frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{e^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(n+1)}$$

```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(2*a*c*n*(a +
 c*x^(2*n))) + (3*d*e^2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-
1))/2, -((c*x^(2*n))/a)])/(a*c) - (d*(c*d^2 - 3*a*e^2)*(1 - 2*n)*
x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)]
)/(2*a^2*c*n) + (e^3*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n)
, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c*(1 + n)) - (e*(3*c*d^2
- a*e^2)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

Rubi [A] time = 0.522391, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{e(1-n)x^{n+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}-\frac{d(1-2n)x\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}+\frac{x\left(ex^n\left(3cd^2-ae^2\right)+d\left(cd^2-3ae^2\right)\right)}{2acn\left(a+cx^{2n}\right)}+\frac{3de^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}+\frac{e^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(2*a*c*n*(a +
 c*x^(2*n))) + (3*d*e^2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-
1))/2, -((c*x^(2*n))/a)])/(a*c) - (d*(c*d^2 - 3*a*e^2)*(1 - 2*n)*
x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)]
)/(2*a^2*c*n) + (e^3*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n)
, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*c*(1 + n)) - (e*(3*c*d^2
- a*e^2)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n))
```

Rubi in Sympy [A] time = 27.384, size = 158, normalized size = 0.55

$$\frac{d^3x\,{}_2F_1\left(\begin{matrix}2,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^2}+\frac{3d^2ex^{n+1}\,{}_2F_1\left(\begin{matrix}2,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^2(n+1)}+\frac{3de^2x^{2n+1}\,{}_2F_1\left(\begin{matrix}2,\frac{n+\frac{1}{2}}{n}\\ 2+\frac{1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^2(2n+1)}+\frac{e^3x^{3n+1}\,{}_2F_1\left(\begin{matrix}2,\frac{3n+1}{2n}\\ \frac{5n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^2(3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**3/(a+c*x**(2*n))**2,x)
```

```
[Out] d**3*x*hyper((2, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/a**2 +
3*d**2*e*x**(n + 1)*hyper((2, (n + 1)/(2*n)), ((3*n + 1)/(2*n),),
 -c*x**(2*n)/a)/(a**2*(n + 1)) + 3*d*e**2*x**(2*n + 1)*hyper((2,
(n + 1/2)/n), (2 + 1/(2*n),), -c*x**(2*n)/a)/(a**2*(2*n + 1)) + e
**3*x**(3*n + 1)*hyper((2, (3*n + 1)/(2*n)), ((5*n + 1)/(2*n),),
-c*x**(2*n)/a)/(a**2*(3*n + 1))
```

Mathematica [A] time = 0.492747, size = 165, normalized size = 0.57

$$\frac{x\left(\left(3ade^2+cd^3(2n-1)\right)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+\frac{ex^n\left(ae^2(n+1)+3cd^2(n-1)\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{a\left(cd^2(d+3ex^n)-ae^2(3d+ex\right.}{a+cx^{2n}}\right.}{2a^2cn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*((a*(-(a*e^2*(3*d + e*x^n)) + c*d^2*(d + 3*e*x^n)))/(a + c*x^(
2*n)) + (3*a*d*e^2 + c*d^3*(-1 + 2*n))*Hypergeometric2F1[1, 1/(2*
n), 1 + 1/(2*n), -((c*x^(2*n))/a)] + (e*(3*c*d^2*(-1 + n) + a*e^2
*(1 + n))*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
 -((c*x^(2*n))/a)])/(1 + n)))/(2*a^2*c*n)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2\,n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,cd^2e-ae^3\right)xx^n+\left(cd^3-3\,ade^2\right)x}{2\,(ac^2nx^{2\,n}+a^2cn)}+\int \frac{cd^3(2\,n-1)+3\,ade^2+\left(ae^3(n+1)+3\,cd^2e(n-1)\right)x^n}{2\,(ac^2nx^{2\,n}+a^2cn)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*c*d^2*e - a*e^3)*x*x^n + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*n*
x^(2*n) + a^2*c*n) + integrate(1/2*(c*d^3*(2*n - 1) + 3*a*d*e^2 +
 (a*e^3*(n + 1) + 3*c*d^2*e*(n - 1))*x^n)/(a*c^2*n*x^(2*n) + a^2*
c*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3\,n}+3\,de^2x^{2\,n}+3\,d^2ex^n+d^3}{c^2x^{4\,n}+2\,acx^{2\,n}+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^2
*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**3/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{(cx^{2n} + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + a)^2, x)
```

3.49 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2} \, dx$

Optimal. Leaf size=203

$$-\frac{(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{de(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)}+\frac{x\left(-ae^2+cd^2+2cdex^n\right)}{2acn\left(a+cx^{2n}\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}$$

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(2*a*c*n*(a + c*x^(2*n))) + (e^
2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a
)])/(a*c) - ((c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(
2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n) - (d*e*(1 -
n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
-((c*x^(2*n))/a)])/(a^2*n*(1 + n))
```

Rubi [A] time = 0.345091, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{de(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)}+\frac{x\left(-ae^2+cd^2+2cdex^n\right)}{2acn\left(a+cx^{2n}\right)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(2*a*c*n*(a + c*x^(2*n))) + (e^
2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a
)])/(a*c) - ((c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(
2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n) - (d*e*(1 -
n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
-((c*x^(2*n))/a)])/(a^2*n*(1 + n))
```

Rubi in Sympy [A] time = 18.8567, size = 109, normalized size = 0.54

$$\frac{d^2x\,{}_2F_1\left(\begin{matrix}2,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^2}+\frac{2dex^{n+1}\,{}_2F_1\left(\begin{matrix}2,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^2\left(n+1\right)}+\frac{e^2x^{2n+1}\,{}_2F_1\left(\begin{matrix}2,\frac{n+\frac{1}{2}}{n}\\ 2+\frac{1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^2\left(2n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

```
[Out] d**2*x*hyper((2, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/a**2 +
2*d*e*x**(n + 1)*hyper((2, (n + 1)/(2*n)), ((3*n + 1)/(2*n),), -c
*x**(2*n)/a)/(a**2*(n + 1)) + e**2*x**(2*n + 1)*hyper((2, (n + 1/
2)/n), (2 + 1/(2*n),), -c*x**(2*n)/a)/(a**2*(2*n + 1))
```

Mathematica [A] time = 0.464009, size = 142, normalized size = 0.7

$$\frac{x\left(\frac{(ae^2+cd^2(2n-1))\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{c}+\frac{a(cd(d+2ex^n)-ae^2)}{c(a+cx^{2n})}+\frac{2de(n-1)x^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}\right)}{2a^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*((a*(-(a*e^2) + c*d*(d + 2*e*x^n)))/(c*(a + c*x^(2*n))) + ((a*
e^2 + c*d^2*(-1 + 2*n))*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n)
, -((c*x^(2*n))/a)])/c + (2*d*e*(-1 + n)*x^n*Hypergeometric2F1[1,
 (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(2*a
^2*n)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,cdexx^n+\left(cd^2-ae^2\right)x}{2\,(ac^2nx^{2\,n}+a^2cn)}+\int \frac{2\,cde(n-1)x^n+cd^2(2\,n-1)+ae^2}{2\,(ac^2nx^{2\,n}+a^2cn)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*d*e*x*x^n + (c*d^2 - a*e^2)*x)/(a*c^2*n*x^(2*n) + a^2*c*
n) + integrate(1/2*(2*c*d*e*(n - 1)*x^n + c*d^2*(2*n - 1) + a*e^2
)/(a*c^2*n*x^(2*n) + a^2*c*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^{2\,n}+2\,dex^n+d^2}{c^2x^{4\,n}+2\,acx^{2\,n}+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^2*x^(4*n) + 2*a*c*x^(
2*n) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{(cx^{2n} + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + a)^2, x)
```

3.50 $\int \frac{d+ex^n}{(a+cx^{2n})^2}\,dx$

Optimal. Leaf size=134

$$-\frac{d(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n}-\frac{e(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)}+\frac{x\left(d+ex^n\right)}{2an\left(a+cx^{2n}\right)}$$

```
[Out] (x*(d + e*x^n))/(2*a*n*(a + c*x^(2*n))) - (d*(1 - 2*n)*x*Hypergeo
metric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*n
) - (e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*n*(1 + n))
```

Rubi [A] time = 0.126051, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{d(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n}-\frac{e(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)}+\frac{x\left(d+ex^n\right)}{2an\left(a+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(d + e*x^n))/(2*a*n*(a + c*x^(2*n))) - (d*(1 - 2*n)*x*Hypergeo
metric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*n
) - (e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*n*(1 + n))
```

Rubi in Sympy [A] time = 19.1854, size = 100, normalized size = 0.75

$$\frac{x\left(d+ex^n\right)}{2an\left(a+cx^{2n}\right)}-\frac{dx\left(-2n+1\right){}_2F_1\left(\begin{matrix}1,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{2a^2n}-\frac{ex^{n+1}\left(-n+1\right){}_2F_1\left(\begin{matrix}1,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{2a^2n\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+c*x**(2*n))**2,x)
```

```
[Out] x*(d + e*x**n)/(2*a*n*(a + c*x**(2*n))) - d*x*(-2*n + 1)*hyper((1
, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/(2*a**2*n) - e*x**(n +
 1)*(-n + 1)*hyper((1, (n + 1)/(2*n)), ((3*n + 1)/(2*n),), -c*x**
(2*n)/a)/(2*a**2*n*(n + 1))
```

Mathematica [A] time = 0.152691, size = 137, normalized size = 1.02

$$\frac{x\left(d\left(2n^2+n-1\right)\left(a+cx^{2n}\right)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+e(n-1)x^n\left(a+cx^{2n}\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+a(n+1)(\right.}{2a^2n(n+1)\left(a+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)/(a + c*x^(2*n))^2,x]
```

```
[Out] (x*(a*(1 + n)*(d + e*x^n) + d*(-1 + n + 2*n^2)*(a + c*x^(2*n))*Hy
pergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)] + e*(-
```

```
1 + n)*x^n*(a + c*x^(2*n))*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)]))/(2*a^2*n*(1 + n)*(a + c*x^(2*n)
))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{d+e x^{n}}{\left(a+c x^{2 n}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{exx^{n}+dx}{2\,(acnx^{2\,n}+a^{2}n)}+\int \frac{e(n-1)x^{n}+d(2\,n-1)}{2\,(acnx^{2\,n}+a^{2}n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x*x^n + d*x)/(a*c*n*x^(2*n) + a^2*n) + integrate(1/2*(e*(n
 - 1)*x^n + d*(2*n - 1))/(a*c*n*x^(2*n) + a^2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^{n}+d}{c^{2}x^{4\,n}+2\,acx^{2\,n}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c^2*x^(4*n) + 2*a*c*x^(2*n) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^{n}+d}{\left(cx^{2\,n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + a)^2, x)
```

3.51 $\int \frac{1}{(d+ex^n)(a+cx^{2n})^2}\,dx$

Optimal. Leaf size=333

$$\frac{ce(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\left(ae^2+cd^2\right)}-\frac{cd(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)}+\frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2}+\frac{cx\left(d-ex^n\right)}{2an\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)}+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)^2}-\frac{ce^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2}$$

```
[Out] (c*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))) + (c*d*
e^2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))
/a)])/(a*(c*d^2 + a*e^2)^2) - (c*d*(1 - 2*n)*x*Hypergeometric2F1[
1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*
e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^
n)/d)])/(d*(c*d^2 + a*e^2)^2) - (c*e^3*x^(1 + n)*Hypergeometric2F
1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2
+ a*e^2)^2*(1 + n)) + (c*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1,
 (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2
+ a*e^2)*n*(1 + n))
```

Rubi [A] time = 0.480938, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{ce(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\left(ae^2+cd^2\right)}-\frac{cd(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)}+\frac{cde^2x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^2}+\frac{cx\left(d-ex^n\right)}{2an\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)}+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)^2}-\frac{ce^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```


```
[Out] (c*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))) + (c*d*
e^2*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))
/a)])/(a*(c*d^2 + a*e^2)^2) - (c*d*(1 - 2*n)*x*Hypergeometric2F1[
1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*
e^2)*n) + (e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^
n)/d)])/(d*(c*d^2 + a*e^2)^2) - (c*e^3*x^(1 + n)*Hypergeometric2F
1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2
+ a*e^2)^2*(1 + n)) + (c*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1,
 (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2
+ a*e^2)*n*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^{2n}\right)^2\left(d+ex^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)/(a+c*x**(2*n))**2,x)
```

```
[Out] Integral(1/((a + c*x**(2*n))**2*(d + e*x**n)), x)
```

Mathematica [A] time = 0.585397, size = 245, normalized size = 0.74

$$\frac{x\left(2a^2e^4n(n+1)\left(a+cx^{2n}\right)\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)+cd^2(n+1)\left(a+cx^{2n}\right)\left(ae^2(4n-1)+cd^2(2n-1)\right)\ {}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\right.\right.}{2a^2dn(n+1)\left(ae^2+\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^2),x]
```

```
[Out] (x*(c*d^2*(1 + n)*(c*d^2*(-1 + 2*n) + a*e^2*(-1 + 4*n))*(a + c*x^
(2*n))*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a
)] + 2*a^2*e^4*n*(1 + n)*(a + c*x^(2*n))*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((e*x^n)/d)] + c*d*(a*(c*d^2 + a*e^2)*(1 + n)*(
d - e*x^n) - e*(c*d^2*(-1 + n) + a*e^2*(-1 + 3*n))*x^n*(a + c*x^(
2*n))*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^
(2*n))/a)])))/(2*a^2*d*(c*d^2 + a*e^2)^2*n*(1 + n)*(a + c*x^(2*n)
))
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)\left(a+cx^{2n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

```
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e^4\int \frac{1}{c^2d^5+2\,acd^3e^2+a^2de^4+(c^2d^4e+2\,acd^2e^3+a^2e^5)x^n}\,dx - \frac{cexx^n-cdx}{2\left(a^2cd^2n+a^3e^2n+(ac^2d^2n+a^2ce^2n)x^{2\,n}\right)} - \int -\frac{acde^2(4\,n-1)+c^2d^3(2\,n-1)-\left(ace^3(3\,n-1)+c^2d^2e(n-1)\right)x^n}{2\left(a^2c^2d^4n+2\,a^3cd^2e^2n+a^4e^4n+(ac^3d^4n+2\,a^2c^2d^2e^2n+a^3ce^4n)x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)),x, algorithm="maxima")
```

```
[Out] e^4*integrate(1/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e
 + 2*a*c*d^2*e^3 + a^2*e^5)*x^n), x) - 1/2*(c*e*x*x^n - c*d*x)/(a
^2*c*d^2*n + a^3*e^2*n + (a*c^2*d^2*n + a^2*c*e^2*n)*x^(2*n)) - i
ntegrate(-1/2*(a*c*d*e^2*(4*n - 1) + c^2*d^3*(2*n - 1) - (a*c*e^3
*(3*n - 1) + c^2*d^2*e*(n - 1))*x^n)/(a^2*c^2*d^4*n + 2*a^3*c*d^2
*e^2*n + a^4*e^4*n + (a*c^3*d^4*n + 2*a^2*c^2*d^2*e^2*n + a^3*c*e
^4*n)*x^(2*n)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2ex^n + a^2d + (c^2ex^n + c^2d)x^{4\,n} + 2\,(acex^n + acd)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e*x^n + a^2*d + (c^2*e*x^n + c^2*d)*x^(4*n) + 2*(
a*c*e*x^n + a*c*d)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2\,n} + a)^2(ex^n + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)), x)
```

3.52 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx$

Optimal. Leaf size=410

$$\frac{c^2de(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)\left(ae^2+cd^2\right)^2}-\frac{c(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^2}-\frac{4c^2de^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^3}+\frac{ce^2x\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^3}+\frac{cx\left(-ae^2+cd^2-2cdex^n\right)}{2an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}+\frac{4ce^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^3}+\frac{e^4x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)^2}$$

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a +
 c*x^(2*n))) + (c*e^2*(3*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/
(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) -
 (c*(c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2
+ n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n) + (4*
c*e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(
c*d^2 + a*e^2)^3 - (4*c^2*d*e^3*x^(1 + n)*Hypergeometric2F1[1, (1
 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2
)^3*(1 + n)) + (c^2*d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1
 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e
^2)^2*n*(1 + n)) + (e^4*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1)
, -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.786815, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{c^2de(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)\left(ae^2+cd^2\right)^2}-\frac{c(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^2}-\frac{4c^2de^3x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^3}+\frac{ce^2x\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^3}+\frac{cx\left(-ae^2+cd^2-2cdex^n\right)}{2an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}+\frac{4ce^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^3}+\frac{e^4x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```


```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a +
 c*x^(2*n))) + (c*e^2*(3*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/
(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) -
 (c*(c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2
+ n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n) + (4*
c*e^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(
c*d^2 + a*e^2)^3 - (4*c^2*d*e^3*x^(1 + n)*Hypergeometric2F1[1, (1
 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2
)^3*(1 + n)) + (c^2*d*e*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1
 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e
^2)^2*n*(1 + n)) + (e^4*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1)
, -((e*x^n)/d)])/(d^2*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+cx^{2n})^2(d+ex^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

```
[Out] Integral(1/((a + c*x**(2*n))**2*(d + e*x**n)**2), x)
```

Mathematica [A] time = 1.32803, size = 495, normalized size = 1.21

$$x\left(\frac{c^3d^4}{a^2n+acnx^{2n}}-\frac{2c^3d^3ex^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2(n+1)}+\frac{2c^3d^3ex^n\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)}-\frac{2c^3d^3ex^n}{a^2n+acnx^{2n}}+\frac{c\left(a^2e^4(1-4n)+6acd^2e^2n+c^2d^4(2n\right.}{a^2n}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^2),x]
```

```
[Out] (x*((2*c*d*e^4)/(d*n + e*n*x^n) + (2*a*e^6)/(d^2*n + d*e*n*x^n) -
 (a*c*e^4)/(a*n + c*n*x^(2*n)) - (2*c^2*d*e^3*x^n)/(a*n + c*n*x^(
2*n)) + (c^3*d^4)/(a^2*n + a*c*n*x^(2*n)) - (2*c^3*d^3*e*x^n)/(a^
2*n + a*c*n*x^(2*n)) + (c*(a^2*e^4*(1 - 4*n) + 6*a*c*d^2*e^2*n +
c^2*d^4*(-1 + 2*n))*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -(
(c*x^(2*n))/a)])/(a^2*n) + (2*e^4*(a*e^2*(-1 + n) + c*d^2*(-1 + 5
*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2
*n) - (2*c^3*d^3*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n
^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1 + n)) - (10*c^2*d*e^3*x^n*Hy
pergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a
)])/(a*(1 + n)) + (2*c^3*d^3*e*x^n*Hypergeometric2F1[1, (1 + n)/(
2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*n*(1 + n)) + (2*c^2
*d*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -(
(c*x^(2*n))/a)])/(a*n*(1 + n))))/(2*(c*d^2 + a*e^2)^3)
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(cd^2e^4(5n-1)+ae^6(n-1)\right)\int \frac{1}{c^3d^8n+3ac^2d^6e^2n+3a^2cd^4e^4n+a^3d^2e^6n+(c^3d^7en+3ac^2d^5e^3n+3a^2cd^3e^5n+a^3de^7n)x^n}$$
$$-\frac{2\left(c^2d^2e^2-ace^4\right)xx^{2n}+\left(c^2d^3e+acde^3\right)xx^n-\left(c^2d^4-acd^2e^2+2a^2e^4\right)x}{2\left(a^2c^2d^6n+2a^3cd^4e^2n+a^4d^2e^4n+(ac^3d^5en+2a^2c^2d^3e^3n+a^3cde^5n)x^{3n}+(ac^3d^6n+2a^2c^2d^4e^2n+a^3cd^2e^4n)x^{2n}+(a^2c\right.}$$
$$-\int \frac{a^2ce^4(4n-1)-c^3d^4(2n-1)-6ac^2d^2e^2n+2\left(ac^2de^3(5n-1)+c^3d^3e(n-1)\right)x^n}{2\left(a^2c^3d^6n+3a^3c^2d^4e^2n+3a^4cd^2e^4n+a^5e^6n+(ac^4d^6n+3a^2c^3d^4e^2n+3a^3c^2d^2e^4n+a^4ce^6n)x^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)^2),x, algorithm="maxima")
```

```
[Out] (c*d^2*e^4*(5*n - 1) + a*e^6*(n - 1))*integrate(1/(c^3*d^8*n + 3*
a*c^2*d^6*e^2*n + 3*a^2*c*d^4*e^4*n + a^3*d^2*e^6*n + (c^3*d^7*e*
n + 3*a*c^2*d^5*e^3*n + 3*a^2*c*d^3*e^5*n + a^3*d*e^7*n)*x^n), x)
 - 1/2*(2*(c^2*d^2*e^2 - a*c*e^4)*x*x^(2*n) + (c^2*d^3*e + a*c*d*
e^3)*x*x^n - (c^2*d^4 - a*c*d^2*e^2 + 2*a^2*e^4)*x)/(a^2*c^2*d^6*
n + 2*a^3*c*d^4*e^2*n + a^4*d^2*e^4*n + (a*c^3*d^5*e*n + 2*a^2*c^
2*d^3*e^3*n + a^3*c*d*e^5*n)*x^(3*n) + (a*c^3*d^6*n + 2*a^2*c^2*d
^4*e^2*n + a^3*c*d^2*e^4*n)*x^(2*n) + (a^2*c^2*d^5*e*n + 2*a^3*c*
d^3*e^3*n + a^4*d*e^5*n)*x^n) - integrate(1/2*(a^2*c*e^4*(4*n - 1
) - c^3*d^4*(2*n - 1) - 6*a*c^2*d^2*e^2*n + 2*(a*c^2*d*e^3*(5*n -
 1) + c^3*d^3*e*(n - 1))*x^n)/(a^2*c^3*d^6*n + 3*a^3*c^2*d^4*e^2*
n + 3*a^4*c*d^2*e^4*n + a^5*e^6*n + (a*c^4*d^6*n + 3*a^2*c^3*d^4*
e^2*n + 3*a^3*c^2*d^2*e^4*n + a^4*c*e^6*n)*x^(2*n)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{2\,a^2dex^n + a^2d^2 + (c^2e^2x^{2\,n} + 2\,c^2dex^n + c^2d^2 + 2\,ace^2)x^{4\,n} + (4\,acdex^n + 2\,acd^2 + a^2e^2)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^2*d*e*x^n + a^2*d^2 + (c^2*e^2*x^(2*n) + 2*c^2*d*
e*x^n + c^2*d^2 + 2*a*c*e^2)*x^(4*n) + (4*a*c*d*e*x^n + 2*a*c*d^2
 + a^2*e^2)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2\,n} + a)^2(ex^n + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^2*(e*x^n + d)^2), x)
```

3.53 $\int \frac{(d+ex^n)^3}{(a+cx^{2n})^3}\,dx$

Optimal. Leaf size=424

$$\frac{e(1-3n)(1-n)x^{n+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(n+1)}+\frac{d(1-4n)(1-2n)x\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}-\frac{x\left(e(1-3n)x^n\left(3cd^2-ae^2\right)+d(1-4n)\left(cd^2-3ae^2\right)\right)}{8a^2cn^2\left(a+cx^{2n}\right)}-\frac{3de^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{e^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}+\frac{x\left(ex^n\left(3cd^2-ae^2\right)+d\left(cd^2-3ae^2\right)\right)}{4acn\left(a+cx^{2n}\right)^2}+\frac{e^2x\left(3d+ex^n\right)}{2acn\left(a+cx^{2n}\right)}$$

```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(4*a*c*n*(a +
 c*x^(2*n))^2) + (e^2*x*(3*d + e*x^n))/(2*a*c*n*(a + c*x^(2*n)))
- (x*(d*(c*d^2 - 3*a*e^2)*(1 - 4*n) + e*(3*c*d^2 - a*e^2)*(1 - 3*
n)*x^n))/(8*a^2*c*n^2*(a + c*x^(2*n))) + (d*(c*d^2 - 3*a*e^2)*(1
- 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2,
-((c*x^(2*n))/a)])/(8*a^3*c*n^2) - (3*d*e^2*(1 - 2*n)*x*Hypergeom
etric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*
n) + (e*(3*c*d^2 - a*e^2)*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeomet
ric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^
3*c*n^2*(1 + n)) - (e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1
 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n
))
```

Rubi [A] time = 0.829315, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{e(1-3n)(1-n)x^{n+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2(n+1)}+\frac{d(1-4n)(1-2n)x\left(cd^2-3ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}-\frac{x\left(e(1-3n)x^n\left(3cd^2-ae^2\right)+d(1-4n)\left(cd^2-3ae^2\right)\right)}{8a^2cn^2\left(a+cx^{2n}\right)}-\frac{3de^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn}-\frac{e^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2cn(n+1)}+\frac{x\left(ex^n\left(3cd^2-ae^2\right)+d\left(cd^2-3ae^2\right)\right)}{4acn\left(a+cx^{2n}\right)^2}+\frac{e^2x\left(3d+ex^n\right)}{2acn\left(a+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```


```
[Out] (x*(d*(c*d^2 - 3*a*e^2) + e*(3*c*d^2 - a*e^2)*x^n))/(4*a*c*n*(a +
 c*x^(2*n))^2) + (e^2*x*(3*d + e*x^n))/(2*a*c*n*(a + c*x^(2*n)))
- (x*(d*(c*d^2 - 3*a*e^2)*(1 - 4*n) + e*(3*c*d^2 - a*e^2)*(1 - 3*
n)*x^n))/(8*a^2*c*n^2*(a + c*x^(2*n))) + (d*(c*d^2 - 3*a*e^2)*(1
- 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2,
-((c*x^(2*n))/a)])/(8*a^3*c*n^2) - (3*d*e^2*(1 - 2*n)*x*Hypergeom
etric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*
n) + (e*(3*c*d^2 - a*e^2)*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeomet
ric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^
3*c*n^2*(1 + n)) - (e^3*(1 - n)*x^(1 + n)*Hypergeometric2F1[1, (1
```

```
+ n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*c*n*(1 + n
))
```

Rubi in Sympy [A] time = 27.8458, size = 158, normalized size = 0.37

$$\frac{d^3 x\,{}_2F_1\left(\begin{matrix}3, \frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle| -\frac{cx^{2n}}{a}\right)}{a^3} + \frac{3d^2 e x^{n+1}\,{}_2F_1\left(\begin{matrix}3, \frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle| -\frac{cx^{2n}}{a}\right)}{a^3(n+1)} + \frac{3de^2 x^{2n+1}\,{}_2F_1\left(\begin{matrix}3, \frac{n+\frac{1}{2}}{n}\\ 2+\frac{1}{2n}\end{matrix}\middle| -\frac{cx^{2n}}{a}\right)}{a^3(2n+1)} + \frac{e^3 x^{3n+1}\,{}_2F_1\left(\begin{matrix}3, \frac{3n+1}{2n}\\ \frac{5n+1}{2n}\end{matrix}\middle| -\frac{cx^{2n}}{a}\right)}{a^3(3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((d+e*x**n)**3/(a+c*x**(2*n))**3,x)
```

```
[Out] d**3*x*hyper((3, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/a**3 +
3*d**2*e*x**(n + 1)*hyper((3, (n + 1)/(2*n)), ((3*n + 1)/(2*n),),
 -c*x**(2*n)/a)/(a**3*(n + 1)) + 3*d*e**2*x**(2*n + 1)*hyper((3,
(n + 1/2)/n), (2 + 1/(2*n),), -c*x**(2*n)/a)/(a**3*(2*n + 1)) + e
**3*x**(3*n + 1)*hyper((3, (3*n + 1)/(2*n)), ((5*n + 1)/(2*n),),
-c*x**(2*n)/a)/(a**3*(3*n + 1))
```

Mathematica [A] time = 1.22617, size = 252, normalized size = 0.59

$$\frac{x\left(\frac{a\left(-a^2e^2(d(6n-3)+e(n-1)x^n)+ac\left(d^3(6n-1)+3d^2e(5n-1)x^n+3de^2x^{2n}+e^3(n+1)x^{3n}\right)+c^2d^2x^{2n}(d(4n-1)+3e(3n-1)x^n)\right)}{(a+cx^{2n})^2} + d(2n-1)\left(3ae^2+cd^2(4n-\right.\right.}{8a^3cn^2}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(d + e*x^n)^3/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*((a*(-(a^2*e^2*(d*(-3 + 6*n) + e*(-1 + n)*x^n)) + c^2*d^2*x^(2
*n)*(d*(-1 + 4*n) + 3*e*(-1 + 3*n)*x^n) + a*c*(d^3*(-1 + 6*n) + 3
*d^2*e*(-1 + 5*n)*x^n + 3*d*e^2*x^(2*n) + e^3*(1 + n)*x^(3*n))))/
(a + c*x^(2*n))^2 + d*(-1 + 2*n)*(3*a*e^2 + c*d^2*(-1 + 4*n))*Hyp
ergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)] + (e*(-
1 + n)*(a*e^2*(1 + n) + 3*c*d^2*(-1 + 3*n))*x^n*Hypergeometric2F1
[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(
8*a^3*c*n^2)
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^3}{(a+cx^{2\,n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^3/(a+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,c^2d^2e(3\,n-1)+ace^3(n+1)\right)xx^{3\,n}+\left(c^2d^3(4\,n-1)+3\,acde^2\right)xx^{2\,n}+\left(3\,acd^2e(5\,n-1)-a^2e^3(n-1)\right)xx^n+\left(acd^3(6\,n-1\right.}{8\,(a^2c^3n^2x^{4\,n}+2\,a^3c^2n^2x^{2\,n}+a^4cn^2)}$$
$$+\int\frac{\left(8\,n^2-6\,n+1\right)cd^3+3\,ade^2(2\,n-1)+\left(3\left(3\,n^2-4\,n+1\right)cd^2e+\left(n^2-1\right)ae^3\right)x^n}{8\,(a^2c^2n^2x^{2\,n}+a^3cn^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*c^2*d^2*e*(3*n - 1) + a*c*e^3*(n + 1))*x*x^(3*n) + (c^2*d
^3*(4*n - 1) + 3*a*c*d*e^2)*x*x^(2*n) + (3*a*c*d^2*e*(5*n - 1) -
a^2*e^3*(n - 1))*x*x^n + (a*c*d^3*(6*n - 1) - 3*a^2*d*e^2*(2*n -
1))*x)/(a^2*c^3*n^2*x^(4*n) + 2*a^3*c^2*n^2*x^(2*n) + a^4*c*n^2)
+ integrate(1/8*((8*n^2 - 6*n + 1)*c*d^3 + 3*a*d*e^2*(2*n - 1) +
(3*(3*n^2 - 4*n + 1)*c*d^2*e + (n^2 - 1)*a*e^3)*x^n)/(a^2*c^2*n^2
*x^(2*n) + a^3*c*n^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3\,n}+3\,de^2x^{2\,n}+3\,d^2ex^n+d^3}{c^3x^{6\,n}+3\,ac^2x^{4\,n}+3\,a^2cx^{2\,n}+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a)^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^3
*x^(6*n) + 3*a*c^2*x^(4*n) + 3*a^2*c*x^(2*n) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**3/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex^n+d)^3}{(cx^{2\,n}+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + a)^3, x)
```

3.54 $\int \frac{(d+ex^n)^2}{(a+cx^{2n})^3}\,dx$

Optimal. Leaf size=272

$$\frac{(1-4n)(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}+\frac{de(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)}-\frac{x\left((1-4n)\left(cd^2-ae^2\right)+2cde(1-3n)x^n\right)}{8a^2cn^2\left(a+cx^{2n}\right)}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}+\frac{x\left(-ae^2+cd^2+2cdex^n\right)}{4acn\left(a+cx^{2n}\right)^2}$$

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(4*a*c*n*(a + c*x^(2*n))^2) - (
x*((c*d^2 - a*e^2)*(1 - 4*n) + 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*c*n
^2*(a + c*x^(2*n))) + ((c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hype
rgeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a
^3*c*n^2) + (d*e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1,
 (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(4*a^3*n^2*(1
+ n)) + (e^2*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c
*x^(2*n))/a)])/(a^2*c)
```

Rubi [A] time = 0.516454, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{(1-4n)(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3cn^2}+\frac{de(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)}-\frac{x\left((1-4n)\left(cd^2-ae^2\right)+2cde(1-3n)x^n\right)}{8a^2cn^2\left(a+cx^{2n}\right)}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2c}+\frac{x\left(-ae^2+cd^2+2cdex^n\right)}{4acn\left(a+cx^{2n}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(c*d^2 - a*e^2 + 2*c*d*e*x^n))/(4*a*c*n*(a + c*x^(2*n))^2) - (
x*((c*d^2 - a*e^2)*(1 - 4*n) + 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*c*n
^2*(a + c*x^(2*n))) + ((c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hype
rgeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a
^3*c*n^2) + (d*e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2F1[1,
 (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(4*a^3*n^2*(1
+ n)) + (e^2*x*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((c
*x^(2*n))/a)])/(a^2*c)
```

Rubi in Sympy [A] time = 19.3914, size = 109, normalized size = 0.4

$$\frac{d^2x\,{}_2F_1\left(\begin{matrix}3,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^3}+\frac{2dex^{n+1}\,{}_2F_1\left(\begin{matrix}3,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^3\left(n+1\right)}+\frac{e^2x^{2n+1}\,{}_2F_1\left(\begin{matrix}3,\frac{n+\frac{1}{2}}{n}\\ 2+\frac{1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{a^3\left(2n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

```
[Out] d**2*x*hyper((3, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/a**3 +
2*d*e*x**(n + 1)*hyper((3, (n + 1)/(2*n)), ((3*n + 1)/(2*n),), -c
*x**(2*n)/a)/(a**3*(n + 1)) + e**2*x**(2*n + 1)*hyper((3, (n + 1/
2)/n), (2 + 1/(2*n),), -c*x**(2*n)/a)/(a**3*(2*n + 1))
```

Mathematica [A] time = 0.825264, size = 212, normalized size = 0.78

$$\frac{x\left(\frac{a\left(a^2e^2(1-2n)+ac\left(d^2(6n-1)+2de(5n-1)x^n+e^2x^{2n}\right)+c^2dx^{2n}(d(4n-1)+2e(3n-1)x^n)\right)}{c(a+cx^{2n})^2}+\frac{(2n-1)\left(ae^2+cd^2(4n-1)\right)\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{c}+\frac{2de\left(3n^2-4n\cdots\right.}{\cdots}\right.}{8a^3n^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^2/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*((a*(a^2*e^2*(1 - 2*n) + c^2*d*x^(2*n)*(d*(-1 + 4*n) + 2*e*(-1
 + 3*n)*x^n) + a*c*(d^2*(-1 + 6*n) + 2*d*e*(-1 + 5*n)*x^n + e^2*x
^(2*n))))/(c*(a + c*x^(2*n))^2) + ((-1 + 2*n)*(a*e^2 + c*d^2*(-1
+ 4*n))*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/
a)])/c + (2*d*e*(1 - 4*n + 3*n^2)*x^n*Hypergeometric2F1[1, (1 + n
)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(8*a^3*n^2)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^2}{(a+cx^{2\,n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,c^2de(3\,n-1)xx^{3\,n}+2\,acde(5\,n-1)xx^n+\left(c^2d^2(4\,n-1)+ace^2\right)xx^{2\,n}+\left(acd^2(6\,n-1)-a^2e^2(2\,n-1)\right)x}{8\left(a^2c^3n^2x^{4\,n}+2\,a^3c^2n^2x^{2\,n}+a^4cn^2\right)}+\int \frac{2\left(3\,n^2-4\,n+1\right)cdex^n+\left(8\,n^2-6\,n+1\right)cd^2+ae^2(2\,n-1)}{8\left(a^2c^2n^2x^{2\,n}+a^3cn^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*c^2*d*e*(3*n - 1)*x*x^(3*n) + 2*a*c*d*e*(5*n - 1)*x*x^n +
(c^2*d^2*(4*n - 1) + a*c*e^2)*x*x^(2*n) + (a*c*d^2*(6*n - 1) - a^
2*e^2*(2*n - 1))*x)/(a^2*c^3*n^2*x^(4*n) + 2*a^3*c^2*n^2*x^(2*n)
+ a^4*c*n^2) + integrate(1/8*(2*(3*n^2 - 4*n + 1)*c*d*e*x^n + (8*
n^2 - 6*n + 1)*c*d^2 + a*e^2*(2*n - 1))/(a^2*c^2*n^2*x^(2*n) + a^
3*c*n^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2\,n}+2\,dex^n+d^2}{c^3x^{6\,n}+3\,ac^2x^{4\,n}+3\,a^2cx^{2\,n}+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a)^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + 3*a*c^2*x
^(4*n) + 3*a^2*c*x^(2*n) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{(cx^{2n} + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + a)^3, x)
```

3.55 $\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx$

Optimal. Leaf size=184

$$\frac{d(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2}+\frac{e(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)}-\frac{x\,(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})}+\frac{x\,(d+ex^n)}{4an\,(a+cx^{2n})^2}$$

```
[Out] (x*(d + e*x^n))/(4*a*n*(a + c*x^(2*n))^2) - (x*(d*(1 - 4*n) + e*(
1 - 3*n)*x^n))/(8*a^2*n^2*(a + c*x^(2*n))) + (d*(1 - 4*n)*(1 - 2*
n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/
a)])/(8*a^3*n^2) + (e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2
F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*n^
2*(1 + n))
```

Rubi [A] time = 0.216182, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{d(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2}+\frac{e(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)}-\frac{x\,(d(1-4n)+e(1-3n)x^n)}{8a^2n^2\,(a+cx^{2n})}+\frac{x\,(d+ex^n)}{4an\,(a+cx^{2n})^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*(d + e*x^n))/(4*a*n*(a + c*x^(2*n))^2) - (x*(d*(1 - 4*n) + e*(
1 - 3*n)*x^n))/(8*a^2*n^2*(a + c*x^(2*n))) + (d*(1 - 4*n)*(1 - 2*
n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/
a)])/(8*a^3*n^2) + (e*(1 - 3*n)*(1 - n)*x^(1 + n)*Hypergeometric2
F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*n^
2*(1 + n))
```

Rubi in Sympy [A] time = 31.4519, size = 151, normalized size = 0.82

$$\frac{x\,(d+ex^n)}{4an\,(a+cx^{2n})^2}-\frac{x\,(d\,(-4n+1)+ex^n\,(-3n+1))}{8a^2n^2\,(a+cx^{2n})}+\frac{dx\,(-4n+1)\,(-2n+1)\,{}_2F_1\left(\begin{matrix}1,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{8a^3n^2}+\frac{ex^{n+1}\,(-3n+1)\,(-n+1)\,{}_2F_1\left(\begin{matrix}1,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{8a^3n^2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+c*x**(2*n))**3,x)
```

```
[Out] x*(d + e*x**n)/(4*a*n*(a + c*x**(2*n))**2) - x*(d*(-4*n + 1) + e*
x**n*(-3*n + 1))/(8*a**2*n**2*(a + c*x**(2*n))) + d*x*(-4*n + 1)*
(-2*n + 1)*hyper((1, 1/(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a)/(8*
a**3*n**2) + e*x**(n + 1)*(-3*n + 1)*(-n + 1)*hyper((1, (n + 1)/(
2*n)), ((3*n + 1)/(2*n),), -c*x**(2*n)/a)/(8*a**3*n**2*(n + 1))
```

Mathematica [A] time = 0.43725, size = 164, normalized size = 0.89

$$\frac{x\left(\frac{a(a(d(6n-1)+e(5n-1)x^n)+cx^{2n}(d(4n-1)+e(3n-1)x^n))}{(a+cx^{2n})^2}+d\left(8n^2-6n+1\right)\ {}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+\frac{e\left(3n^2-4n+1\right)x^n\ {}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);\right.}{n+1}}{8a^3n^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)/(a + c*x^(2*n))^3,x]
```

```
[Out] (x*((a*(c*x^(2*n)*(d*(-1 + 4*n) + e*(-1 + 3*n)*x^n) + a*(d*(-1 +
6*n) + e*(-1 + 5*n)*x^n)))/(a + c*x^(2*n))^2 + d*(1 - 6*n + 8*n^2
)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)] +
(e*(1 - 4*n + 3*n^2)*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 +
 n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n)))/(8*a^3*n^2)
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \frac{d+ex^n}{(a+cx^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)/(a+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce(3\,n-1)xx^{3\,n}+cd(4\,n-1)xx^{2\,n}+ae(5\,n-1)xx^{n}+ad(6\,n-1)x}{8\left(a^2c^2n^2x^{4\,n}+2\,a^3cn^2x^{2\,n}+a^4n^2\right)}+\int \frac{\left(3\,n^2-4\,n+1\right)ex^n+\left(8\,n^2-6\,n+1\right)d}{8\left(a^2cn^2x^{2\,n}+a^3n^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(c*e*(3*n - 1)*x*x^(3*n) + c*d*(4*n - 1)*x*x^(2*n) + a*e*(5*n
 - 1)*x*x^n + a*d*(6*n - 1)*x)/(a^2*c^2*n^2*x^(4*n) + 2*a^3*c*n^2
*x^(2*n) + a^4*n^2) + integrate(1/8*((3*n^2 - 4*n + 1)*e*x^n + (8
*n^2 - 6*n + 1)*d)/(a^2*c*n^2*x^(2*n) + a^3*n^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^n+d}{c^3x^{6\,n}+3\,ac^2x^{4\,n}+3\,a^2cx^{2\,n}+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c^3*x^(6*n) + 3*a*c^2*x^(4*n) + 3*a^2*c*x^(
2*n) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + a)^3, x)
```

3.56 $\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx$

Optimal. Leaf size=582

$$-\frac{ce(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)\left(ae^2+cd^2\right)}+\frac{cd(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)}-\frac{cde^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^2}-\frac{cx\left(d(1-4n)-e(1-3n)x^n\right)}{8a^2n^2\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)}+\frac{ce^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\left(ae^2+cd^2\right)^2}+\frac{ce^2x\left(d-ex^n\right)}{2an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}+\frac{cx\left(d-ex^n\right)}{4an\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)^2}+\frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)^3}-\frac{ce^5x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^3}+\frac{cde^4x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^3}$$

```
[Out] (c*x*(d - e*x^n))/(4*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))^2) + (c*
e^2*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n))) - (c
*x*(d*(1 - 4*n) - e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)*n^2*(a
 + c*x^(2*n))) + (c*d*e^4*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^
(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) + (c*d*(1 - 4*n
)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*
x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2) - (c*d*e^2*(1 - 2*n)*x*
Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/
(2*a^2*(c*d^2 + a*e^2)^2*n) + (e^6*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)^3) - (c*e^5*x^(1 +
 n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a*(c*d^2 + a*e^2)^3*(1 + n)) - (c*e*(1 - 3*n)*(1 - n)*
x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((
c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2*(1 + n)) + (c*e^3*(1 -
 n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
 -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n*(1 + n))
```

Rubi [A] time = 0.941329, antiderivative size = 582, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{ce(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2(n+1)\left(ae^2+cd^2\right)}+\frac{cd(1-4n)(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)}-\frac{cde^2(1-2n)x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^2}-\frac{cx\left(d(1-4n)-e(1-3n)x^n\right)}{8a^2n^2\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)}+\frac{ce^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n(n+1)\left(ae^2+cd^2\right)^2}+\frac{ce^2x\left(d-ex^n\right)}{2an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}+\frac{cx\left(d-ex^n\right)}{4an\left(ae^2+cd^2\right)\left(a+cx^{2n}\right)^2}+\frac{e^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2+cd^2\right)^3}-\frac{ce^5x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^3}+\frac{cde^4x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]
```


```
[Out] (c*x*(d - e*x^n))/(4*a*(c*d^2 + a*e^2)*n*(a + c*x^(2*n))^2) + (c*
e^2*x*(d - e*x^n))/(2*a*(c*d^2 + a*e^2)^2*n*(a + c*x^(2*n))) - (c
*x*(d*(1 - 4*n) - e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)*n^2*(a
```

```
 + c*x^(2*n))) + (c*d*e^4*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^
(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^3) + (c*d*(1 - 4*n
)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*
x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2) - (c*d*e^2*(1 - 2*n)*x*
Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/
(2*a^2*(c*d^2 + a*e^2)^2*n) + (e^6*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 + a*e^2)^3) - (c*e^5*x^(1 +
 n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a*(c*d^2 + a*e^2)^3*(1 + n)) - (c*e*(1 - 3*n)*(1 - n)*
x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((
c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)*n^2*(1 + n)) + (c*e^3*(1 -
 n)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2,
 -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^2)^2*n*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^{2n}\right)^3\left(d+ex^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)/(a+c*x**(2*n))**3,x)

[Out] Integral(1/((a + c*x**(2*n))**3*(d + e*x**n)), x)
```

Mathematica [A] time = 2.25215, size = 1031, normalized size = 1.77

$$x\left(-\frac{15ce^5\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)x^n}{a(n+1)}-\frac{10c^2d^2e^3\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)x^n}{a^2(n+1)}-\frac{3c^3d^4e\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)x^n}{a^3(n+1)}+\frac{8ce^5\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right)\right.}{an(n+1)}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)*(a + c*x^(2*n))^3),x]

[Out] (x*((2*c*(c*d^2 + a*e^2)^2*(d - e*x^n))/(a*n*(a + c*x^(2*n))^2) +
 (c*(c*d^2 + a*e^2)*(c*d^2*(d*(-1 + 4*n) - e*(-1 + 3*n)*x^n) + a*
e^2*(d*(-1 + 8*n) - e*(-1 + 7*n)*x^n)))/(a^2*n^2*(a + c*x^(2*n)))
 + (8*c^3*d^5*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(
2*n))/a)])/a^3 + (24*c^2*d^3*e^2*Hypergeometric2F1[1, 1/(2*n), 1
+ 1/(2*n), -((c*x^(2*n))/a)])/a^2 + (24*c*d*e^4*Hypergeometric2F1
[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a + (c^3*d^5*Hyperge
ometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^3*n^2)
+ (2*c^2*d^3*e^2*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*
x^(2*n))/a)])/(a^2*n^2) + (c*d*e^4*Hypergeometric2F1[1, 1/(2*n),
1 + 1/(2*n), -((c*x^(2*n))/a)])/(a*n^2) - (6*c^3*d^5*Hypergeometr
ic2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^3*n) - (16*c
^2*d^3*e^2*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n
))/a)])/(a^2*n) - (10*c*d*e^4*Hypergeometric2F1[1, 1/(2*n), 1 + 1
/(2*n), -((c*x^(2*n))/a)])/(a*n) + (8*e^6*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((e*x^n)/d)])/d - (3*c^3*d^4*e*x^n*Hypergeomet
ric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*
(1 + n)) - (10*c^2*d^2*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n)
, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^2*(1 + n)) - (15*c*e^5*x^
n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n
))/a)])/(a*(1 + n)) - (c^3*d^4*e*x^n*Hypergeometric2F1[1, (1 + n)
/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*n^2*(1 + n)) - (2
*c^2*d^2*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))
/2, -((c*x^(2*n))/a)])/(a^2*n^2*(1 + n)) - (c*e^5*x^n*Hypergeomet
ric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n^
2*(1 + n)) + (4*c^3*d^4*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n),
```

```
(3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*n*(1 + n)) + (12*c^2*d^2
*e^3*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c
*x^(2*n))/a)])/(a^2*n*(1 + n)) + (8*c*e^5*x^n*Hypergeometric2F1[1
, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n*(1 + n))
))/(8*(c*d^2 + a*e^2)^3)
```

Maple [F] time = 0.242, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)(a+cx^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e^6 \int \frac{1}{c^3d^7+3\,ac^2d^5e^2+3\,a^2cd^3e^4+a^3de^6+(c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7)x^n}\,dx - \frac{\left(ac^2e^3(7\,n-1)+c^3d^2e(3\,n-1)\right)xx^{3\,n}-\left(ac^2de^2(8\,n-1)+c^3d^3(4\,n-1)\right)xx^{2\,n}+\left(a^2ce^3(9\,n-1)+ac^2d^2e(5\,n-1)\right)xx^n-}{8\left(a^4c^2d^4n^2+2\,a^5cd^2e^2n^2+a^6e^4n^2+(a^2c^4d^4n^2+2\,a^3c^3d^2e^2n^2+a^4c^2e^4n^2)x^{4\,n}+2\,(a^3c^3d^4n^2+2\,a^4c^2d^2e\right.}$$
$$-\int -\frac{\left(8\,n^2-6\,n+1\right)c^3d^5+2\left(12\,n^2-8\,n+1\right)ac^2d^3e^2+\left(24\,n^2-10\,n+1\right)a^2cde^4-\left(\left(3\,n^2-4\,n+1\right)c^3d^4e+2\left(5\,n^2-6\,n+1\right)a\right.}{8\,(a^3c^3d^6n^2+3\,a^4c^2d^4e^2n^2+3\,a^5cd^2e^4n^2+a^6e^6n^2+(a^2c^4d^6n^2+3\,a^3c^3d^4e^2n^2+3\,a^4c^2d^2e^4n^2+a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)),x, algorithm="maxima")
```

```
[Out] e^6*integrate(1/(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^
3*d*e^6 + (c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^
7)*x^n), x) - 1/8*((a*c^2*e^3*(7*n - 1) + c^3*d^2*e*(3*n - 1))*x*
x^(3*n) - (a*c^2*d*e^2*(8*n - 1) + c^3*d^3*(4*n - 1))*x*x^(2*n) +
 (a^2*c*e^3*(9*n - 1) + a*c^2*d^2*e*(5*n - 1))*x*x^n - (a^2*c*d*e
^2*(10*n - 1) + a*c^2*d^3*(6*n - 1))*x)/(a^4*c^2*d^4*n^2 + 2*a^5*
c*d^2*e^2*n^2 + a^6*e^4*n^2 + (a^2*c^4*d^4*n^2 + 2*a^3*c^3*d^2*e^
2*n^2 + a^4*c^2*e^4*n^2)*x^(4*n) + 2*(a^3*c^3*d^4*n^2 + 2*a^4*c^2
*d^2*e^2*n^2 + a^5*c*e^4*n^2)*x^(2*n)) - integrate(-1/8*((8*n^2 -
 6*n + 1)*c^3*d^5 + 2*(12*n^2 - 8*n + 1)*a*c^2*d^3*e^2 + (24*n^2
- 10*n + 1)*a^2*c*d*e^4 - ((3*n^2 - 4*n + 1)*c^3*d^4*e + 2*(5*n^2
 - 6*n + 1)*a*c^2*d^2*e^3 + (15*n^2 - 8*n + 1)*a^2*c*e^5)*x^n)/(a
^3*c^3*d^6*n^2 + 3*a^4*c^2*d^4*e^2*n^2 + 3*a^5*c*d^2*e^4*n^2 + a^
6*e^6*n^2 + (a^2*c^4*d^6*n^2 + 3*a^3*c^3*d^4*e^2*n^2 + 3*a^4*c^2*
d^2*e^4*n^2 + a^5*c*e^6*n^2)*x^(2*n)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{a^3ex^n+a^3d+(c^3ex^n+c^3d)x^{6\,n}+3\,(ac^2ex^n+ac^2d)x^{4\,n}+3\,(a^2cex^n+a^2cd)x^{2\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*e*x^n + a^3*d + (c^3*e*x^n + c^3*d)*x^(6*n) + 3*(
a*c^2*e*x^n + a*c^2*d)*x^(4*n) + 3*(a^2*c*e*x^n + a^2*c*d)*x^(2*n
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2\,n}+a\right)^{3}\left(ex^{n}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)), x)
```

3.57 $\int \frac{1}{(d+ex^n)^2(a+cx^{2n})^3}\,dx$

Optimal. Leaf size=701

$$-\frac{c^2de(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)\left(ae^2+cd^2\right)^2}+\frac{c(1-4n)(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)^2}+\frac{2c^2de^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)\left(ae^2+cd^2\right)^3}-\frac{ce^2(1-2n)x\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^3}-\frac{cx\left((1-4n)\left(cd^2-ae^2\right)-2cde(1-3n)x^n\right)}{8a^2n^2\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}-\frac{6c^2de^5x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^4}+\frac{ce^2x\left(-ae^2+3cd^2-4cdex^n\right)}{2an\left(ae^2+cd^2\right)^3\left(a+cx^{2n}\right)}+\frac{cx\left(-ae^2+cd^2-2cdex^n\right)}{4an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)^2}+\frac{6ce^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^4}+\frac{e^6x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)^3}+\frac{ce^4x\left(5cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^4}$$

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(4*a*(c*d^2 + a*e^2)^2*n*(a +
 c*x^(2*n))^2) + (c*e^2*x*(3*c*d^2 - a*e^2 - 4*c*d*e*x^n))/(2*a*(
c*d^2 + a*e^2)^3*n*(a + c*x^(2*n))) - (c*x*((c*d^2 - a*e^2)*(1 -
4*n) - 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)^2*n^2*(a +
c*x^(2*n))) + (c*e^4*(5*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(
2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4) +
(c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(
2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)^2
*n^2) - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1,
 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^
2)^3*n) + (6*c*e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((
e*x^n)/d)])/(c*d^2 + a*e^2)^4 - (6*c^2*d*e^5*x^(1 + n)*Hypergeome
tric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(
c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3*n)*(1 - n)*x^(1 + n)*
Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))
/a)])/(4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1 - n
)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -
((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hype
rgeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 +
 a*e^2)^3)
```

Rubi [A] time = 1.49197, antiderivative size = 701, normalized size of antiderivative = 1., number of

steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{c^2de(1-3n)(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{4a^3n^2(n+1)\left(ae^2+cd^2\right)^2}+\frac{c(1-4n)(1-2n)x\left(cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{8a^3n^2\left(ae^2+cd^2\right)^2}+\frac{2c^2de^3(1-n)x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a^2n(n+1)\left(ae^2+cd^2\right)^3}-\frac{ce^2(1-2n)x\left(3cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2a^2n\left(ae^2+cd^2\right)^3}-\frac{cx\left((1-4n)\left(cd^2-ae^2\right)-2cde(1-3n)x^n\right)}{8a^2n^2\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)}-\frac{6c^2de^5x^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a(n+1)\left(ae^2+cd^2\right)^4}+\frac{ce^2x\left(-ae^2+3cd^2-4cdex^n\right)}{2an\left(ae^2+cd^2\right)^3\left(a+cx^{2n}\right)}+\frac{cx\left(-ae^2+cd^2-2cdex^n\right)}{4an\left(ae^2+cd^2\right)^2\left(a+cx^{2n}\right)^2}+\frac{6ce^6x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{\left(ae^2+cd^2\right)^4}+\frac{e^6x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2+cd^2\right)^3}+\frac{ce^4x\left(5cd^2-ae^2\right)\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{a\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

```
[Out] (c*x*(c*d^2 - a*e^2 - 2*c*d*e*x^n))/(4*a*(c*d^2 + a*e^2)^2*n*(a +
 c*x^(2*n))^2) + (c*e^2*x*(3*c*d^2 - a*e^2 - 4*c*d*e*x^n))/(2*a*(
c*d^2 + a*e^2)^3*n*(a + c*x^(2*n))) - (c*x*((c*d^2 - a*e^2)*(1 -
4*n) - 2*c*d*e*(1 - 3*n)*x^n))/(8*a^2*(c*d^2 + a*e^2)^2*n^2*(a +
c*x^(2*n))) + (c*e^4*(5*c*d^2 - a*e^2)*x*Hypergeometric2F1[1, 1/(
2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(c*d^2 + a*e^2)^4) +
(c*(c*d^2 - a*e^2)*(1 - 4*n)*(1 - 2*n)*x*Hypergeometric2F1[1, 1/(
2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(8*a^3*(c*d^2 + a*e^2)^2
*n^2) - (c*e^2*(3*c*d^2 - a*e^2)*(1 - 2*n)*x*Hypergeometric2F1[1,
 1/(2*n), (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(2*a^2*(c*d^2 + a*e^
2)^3*n) + (6*c*e^6*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((
e*x^n)/d)])/(c*d^2 + a*e^2)^4 - (6*c^2*d*e^5*x^(1 + n)*Hypergeome
tric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(
c*d^2 + a*e^2)^4*(1 + n)) - (c^2*d*e*(1 - 3*n)*(1 - n)*x^(1 + n)*
Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))
/a)])/(4*a^3*(c*d^2 + a*e^2)^2*n^2*(1 + n)) + (2*c^2*d*e^3*(1 - n
)*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -
((c*x^(2*n))/a)])/(a^2*(c*d^2 + a*e^2)^3*n*(1 + n)) + (e^6*x*Hype
rgeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 +
 a*e^2)^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^{2n}\right)^3\left(d+ex^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

```
[Out] Integral(1/((a + c*x**(2*n))**3*(d + e*x**n)**2), x)
```

Mathematica [A] time = 4.60612, size = 1241, normalized size = 1.77

$$x\left(-\frac{70c^2de^5\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)x^n}{a(n+1)}-\frac{28c^3d^3e^3\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)x^n}{a^2(n+1)}-\frac{6c^4d^5e\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)x^n}{a^3(n+1)}+\frac{24c^2de^5\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\right.}{an(n}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)^2*(a + c*x^(2*n))^3),x]
```

```
[Out] (x*((8*c*d^2*e^6 + 8*a*e^8)/(d^2*n + d*e*n*x^n) + (2*c*(c*d^2 + a
*e^2)^2*(-(a*e^2) + c*d*(d - 2*e*x^n)))/(a*n*(a + c*x^(2*n))^2) +
 (c*(c*d^2 + a*e^2)*(a^2*e^4*(1 - 8*n) + c^2*d^3*(d*(-1 + 4*n) -
2*e*(-1 + 3*n)*x^n) + 2*a*c*d*e^2*(6*d*n - e*(-1 + 11*n)*x^n)))/(
a^2*n^2*(a + c*x^(2*n))) + (8*c^4*d^6*Hypergeometric2F1[1, 1/(2*n
), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a^3 + (32*c^3*d^4*e^2*Hypergeo
metric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/a^2 + (48*c
^2*d^2*e^4*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n
))/a)])/a - 24*c*e^6*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -
((c*x^(2*n))/a)] + (c^4*d^6*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(
2*n), -((c*x^(2*n))/a)])/(a^3*n^2) + (c^3*d^4*e^2*Hypergeometric2
F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^2*n^2) - (c^2*d
^2*e^4*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a
)])/(a*n^2) - (c*e^6*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -
((c*x^(2*n))/a)])/n^2 - (6*c^4*d^6*Hypergeometric2F1[1, 1/(2*n),
1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^3*n) - (18*c^3*d^4*e^2*Hyperge
ometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^(2*n))/a)])/(a^2*n) -
(2*c^2*d^2*e^4*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -((c*x^
(2*n))/a)])/(a*n) + (10*c*e^6*Hypergeometric2F1[1, 1/(2*n), 1 + 1
/(2*n), -((c*x^(2*n))/a)])/n + (8*e^6*(a*e^2*(-1 + n) + c*d^2*(-1
 + 7*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/
(d^2*n) - (6*c^4*d^5*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3
 + n^(-1))/2, -((c*x^(2*n))/a)])/(a^3*(1 + n)) - (28*c^3*d^3*e^3*
x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2
*n))/a)])/(a^2*(1 + n)) - (70*c^2*d*e^5*x^n*Hypergeometric2F1[1,
(1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*(1 + n)) - (
2*c^4*d^5*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/
2, -((c*x^(2*n))/a)])/(a^3*n^2*(1 + n)) - (4*c^3*d^3*e^3*x^n*Hype
rgeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)]
)/(a^2*n^2*(1 + n)) - (2*c^2*d*e^5*x^n*Hypergeometric2F1[1, (1 +
n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n^2*(1 + n)) + (8
*c^4*d^5*e*x^n*Hypergeometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2
, -((c*x^(2*n))/a)])/(a^3*n*(1 + n)) + (32*c^3*d^3*e^3*x^n*Hyperg
eometric2F1[1, (1 + n)/(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/
(a^2*n*(1 + n)) + (24*c^2*d*e^5*x^n*Hypergeometric2F1[1, (1 + n)/
(2*n), (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(a*n*(1 + n))))/(8*(c*d
^2 + a*e^2)^4)
```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2\,(a+cx^{2\,n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)^2),x, algorithm="maxima")
```

```
[Out] (c*d^2*e^6*(7*n - 1) + a*e^8*(n - 1))*integrate(1/(c^4*d^10*n + 4
*a*c^3*d^8*e^2*n + 6*a^2*c^2*d^6*e^4*n + 4*a^3*c*d^4*e^6*n + a^4*
d^2*e^8*n + (c^4*d^9*e*n + 4*a*c^3*d^7*e^3*n + 6*a^2*c^2*d^5*e^5*
n + 4*a^3*c*d^3*e^7*n + a^4*d*e^9*n)*x^n), x) - 1/8*(2*(a*c^3*d^2
*e^4*(11*n - 1) + c^4*d^4*e^2*(3*n - 1) - 4*a^2*c^2*e^6*n)*x*x^(4
*n) + (a^2*c^2*d*e^5*(8*n - 1) + 2*a*c^3*d^3*e^3*(5*n - 1) + c^4*
d^5*e*(2*n - 1))*x*x^(3*n) + (a^2*c^2*d^2*e^4*(34*n - 3) - c^4*d^
6*(4*n - 1) - 2*a*c^3*d^4*e^2*(n + 1) - 16*a^3*c*e^6*n)*x*x^(2*n)
 + (a^3*c*d*e^5*(10*n - 1) + 2*a^2*c^2*d^3*e^3*(7*n - 1) + a*c^3*
d^5*e*(4*n - 1))*x*x^n + (a^3*c*d^2*e^4*(10*n - 1) - a*c^3*d^6*(6
*n - 1) - 12*a^2*c^2*d^4*e^2*n - 8*a^4*e^6*n)*x)/(a^4*c^3*d^8*n^2
 + 3*a^5*c^2*d^6*e^2*n^2 + 3*a^6*c*d^4*e^4*n^2 + a^7*d^2*e^6*n^2
+ (a^2*c^5*d^7*e*n^2 + 3*a^3*c^4*d^5*e^3*n^2 + 3*a^4*c^3*d^3*e^5*
n^2 + a^5*c^2*d*e^7*n^2)*x^(5*n) + (a^2*c^5*d^8*n^2 + 3*a^3*c^4*d
^6*e^2*n^2 + 3*a^4*c^3*d^4*e^4*n^2 + a^5*c^2*d^2*e^6*n^2)*x^(4*n)
 + 2*(a^3*c^4*d^7*e*n^2 + 3*a^4*c^3*d^5*e^3*n^2 + 3*a^5*c^2*d^3*e
^5*n^2 + a^6*c*d*e^7*n^2)*x^(3*n) + 2*(a^3*c^4*d^8*n^2 + 3*a^4*c^
3*d^6*e^2*n^2 + 3*a^5*c^2*d^4*e^4*n^2 + a^6*c*d^2*e^6*n^2)*x^(2*n
) + (a^4*c^3*d^7*e*n^2 + 3*a^5*c^2*d^5*e^3*n^2 + 3*a^6*c*d^3*e^5*
n^2 + a^7*d*e^7*n^2)*x^n) - integrate(-1/8*((8*n^2 - 6*n + 1)*c^4
*d^6 + (32*n^2 - 18*n + 1)*a*c^3*d^4*e^2 + (48*n^2 - 2*n - 1)*a^2
*c^2*d^2*e^4 - (24*n^2 - 10*n + 1)*a^3*c*e^6 - 2*((3*n^2 - 4*n +
1)*c^4*d^5*e + 2*(7*n^2 - 8*n + 1)*a*c^3*d^3*e^3 + (35*n^2 - 12*n
 + 1)*a^2*c^2*d*e^5)*x^n)/(a^3*c^4*d^8*n^2 + 4*a^4*c^3*d^6*e^2*n^
2 + 6*a^5*c^2*d^4*e^4*n^2 + 4*a^6*c*d^2*e^6*n^2 + a^7*e^8*n^2 + (
a^2*c^5*d^8*n^2 + 4*a^3*c^4*d^6*e^2*n^2 + 6*a^4*c^3*d^4*e^4*n^2 +
 4*a^5*c^2*d^2*e^6*n^2 + a^6*c*e^8*n^2)*x^(2*n)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{2\,a^3dex^n + a^3d^2 + (c^3e^2x^{2\,n} + 2\,c^3dex^n + c^3d^2)x^{6\,n} + 3\,(ac^2e^2x^{2\,n} + 2\,ac^2dex^n + ac^2d^2 + a^2ce^2)x^{4\,n} + (6\,a^2cdex^n +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(2*a^3*d*e*x^n + a^3*d^2 + (c^3*e^2*x^(2*n) + 2*c^3*d*
e*x^n + c^3*d^2)*x^(6*n) + 3*(a*c^2*e^2*x^(2*n) + 2*a*c^2*d*e*x^n
 + a*c^2*d^2 + a^2*c*e^2)*x^(4*n) + (6*a^2*c*d*e*x^n + 3*a^2*c*d^
2 + a^3*e^2)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)**2/(a+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2\,n} + a)^3(ex^n + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)^2),x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + a)^3*(e*x^n + d)^2), x)
```

3.58 $\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$

Optimal. Leaf size=171

$$\frac{x\sqrt{\frac{cx^{2n}}{a}+1}F_1\left(\frac{1}{2n};\frac{1}{2},1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}}-\frac{ex^{n+1}\sqrt{\frac{cx^{2n}}{a}+1}F_1\left(\frac{n+1}{2n};\frac{1}{2},1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)\sqrt{a+cx^{2n}}}$$

```
[Out] (x*Sqrt[1 + (c*x^(2*n))/a]*AppellF1[1/(2*n), 1/2, 1, (2 + n^(-1))
/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*Sqrt[a + c*x^(2*n)])
 - (e*x^(1 + n)*Sqrt[1 + (c*x^(2*n))/a]*AppellF1[(1 + n)/(2*n), 1
/2, 1, (3 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2
*(1 + n)*Sqrt[a + c*x^(2*n)])
```

Rubi [A] time = 0.425131, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{x\sqrt{\frac{cx^{2n}}{a}+1}F_1\left(\frac{1}{2n};\frac{1}{2},1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d\sqrt{a+cx^{2n}}}-\frac{ex^{n+1}\sqrt{\frac{cx^{2n}}{a}+1}F_1\left(\frac{n+1}{2n};\frac{1}{2},1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)\sqrt{a+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
```

```
[Out] (x*Sqrt[1 + (c*x^(2*n))/a]*AppellF1[1/(2*n), 1/2, 1, (2 + n^(-1))
/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*Sqrt[a + c*x^(2*n)])
 - (e*x^(1 + n)*Sqrt[1 + (c*x^(2*n))/a]*AppellF1[(1 + n)/(2*n), 1
/2, 1, (3 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2
*(1 + n)*Sqrt[a + c*x^(2*n)])
```

Rubi in Sympy [A] time = 73.6189, size = 138, normalized size = 0.81

$$\frac{x\sqrt{a+cx^{2n}}\,\text{appellf}_1\left(\frac{1}{2n},\frac{1}{2},1,\frac{n+\frac{1}{2}}{n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{ad\sqrt{1+\frac{cx^{2n}}{a}}}-\frac{ex^{n+1}\sqrt{a+cx^{2n}}\,\text{appellf}_1\left(\frac{n+1}{2n},\frac{1}{2},1,\frac{3n+1}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{ad^2\sqrt{1+\frac{cx^{2n}}{a}}\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)/(a+c*x**(2*n))**(1/2),x)
```

```
[Out] x*sqrt(a + c*x**(2*n))*appellf1(1/(2*n), 1/2, 1, (n + 1/2)/n, -c*
x**(2*n)/a, e**2*x**(2*n)/d**2)/(a*d*sqrt(1 + c*x**(2*n)/a)) - e*
x**(n + 1)*sqrt(a + c*x**(2*n))*appellf1((n + 1)/(2*n), 1/2, 1, (
3*n + 1)/(2*n), -c*x**(2*n)/a, e**2*x**(2*n)/d**2)/(a*d**2*sqrt(1
 + c*x**(2*n)/a)*(n + 1))
```

Mathematica [A] time = 0.120825, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)\sqrt{a+cx^{2n}}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]),x]
```

[Out] Integrate[1/((d + e*x^n)*Sqrt[a + c*x^(2*n)]), x]

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{1}{d+ex^n}\frac{1}{\sqrt{a+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)

[Out] int(1/(d+e*x^n)/(a+c*x^(2*n))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)),x, algorithm="fricas")

[Out] integral(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^{2n}}\,(d+ex^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x**n)/(a+c*x**(2*n))**(1/2),x)

[Out] Integral(1/(sqrt(a + c*x**(2*n))*(d + e*x**n)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{2n}+a}(ex^n+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + a)*(e*x^n + d)), x)
```

3.59 $\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=24

$$\text{Int}\left(\left(a+cx^{2n}\right)^p (d+ex^n)^q, x\right)$$

```
[Out] Unintegrable[(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi [A] time = 0.0207547, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left((d+ex^n)^q \left(a+cx^{2n}\right)^p, x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]

[Out] Defer[Int][(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+cx^{2n}\right)^p (d+ex^n)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**q*(a+c*x**(2*n))**p,x)

[Out] Integral((a + c*x**(2*n))**p*(d + e*x**n)**q, x)
```

Mathematica [A] time = 0.189138, size = 0, normalized size = 0.

$$\int (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p,x]

[Out] Integrate[(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Maple [A] time = 0.226, size = 0, normalized size = 0.

$$\int (d+ex^n)^q \left(a+cx^{2\,n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+a\right)^p (ex^n+d)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2n}+a\right)^p (ex^n+d)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**q*(a+c*x**(2*n))**p,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+a\right)^p (ex^n+d)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q, x)
```

3.60 $\int (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=299

$$d^3x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{3d^2ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{3de^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2n+1}+\frac{e^3x^{3n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{3n+1}$$

```
[Out] (3*d*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(
-1))/2, -p, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c
*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*Hypergeometr
ic2F1[(3 + n^(-1))/2, -p, (5 + n^(-1))/2, -((c*x^(2*n))/a)])/((1
+ 3*n)*(1 + (c*x^(2*n))/a)^p) + (d^3*x*(a + c*x^(2*n))^p*Hypergeo
metric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + (c
*x^(2*n))/a)^p + (3*d^2*e*x^(1 + n)*(a + c*x^(2*n))^p*Hypergeomet
ric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1
+ n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.318933, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$d^3x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{3d^2ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{3de^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2n+1}+\frac{e^3x^{3n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{3n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```


```
[Out] (3*d*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(
-1))/2, -p, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c
*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(a + c*x^(2*n))^p*Hypergeometr
ic2F1[(3 + n^(-1))/2, -p, (5 + n^(-1))/2, -((c*x^(2*n))/a)])/((1
+ 3*n)*(1 + (c*x^(2*n))/a)^p) + (d^3*x*(a + c*x^(2*n))^p*Hypergeo
metric2F1[1/(2*n), -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + (c
*x^(2*n))/a)^p + (3*d^2*e*x^(1 + n)*(a + c*x^(2*n))^p*Hypergeomet
ric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1
+ n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 41.2103, size = 240, normalized size = 0.8

$$d^3x\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p {}_2F_1\left(\begin{matrix}-p,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)+\frac{3d^2ex^{n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p {}_2F_1\left(\begin{matrix}-p,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{3de^2x^{2n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p {}_2F_1\left(\begin{matrix}-p,\frac{n+\frac{1}{2}}{n}\\ 2+\frac{1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{2n+1}+\frac{e^3x^{3n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p {}_2F_1\left(\begin{matrix}-p,\frac{3n+1}{2n}\\ \frac{5n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

```
[Out] d**3*x*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, 1/
(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a) + 3*d**2*e*x**(n + 1)*(1 +
 c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (n + 1)/(2*n)
), ((3*n + 1)/(2*n),), -c*x**(2*n)/a)/(n + 1) + 3*d*e**2*x**(2*n
+ 1)*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (n +
 1/2)/n), (2 + 1/(2*n),), -c*x**(2*n)/a)/(2*n + 1) + e**3*x**(3*n
 + 1)*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (3*
n + 1)/(2*n)), ((5*n + 1)/(2*n),), -c*x**(2*n)/a)/(3*n + 1)
```

Mathematica [A] time = 0.396314, size = 213, normalized size = 0.71

$$x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(d^2\left(d\,{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+\frac{3ex^n\,{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}\right)+\frac{3de^2x^{2n}\,{}_2F_1\left(1+\frac{1}{2n},-p;2+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)}{2n+1}+\frac{e^3x^{3n}\,{}_2F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right),-p;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{3n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + c*x^(2*n))^p*((3*d*e^2*x^(2*n)*Hypergeometric2F1[1 + 1/(2
*n), -p, 2 + 1/(2*n), -((c*x^(2*n))/a)])/(1 + 2*n) + (e^3*x^(3*n)
*Hypergeometric2F1[(3 + n^(-1))/2, -p, (5 + n^(-1))/2, -((c*x^(2*
n))/a)])/(1 + 3*n) + d^2*(d*Hypergeometric2F1[1/(2*n), -p, 1 + 1/
(2*n), -((c*x^(2*n))/a)] + (3*e*x^n*Hypergeometric2F1[(1 + n)/(2*
n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + n))))/(1 + (c*x^(
2*n))/a)^p
```

Maple [F] time = 0.169, size = 0, normalized size = 0.

$$\int \left(d+ex^n\right)^3\left(a+cx^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)

[Out] int((d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^3 (cx^{2n} + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p,x, algorithm="maxima")

[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^{3n} + 3\,de^2x^{2n} + 3\,d^2ex^n + d^3\right)\left(cx^{2n} + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p,x, algorithm="fricas")

[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x
^(2*n) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**3*(a+c*x**(2*n))**p,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.61 $\int (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=217

$$d^2x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{2dex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{e^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2n+1}$$

```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))
/2, -p, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(
2*n))/a)^p) + (d^2*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n),
 -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (
2*d*e*x^(1 + n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + n)/(2*n)
, -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*(1 + (c*x^(2*n)
)/a)^p)
```

Rubi [A] time = 0.216818, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$d^2x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{2dex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{e^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right),-p;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```


```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(2 + n^(-1))
/2, -p, (4 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + 2*n)*(1 + (c*x^(
2*n))/a)^p) + (d^2*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n),
 -p, (2 + n^(-1))/2, -((c*x^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (
2*d*e*x^(1 + n)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + n)/(2*n)
, -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)])/((1 + n)*(1 + (c*x^(2*n)
)/a)^p)
```

Rubi in Sympy [A] time = 29.3167, size = 170, normalized size = 0.78

$$d^2x\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p{}_2F_1\left(\left.\begin{matrix}-p,\frac{1}{2n}\\\frac{n+\frac{1}{2}}{n}\end{matrix}\right|-\frac{cx^{2n}}{a}\right)+\frac{2dex^{n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p{}_2F_1\left(\left.\begin{matrix}-p,\frac{n+1}{2n}\\\frac{3n+1}{2n}\end{matrix}\right|-\frac{cx^{2n}}{a}\right)}{n+1}+\frac{e^2x^{2n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p{}_2F_1\left(\left.\begin{matrix}-p,\frac{n+\frac{1}{2}}{n}\\2+\frac{1}{2n}\end{matrix}\right|-\frac{cx^{2n}}{a}\right)}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] d**2*x*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, 1/
(2*n)), ((n + 1/2)/n,), -c*x**(2*n)/a) + 2*d*e*x**(n + 1)*(1 + c*
x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (n + 1)/(2*n)),
((3*n + 1)/(2*n),), -c*x**(2*n)/a)/(n + 1) + e**2*x**(2*n + 1)*(1
 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (n + 1/2)/n
), (2 + 1/(2*n),), -c*x**(2*n)/a)/(2*n + 1)
```

Mathematica [A] time = 0.206929, size = 171, normalized size = 0.79

$$\frac{x\left(a+cx^{2n}\right)^{p}\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(d(2n+1)\left(d(n+1)\,{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+2ex^{n}\,{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)+e^{2}(n}{(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + c*x^(2*n))^p*(e^2*(1 + n)*x^(2*n)*Hypergeometric2F1[1 + 1
/(2*n), -p, 2 + 1/(2*n), -((c*x^(2*n))/a)] + d*(1 + 2*n)*(d*(1 +
n)*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n), -((c*x^(2*n))/a)]
+ 2*e*x^n*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2, -(
(c*x^(2*n))/a)])))/((1 + n)*(1 + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \left(d+ex^{n}\right)^{2}\left(a+cx^{2\,n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^{n}+d\right)^{2}\left(cx^{2\,n}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^{2}x^{2\,n}+2\,dex^{n}+d^{2}\right)\left(cx^{2\,n}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.62 $\int (d + ex^n)\left(a + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=135

$$dx\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}$$

```
[Out] (d*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1)
)/2, -((c*x^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (e*x^(1 + n)*(a +
 c*x^(2*n))^p*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2
, -((c*x^(2*n))/a)])/((1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.128689, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$dx\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)+\frac{ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (d*x*(a + c*x^(2*n))^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1)
)/2, -((c*x^(2*n))/a)])/(1 + (c*x^(2*n))/a)^p + (e*x^(1 + n)*(a +
 c*x^(2*n))^p*Hypergeometric2F1[(1 + n)/(2*n), -p, (3 + n^(-1))/2
, -((c*x^(2*n))/a)])/((1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 17.0747, size = 104, normalized size = 0.77

$$dx\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p{}_2F_1\left(\left.\begin{matrix}-p,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\right|-\frac{cx^{2n}}{a}\right)+\frac{ex^{n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p{}_2F_1\left(\left.\begin{matrix}-p,\frac{n+1}{2n}\\ \frac{3n+1}{2n}\end{matrix}\right|-\frac{cx^{2n}}{a}\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)*(a+c*x**(2*n))**p,x)
```

```
[Out] d*x*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, 1/(2*
n)), ((n + 1/2)/n,), -c*x**(2*n)/a) + e*x**(n + 1)*(1 + c*x**(2*n
)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (n + 1)/(2*n)), ((3*n +
 1)/(2*n),), -c*x**(2*n)/a)/(n + 1)
```

Mathematica [A] time = 0.0931384, size = 110, normalized size = 0.81

$$\frac{x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(d(n+1)\,{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};-\frac{cx^{2n}}{a}\right)+ex^n\,{}_2F_1\left(\frac{n+1}{2n},-p;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a}\right)\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(a + c*x^(2*n))^p*(d*(1 + n)*Hypergeometric2F1[1/(2*n), -p, 1
+ 1/(2*n), -((c*x^(2*n))/a)] + e*x^n*Hypergeometric2F1[(1 + n)/(2
*n), -p, (3 + n^(-1))/2, -((c*x^(2*n))/a)]))/((1 + n)*(1 + (c*x^(
2*n))/a)^p)
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (d + e x^{n}) \left(a + c x^{2 n}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)*(a+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e x^{n} + d)\left(c x^{2 n} + a\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(c*x^(2*n) + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e x^{n} + d)\left(c x^{2 n} + a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(c*x^(2*n) + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)(cx^{2n} + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(c*x^(2*n) + a)^p,x, algorithm="giac")

[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p, x)
```

3.63 $\int \frac{(a+cx^{2n})^p}{d+ex^n}\,dx$

Optimal. Leaf size=167

$$\frac{x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d}-\frac{ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)}$$

```
[Out] (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, (2 + n^(-1))/2, -((
c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + (c*x^(2*n))/a)^p) - (e
*x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 1, (3 +
n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + n)*(1
+ (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.331645, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,1;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d}-\frac{ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,1;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^(2*n))^p/(d + e*x^n),x]
```

```
[Out] (x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, (2 + n^(-1))/2, -((
c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + (c*x^(2*n))/a)^p) - (e
*x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 1, (3 +
n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + n)*(1
+ (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 70.9471, size = 128, normalized size = 0.77

$$\frac{x\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p\operatorname{appellf_1}\left(\frac{1}{2n},1,-p,\frac{n+\frac{1}{2}}{n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d}-\frac{ex^{n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p\operatorname{appellf_1}\left(\frac{n+1}{2n},1,-p,\frac{3n+1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^2\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] x*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*appellf1(1/(2*n),
1, -p, (n + 1/2)/n, e**2*x**(2*n)/d**2, -c*x**(2*n)/a)/d - e*x**(
n + 1)*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*appellf1((n +
 1)/(2*n), 1, -p, (3*n + 1)/(2*n), e**2*x**(2*n)/d**2, -c*x**(2*n
)/a)/(d**2*(n + 1))
```

Mathematica [A] time = 0.069954, size = 0, normalized size = 0.

$$\int \frac{\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + c*x^(2*n))^p/(d + e*x^n),x]
```

```
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n), x]
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \frac{(a + c x^{2n})^p}{d + e x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c*x^(2*n))^p/(d+e*x^n),x)
```

```
[Out] int((a+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c x^{2n} + a)^p}{e x^n + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c x^{2n} + a)^p}{e x^n + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d),x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + a\right)^p}{ex^n + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d),x, algorithm="giac")

[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d), x)
```

3.64 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=261

$$\frac{x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,2;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2}+\frac{e^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,2;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(2n+1)}-\frac{2ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,2;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(n+1)}$$

```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 2
, (4 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 +
 2*n)*(1 + (c*x^(2*n))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2
*n), -p, 2, (2 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])
/(d^2*(1 + (c*x^(2*n))/a)^p) - (2*e*x^(1 + n)*(a + c*x^(2*n))^p*A
ppellF1[(1 + n)/(2*n), -p, 2, (3 + n^(-1))/2, -((c*x^(2*n))/a), (
e^2*x^(2*n))/d^2])/(d^3*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.55777, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,2;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2}+\frac{e^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,2;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(2n+1)}-\frac{2ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,2;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

```
[Out] (e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p, 2
, (4 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 +
 2*n)*(1 + (c*x^(2*n))/a)^p) + (x*(a + c*x^(2*n))^p*AppellF1[1/(2
*n), -p, 2, (2 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])
/(d^2*(1 + (c*x^(2*n))/a)^p) - (2*e*x^(1 + n)*(a + c*x^(2*n))^p*A
ppellF1[(1 + n)/(2*n), -p, 2, (3 + n^(-1))/2, -((c*x^(2*n))/a), (
e^2*x^(2*n))/d^2])/(d^3*(1 + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 137.215, size = 206, normalized size = 0.79

$$\frac{x\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p\operatorname{appellf}_1\left(\frac{1}{2n},2,-p,\frac{n+\frac{1}{2}}{n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^2}-\frac{2ex^{n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p\operatorname{appellf}_1\left(\frac{n+1}{2n},2,-p,\frac{3n+1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^3\left(n+1\right)}+\frac{e^2x^{2n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p\operatorname{appellf}_1\left(\frac{n+\frac{1}{2}}{n},2,-p,2+\frac{1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^4\left(2n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] x*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*appellf1(1/(2*n),
2, -p, (n + 1/2)/n, e**2*x**(2*n)/d**2, -c*x**(2*n)/a)/d**2 - 2*e
*x**(n + 1)*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*appellf1
((n + 1)/(2*n), 2, -p, (3*n + 1)/(2*n), e**2*x**(2*n)/d**2, -c*x*
*(2*n)/a)/(d**3*(n + 1)) + e**2*x**(2*n + 1)*(1 + c*x**(2*n)/a)**
(-p)*(a + c*x**(2*n))**p*appellf1((n + 1/2)/n, 2, -p, 2 + 1/(2*n)
, e**2*x**(2*n)/d**2, -c*x**(2*n)/a)/(d**4*(2*n + 1))
```

Mathematica [A] time = 0.117383, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2n})^p}{(d + ex^n)^2}\, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2,x]
```

```
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \frac{(a + cx^{2\,n})^p}{(d + ex^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

```
[Out] int((a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2\,n} + a)^p}{(ex^n + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx^{2\,n} + a)^p}{e^2x^{2\,n} + 2\,dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2\,n}+a\right)^{p}}{\left(ex^{n}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^2, x)
```

3.65 $\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$

Optimal. Leaf size=357

$$-\frac{e^3x^{3n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(3n+1)}+\frac{3e^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(2n+1)}-\frac{3ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,3;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(n+1)}+\frac{x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,3;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}$$

```
[Out] (3*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p,
 3, (4 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^5*(1
 + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^3*x^(1 + 3*n)*(a + c*x^(2*n))
^p*AppellF1[(3 + n^(-1))/2, -p, 3, (5 + n^(-1))/2, -((c*x^(2*n))/
a), (e^2*x^(2*n))/d^2])/(d^6*(1 + 3*n)*(1 + (c*x^(2*n))/a)^p) + (
x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 3, (2 + n^(-1))/2, -((c
*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + (c*x^(2*n))/a)^p) - (
3*e*x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 3, (3
 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.795284, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{e^3x^{3n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(3+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(5+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(3n+1)}+\frac{3e^2x^{2n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2}\left(2+\frac{1}{n}\right);-p,3;\frac{1}{2}\left(4+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(2n+1)}-\frac{3ex^{n+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2n};-p,3;\frac{1}{2}\left(3+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(n+1)}+\frac{x\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{1}{2n};-p,3;\frac{1}{2}\left(2+\frac{1}{n}\right);-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
```


```
[Out] (3*e^2*x^(1 + 2*n)*(a + c*x^(2*n))^p*AppellF1[(2 + n^(-1))/2, -p,
 3, (4 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^5*(1
 + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^3*x^(1 + 3*n)*(a + c*x^(2*n))
^p*AppellF1[(3 + n^(-1))/2, -p, 3, (5 + n^(-1))/2, -((c*x^(2*n))/
a), (e^2*x^(2*n))/d^2])/(d^6*(1 + 3*n)*(1 + (c*x^(2*n))/a)^p) + (
x*(a + c*x^(2*n))^p*AppellF1[1/(2*n), -p, 3, (2 + n^(-1))/2, -((c
*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + (c*x^(2*n))/a)^p) - (
3*e*x^(1 + n)*(a + c*x^(2*n))^p*AppellF1[(1 + n)/(2*n), -p, 3, (3
 + n^(-1))/2, -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + n)*
(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.411012, size = 0, normalized size = 0.

$$\int \frac{(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] Integrate[(a + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{(a+cx^{2\,n})^p}{(d+ex^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

```
[Out] int((a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2\,n}+a)^p}{(ex^n+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2\,n}+a)^p}{e^3x^{3\,n}+3\,de^2x^{2\,n}+3\,d^2ex^n+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2
*e*x^n + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n}+a)^p}{(ex^n+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p/(e*x^n + d)^3, x)
```

3.66 $\int (d + ex^n)\left(a + bx^n + cx^{2n}\right)\,dx$

Optimal. Leaf size=62

$$\frac{x^{n+1}(ae+bd)}{n+1}+adx+\frac{x^{2n+1}(be+cd)}{2n+1}+\frac{cex^{3n+1}}{3n+1}$$

```
[Out] a*d*x + ((b*d + a*e)*x^(1 + n))/(1 + n) + ((c*d + b*e)*x^(1 + 2*n
))/(1 + 2*n) + (c*e*x^(1 + 3*n))/(1 + 3*n)
```

Rubi [A] time = 0.0827313, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{x^{n+1}(ae+bd)}{n+1}+adx+\frac{x^{2n+1}(be+cd)}{2n+1}+\frac{cex^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^(1 + n))/(1 + n) + ((c*d + b*e)*x^(1 + 2*n
))/(1 + 2*n) + (c*e*x^(1 + 3*n))/(1 + 3*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{cex^{3n+1}}{3n+1}+d\int a\,dx+\frac{x^{n+1}\left(ae+bd\right)}{n+1}+\frac{x^{2n+1}\left(be+cd\right)}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] c*e*x**(3*n + 1)/(3*n + 1) + d*Integral(a, x) + x**(n + 1)*(a*e +
 b*d)/(n + 1) + x**(2*n + 1)*(b*e + c*d)/(2*n + 1)
```

Mathematica [A] time = 0.198996, size = 57, normalized size = 0.92

$$x\left(\frac{x^n(ae+bd)}{n+1}+ad+\frac{x^{2n}(be+cd)}{2n+1}+\frac{cex^{3n}}{3n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(a*d + ((b*d + a*e)*x^n)/(1 + n) + ((c*d + b*e)*x^(2*n))/(1 + 2
*n) + (c*e*x^(3*n))/(1 + 3*n))
```

Maple [A] time = 0.018, size = 66, normalized size = 1.1

$$adx+\frac{(ae+bd)\,x\mathrm{e}^{n\ln(x)}}{1+n}+\frac{(be+cd)\,x\left(\mathrm{e}^{n\ln(x)}\right)^{2}}{1+2\,n}+\frac{cex\left(\mathrm{e}^{n\ln(x)}\right)^{3}}{1+3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)*(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] a*d*x+(a*e+b*d)/(1+n)*x*exp(n*ln(x))+(b*e+c*d)/(1+2*n)*x*exp(n*ln
(x))^2+c*e/(1+3*n)*x*exp(n*ln(x))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)*(e*x^n + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271977, size = 185, normalized size = 2.98

$$\frac{\left(2\,cen^2 + 3\,cen + ce\right)xx^{3\,n} + \left(3\,(cd + be)n^2 + cd + be + 4\,(cd + be)n\right)xx^{2\,n} + \left(6\,(bd + ae)n^2 + bd + ae + 5\,(bd + ae)n\right)xx^{n} + (6}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)*(e*x^n + d),x, algorithm="fricas")
```

```
[Out] ((2*c*e*n^2 + 3*c*e*n + c*e)*x*x^(3*n) + (3*(c*d + b*e)*n^2 + c*d
 + b*e + 4*(c*d + b*e)*n)*x*x^(2*n) + (6*(b*d + a*e)*n^2 + b*d +
a*e + 5*(b*d + a*e)*n)*x*x^n + (6*a*d*n^3 + 11*a*d*n^2 + 6*a*d*n
+ a*d)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [A] time = 3.84148, size = 656, normalized size = 10.58

$$\begin{cases} adx + ae\log(x) + bd\log(x) - \frac{be}{x} - \frac{cd}{x} - \frac{ce}{2x^2} \\ adx + 2ae\sqrt{x} + 2bd\sqrt{x} + be\log(x) + cd\log(x) - \frac{2ce}{\sqrt{x}} \\ adx + \frac{3aex^{\frac{2}{3}}}{2} + \frac{3bdx^{\frac{2}{3}}}{2} + 3be\sqrt[3]{x} + 3cd\sqrt[3]{x} + ce\log(x) \\ \frac{6adn^3x}{6n^3+11n^2+6n+1} + \frac{11adn^2x}{6n^3+11n^2+6n+1} + \frac{6adnx}{6n^3+11n^2+6n+1} + \frac{adx}{6n^3+11n^2+6n+1} + \frac{6aen^2xx^n}{6n^3+11n^2+6n+1} + \frac{5aenxx^n}{6n^3+11n^2+6n+1} + \frac{aexx^n}{6n^3+11n^2+6n+1} + \frac{6bdn^2xx^n}{6n^3+11n^2+6n+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((a*d*x + a*e*log(x) + b*d*log(x) - b*e/x - c*d/x - c*e/
(2*x**2), Eq(n, -1)), (a*d*x + 2*a*e*sqrt(x) + 2*b*d*sqrt(x) + b*
e*log(x) + c*d*log(x) - 2*c*e/sqrt(x), Eq(n, -1/2)), (a*d*x + 3*a
*e*x**(2/3)/2 + 3*b*d*x**(2/3)/2 + 3*b*e*x**(1/3) + 3*c*d*x**(1/3
) + c*e*log(x), Eq(n, -1/3)), (6*a*d*n**3*x/(6*n**3 + 11*n**2 + 6
*n + 1) + 11*a*d*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*d*n*x/
(6*n**3 + 11*n**2 + 6*n + 1) + a*d*x/(6*n**3 + 11*n**2 + 6*n + 1)
 + 6*a*e*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a*e*n*x*x**
n/(6*n**3 + 11*n**2 + 6*n + 1) + a*e*x*x**n/(6*n**3 + 11*n**2 + 6
*n + 1) + 6*b*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*d*
n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + b*d*x*x**n/(6*n**3 + 11*n
**2 + 6*n + 1) + 3*b*e*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 4*b*e*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*e*x*x**(
2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*d*n**2*x*x**(2*n)/(6*n**3
 + 11*n**2 + 6*n + 1) + 4*c*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*
```

```
n + 1) + c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*c*e*n**2
*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*c*e*n*x*x**(3*n)/(6*
n**3 + 11*n**2 + 6*n + 1) + c*e*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*
n + 1), True))
```

GIAC/XCAS [A] time = 0.269814, size = 308, normalized size = 4.97

$$\frac{6\,adn^3x + 3\,cdn^2xe^{(2\,n\ln(x))} + 6\,bdn^2xe^{(n\ln(x))} + 11\,adn^2x + 4\,cdnxe^{(2\,n\ln(x))} + 2\,cn^2xe^{(3\,n\ln(x)+1)} + 3\,bn^2xe^{(2\,n\ln(x)+1)} + 6\,an^2xe^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)*(e*x^n + d),x, algorithm="giac")
```

```
[Out] (6*a*d*n^3*x + 3*c*d*n^2*x*e^(2*n*ln(x)) + 6*b*d*n^2*x*e^(n*ln(x)
) + 11*a*d*n^2*x + 4*c*d*n*x*e^(2*n*ln(x)) + 2*c*n^2*x*e^(3*n*ln(
x) + 1) + 3*b*n^2*x*e^(2*n*ln(x) + 1) + 6*a*n^2*x*e^(n*ln(x) + 1)
 + 5*b*d*n*x*e^(n*ln(x)) + 6*a*d*n*x + c*d*x*e^(2*n*ln(x)) + 3*c*
n*x*e^(3*n*ln(x) + 1) + 4*b*n*x*e^(2*n*ln(x) + 1) + 5*a*n*x*e^(n*
ln(x) + 1) + b*d*x*e^(n*ln(x)) + a*d*x + c*x*e^(3*n*ln(x) + 1) +
b*x*e^(2*n*ln(x) + 1) + a*x*e^(n*ln(x) + 1))/(6*n^3 + 11*n^2 + 6*
n + 1)
```

3.67 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^2\,dx$

Optimal. Leaf size=132

$$a^2dx+\frac{x^{2n+1}\left(2abe+2acd+b^2d\right)}{2n+1}+\frac{x^{3n+1}\left(2ace+b^2e+2bcd\right)}{3n+1}+\frac{ax^{n+1}(ae+2bd)}{n+1}+\frac{cx^{4n+1}(2be+cd)}{4n+1}+\frac{c^2ex^{5n+1}}{5n+1}$$

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d + 2*a*c*d
 + 2*a*b*e)*x^(1 + 2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)
*x^(1 + 3*n))/(1 + 3*n) + (c*(c*d + 2*b*e)*x^(1 + 4*n))/(1 + 4*n)
 + (c^2*e*x^(1 + 5*n))/(1 + 5*n)
```

Rubi [A] time = 0.202595, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$a^2dx+\frac{x^{2n+1}\left(2abe+2acd+b^2d\right)}{2n+1}+\frac{x^{3n+1}\left(2ace+b^2e+2bcd\right)}{3n+1}+\frac{ax^{n+1}(ae+2bd)}{n+1}+\frac{cx^{4n+1}(2be+cd)}{4n+1}+\frac{c^2ex^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d + 2*a*c*d
 + 2*a*b*e)*x^(1 + 2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)
*x^(1 + 3*n))/(1 + 3*n) + (c*(c*d + 2*b*e)*x^(1 + 4*n))/(1 + 4*n)
 + (c^2*e*x^(1 + 5*n))/(1 + 5*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\int d\,dx+\frac{ax^{n+1}(ae+2bd)}{n+1}+\frac{c^2ex^{5n+1}}{5n+1}+\frac{cx^{4n+1}(2be+cd)}{4n+1}+\frac{x^{2n+1}\left(2a(be+cd)+b^2d\right)}{2n+1}+\frac{x^{3n+1}\left(b^2e+2c(ae+bd)\right)}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] a**2*Integral(d, x) + a*x**(n + 1)*(a*e + 2*b*d)/(n + 1) + c**2*e
*x**(5*n + 1)/(5*n + 1) + c*x**(4*n + 1)*(2*b*e + c*d)/(4*n + 1)
+ x**(2*n + 1)*(2*a*(b*e + c*d) + b**2*d)/(2*n + 1) + x**(3*n + 1
)*(b**2*e + 2*c*(a*e + b*d))/(3*n + 1)
```

Mathematica [A] time = 0.363756, size = 123, normalized size = 0.93

$$x\left(a^2d+\frac{x^{2n}\left(2abe+2acd+b^2d\right)}{2n+1}+\frac{x^{3n}\left(2ace+b^2e+2bcd\right)}{3n+1}+\frac{ax^n(ae+2bd)}{n+1}+\frac{cx^{4n}(2be+cd)}{4n+1}+\frac{c^2ex^{5n}}{5n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(a^2*d + (a*(2*b*d + a*e)*x^n)/(1 + n) + ((b^2*d + 2*a*c*d + 2*
a*b*e)*x^(2*n))/(1 + 2*n) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^(3*n))
/(1 + 3*n) + (c*(c*d + 2*b*e)*x^(4*n))/(1 + 4*n) + (c^2*e*x^(5*n)
)/(1 + 5*n))
```

Maple [A] time = 0.022, size = 138, normalized size = 1.1

$$a^2dx + \frac{(2\,ace + b^2e + 2\,bcd)\,x\left(e^{n\ln(x)}\right)^3}{1+3\,n} + \frac{(2\,abe + 2\,acd + b^2d)\,x\left(e^{n\ln(x)}\right)^2}{1+2\,n} + \frac{a\,(ae + 2\,bd)\,xe^{n\ln(x)}}{1+n} + \frac{c\,(2\,be + cd)\,x\left(e^{n\ln(x)}\right)^4}{1+4\,n} + \frac{c^2ex\left(e^{n\ln(x)}\right)^5}{1+5\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] a^2*d*x+(2*a*c*e+b^2*e+2*b*c*d)/(1+3*n)*x*exp(n*ln(x))^3+(2*a*b*e
+2*a*c*d+b^2*d)/(1+2*n)*x*exp(n*ln(x))^2+a*(a*e+2*b*d)/(1+n)*x*ex
p(n*ln(x))+c*(2*b*e+c*d)/(1+4*n)*x*exp(n*ln(x))^4+c^2*e/(1+5*n)*x
*exp(n*ln(x))^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290458, size = 668, normalized size = 5.06

$$\frac{\left(24\,c^2en^4 + 50\,c^2en^3 + 35\,c^2en^2 + 10\,c^2en + c^2e\right)xx^{5\,n} + \left(30\,\left(c^2d + 2\,bce\right)n^4 + 61\,\left(c^2d + 2\,bce\right)n^3 + c^2d + 2\,bce + 41\,\left(c^2d + 2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d),x, algorithm="fricas")
```

```
[Out] ((24*c^2*e*n^4 + 50*c^2*e*n^3 + 35*c^2*e*n^2 + 10*c^2*e*n + c^2*e
)*x*x^(5*n) + (30*(c^2*d + 2*b*c*e)*n^4 + 61*(c^2*d + 2*b*c*e)*n^
3 + c^2*d + 2*b*c*e + 41*(c^2*d + 2*b*c*e)*n^2 + 11*(c^2*d + 2*b*
c*e)*n)*x*x^(4*n) + (40*(2*b*c*d + (b^2 + 2*a*c)*e)*n^4 + 78*(2*b
*c*d + (b^2 + 2*a*c)*e)*n^3 + 2*b*c*d + 49*(2*b*c*d + (b^2 + 2*a*
c)*e)*n^2 + (b^2 + 2*a*c)*e + 12*(2*b*c*d + (b^2 + 2*a*c)*e)*n)*x
*x^(3*n) + (60*(2*a*b*e + (b^2 + 2*a*c)*d)*n^4 + 107*(2*a*b*e + (
b^2 + 2*a*c)*d)*n^3 + 2*a*b*e + 59*(2*a*b*e + (b^2 + 2*a*c)*d)*n^
2 + (b^2 + 2*a*c)*d + 13*(2*a*b*e + (b^2 + 2*a*c)*d)*n)*x*x^(2*n)
 + (120*(2*a*b*d + a^2*e)*n^4 + 154*(2*a*b*d + a^2*e)*n^3 + 2*a*b
*d + a^2*e + 71*(2*a*b*d + a^2*e)*n^2 + 14*(2*a*b*d + a^2*e)*n)*x
*x^n + (120*a^2*d*n^5 + 274*a^2*d*n^4 + 225*a^2*d*n^3 + 85*a^2*d*
n^2 + 15*a^2*d*n + a^2*d)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^
2 + 15*n + 1)
```

Sympy [A] time = 42.6775, size = 3128, normalized size = 23.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*log(x) + 2*a*b*d*log(x) - 2*a*b*e/x
- 2*a*c*d/x - a*c*e/x**2 - b**2*d/x - b**2*e/(2*x**2) - b*c*d/x**
2 - 2*b*c*e/(3*x**3) - c**2*d/(3*x**3) - c**2*e/(4*x**4), Eq(n, -
1)), (a**2*d*x + 2*a**2*e*sqrt(x) + 4*a*b*d*sqrt(x) + 2*a*b*e*log
(x) + 2*a*c*d*log(x) - 4*a*c*e/sqrt(x) + b**2*d*log(x) - 2*b**2*e
/sqrt(x) - 4*b*c*d/sqrt(x) - 2*b*c*e/x - c**2*d/x - 2*c**2*e/(3*x
**(3/2)), Eq(n, -1/2)), (a**2*d*x + 3*a**2*e*x**(2/3)/2 + 3*a*b*d
*x**(2/3) + 6*a*b*e*x**(1/3) + 6*a*c*d*x**(1/3) + 2*a*c*e*log(x)
+ 3*b**2*d*x**(1/3) + b**2*e*log(x) + 2*b*c*d*log(x) - 6*b*c*e/x*
*(1/3) - 3*c**2*d/x**(1/3) - 3*c**2*e/(2*x**(2/3)), Eq(n, -1/3)),
 (a**2*d*x + 4*a**2*e*x**(3/4)/3 + 8*a*b*d*x**(3/4)/3 + 4*a*b*e*s
qrt(x) + 4*a*c*d*sqrt(x) + 8*a*c*e*x**(1/4) + 2*b**2*d*sqrt(x) +
4*b**2*e*x**(1/4) + 8*b*c*d*x**(1/4) + 2*b*c*e*log(x) + c**2*d*lo
g(x) - 4*c**2*e/x**(1/4), Eq(n, -1/4)), (a**2*d*x + 5*a**2*e*x**(
4/5)/4 + 5*a*b*d*x**(4/5)/2 + 10*a*b*e*x**(3/5)/3 + 10*a*c*d*x**(
3/5)/3 + 5*a*c*e*x**(2/5) + 5*b**2*d*x**(3/5)/3 + 5*b**2*e*x**(2/
5)/2 + 5*b*c*d*x**(2/5) + 10*b*c*e*x**(1/5) + 5*c**2*d*x**(1/5) +
 c**2*e*log(x), Eq(n, -1/5)), (120*a**2*d*n**5*x/(120*n**5 + 274*
n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 274*a**2*d*n**4*x/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 225*a**2*d*n**3*
x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 85*a**2
*d*n**2*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 15*a**2*d*n*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + a**2*d*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 120*a**2*e*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85
*n**2 + 15*n + 1) + 154*a**2*e*n**3*x*x**n/(120*n**5 + 274*n**4 +
 225*n**3 + 85*n**2 + 15*n + 1) + 71*a**2*e*n**2*x*x**n/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 14*a**2*e*n*x*x**n
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + a**2*e*x
*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240
*a*b*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 308*a*b*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + 142*a*b*d*n**2*x*x**n/(120*n**5 + 274*n**4
 + 225*n**3 + 85*n**2 + 15*n + 1) + 28*a*b*d*n*x*x**n/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*b*d*x*x**n/(120*
n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 120*a*b*e*n**4
*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 214*a*b*e*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85
*n**2 + 15*n + 1) + 118*a*b*e*n**2*x*x**(2*n)/(120*n**5 + 274*n**
4 + 225*n**3 + 85*n**2 + 15*n + 1) + 26*a*b*e*n*x*x**(2*n)/(120*n
**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*b*e*x*x**(2
*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 120*a
*c*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + 214*a*c*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*
n**3 + 85*n**2 + 15*n + 1) + 118*a*c*d*n**2*x*x**(2*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 26*a*c*d*n*x*x**(2*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*c*
d*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1
) + 80*a*c*e*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85
*n**2 + 15*n + 1) + 156*a*c*e*n**3*x*x**(3*n)/(120*n**5 + 274*n**
4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98*a*c*e*n**2*x*x**(3*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*a*c*e*n*x
*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 2*a*c*e*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 60*b**2*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 107*b**2*d*n**3*x*x**(2*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 59*b**2*d*n**2*x*x*
*(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 13
*b**2*d*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + b**2*d*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 40*b**2*e*n**4*x*x**(3*n)/(120*n**5 + 274*n*
*4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*b**2*e*n**3*x*x**(3*n)/(
```

```
120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 49*b**2*e*
n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 12*b**2*e*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + b**2*e*x*x**(3*n)/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 80*b*c*d*n**4*x*x**(3*n)/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 156*b*c*d*n**3*x*x*
*(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 98
*b*c*d*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 24*b*c*d*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n*
*3 + 85*n**2 + 15*n + 1) + 2*b*c*d*x*x**(3*n)/(120*n**5 + 274*n**
4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b*c*e*n**4*x*x**(4*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 122*b*c*e*n*
*3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 82*b*c*e*n**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 22*b*c*e*n*x*x**(4*n)/(120*n**5 + 274*n**4 +
 225*n**3 + 85*n**2 + 15*n + 1) + 2*b*c*e*x*x**(4*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 30*c**2*d*n**4*x*x**(
4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 61*c
**2*d*n**3*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 41*c**2*d*n**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 11*c**2*d*n*x*x**(4*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*d*x*x**(4*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*c**2*e*n*
*4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 50*c**2*e*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + 35*c**2*e*n**2*x*x**(5*n)/(120*n**5 + 274*n
**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 10*c**2*e*n*x*x**(5*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + c**2*e*x*x**
(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1), True
))
```

GIAC/XCAS [A] time = 0.289965, size = 1206, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d),x, algorithm="giac")
```

```
[Out] (120*a^2*d*n^5*x + 30*c^2*d*n^4*x*e^(4*n*ln(x)) + 80*b*c*d*n^4*x*
e^(3*n*ln(x)) + 60*b^2*d*n^4*x*e^(2*n*ln(x)) + 120*a*c*d*n^4*x*e^
(2*n*ln(x)) + 240*a*b*d*n^4*x*e^(n*ln(x)) + 274*a^2*d*n^4*x + 61*
c^2*d*n^3*x*e^(4*n*ln(x)) + 156*b*c*d*n^3*x*e^(3*n*ln(x)) + 107*b
^2*d*n^3*x*e^(2*n*ln(x)) + 214*a*c*d*n^3*x*e^(2*n*ln(x)) + 24*c^2
*n^4*x*e^(5*n*ln(x) + 1) + 60*b*c*n^4*x*e^(4*n*ln(x) + 1) + 40*b^
2*n^4*x*e^(3*n*ln(x) + 1) + 80*a*c*n^4*x*e^(3*n*ln(x) + 1) + 120*
a*b*n^4*x*e^(2*n*ln(x) + 1) + 120*a^2*n^4*x*e^(n*ln(x) + 1) + 308
*a*b*d*n^3*x*e^(n*ln(x)) + 225*a^2*d*n^3*x + 41*c^2*d*n^2*x*e^(4*
n*ln(x)) + 98*b*c*d*n^2*x*e^(3*n*ln(x)) + 59*b^2*d*n^2*x*e^(2*n*l
n(x)) + 118*a*c*d*n^2*x*e^(2*n*ln(x)) + 50*c^2*n^3*x*e^(5*n*ln(x)
 + 1) + 122*b*c*n^3*x*e^(4*n*ln(x) + 1) + 78*b^2*n^3*x*e^(3*n*ln(
x) + 1) + 156*a*c*n^3*x*e^(3*n*ln(x) + 1) + 214*a*b*n^3*x*e^(2*n*
ln(x) + 1) + 154*a^2*n^3*x*e^(n*ln(x) + 1) + 142*a*b*d*n^2*x*e^(n
*ln(x)) + 85*a^2*d*n^2*x + 11*c^2*d*n*x*e^(4*n*ln(x)) + 24*b*c*d*
n*x*e^(3*n*ln(x)) + 13*b^2*d*n*x*e^(2*n*ln(x)) + 26*a*c*d*n*x*e^(
2*n*ln(x)) + 35*c^2*n^2*x*e^(5*n*ln(x) + 1) + 82*b*c*n^2*x*e^(4*n
*ln(x) + 1) + 49*b^2*n^2*x*e^(3*n*ln(x) + 1) + 98*a*c*n^2*x*e^(3*
n*ln(x) + 1) + 118*a*b*n^2*x*e^(2*n*ln(x) + 1) + 71*a^2*n^2*x*e^(
n*ln(x) + 1) + 28*a*b*d*n*x*e^(n*ln(x)) + 15*a^2*d*n*x + c^2*d*x*
e^(4*n*ln(x)) + 2*b*c*d*x*e^(3*n*ln(x)) + b^2*d*x*e^(2*n*ln(x)) +
 2*a*c*d*x*e^(2*n*ln(x)) + 10*c^2*n*x*e^(5*n*ln(x) + 1) + 22*b*c*
n*x*e^(4*n*ln(x) + 1) + 12*b^2*n*x*e^(3*n*ln(x) + 1) + 24*a*c*n*x
*e^(3*n*ln(x) + 1) + 26*a*b*n*x*e^(2*n*ln(x) + 1) + 14*a^2*n*x*e^
(n*ln(x) + 1) + 2*a*b*d*x*e^(n*ln(x)) + a^2*d*x + c^2*x*e^(5*n*ln
(x) + 1) + 2*b*c*x*e^(4*n*ln(x) + 1) + b^2*x*e^(3*n*ln(x) + 1) +
2*a*c*x*e^(3*n*ln(x) + 1) + 2*a*b*x*e^(2*n*ln(x) + 1) + a^2*x*e^(
n*ln(x) + 1))/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

3.68 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^3\,dx$

Optimal. Leaf size=218

$$a^3dx+\frac{x^{3n+1}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{3n+1}+\frac{a^2x^{n+1}(ae+3bd)}{n+1}+\frac{3ax^{2n+1}\left(abe+acd+b^2d\right)}{2n+1}+\frac{3cx^{5n+1}\left(ace+b^2e+bcd\right)}{5n+1}+\frac{x^{4n+1}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{4n+1}+\frac{c^2x^{6n+1}(3be+cd)}{6n+1}+\frac{c^3ex^{7n+1}}{7n+1}$$

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a
*c*d + a*b*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*
b^2*e + 3*a^2*c*e)*x^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2
*d + b^3*e + 6*a*b*c*e)*x^(1 + 4*n))/(1 + 4*n) + (3*c*(b*c*d + b^
2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n) + (c^2*(c*d + 3*b*e)*x^(1 + 6
*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n)
```

Rubi [A] time = 0.378387, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$a^3dx+\frac{x^{3n+1}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{3n+1}+\frac{a^2x^{n+1}(ae+3bd)}{n+1}+\frac{3ax^{2n+1}\left(abe+acd+b^2d\right)}{2n+1}+\frac{3cx^{5n+1}\left(ace+b^2e+bcd\right)}{5n+1}+\frac{x^{4n+1}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{4n+1}+\frac{c^2x^{6n+1}(3be+cd)}{6n+1}+\frac{c^3ex^{7n+1}}{7n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] a^3*d*x + (a^2*(3*b*d + a*e)*x^(1 + n))/(1 + n) + (3*a*(b^2*d + a
*c*d + a*b*e)*x^(1 + 2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*
b^2*e + 3*a^2*c*e)*x^(1 + 3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2
*d + b^3*e + 6*a*b*c*e)*x^(1 + 4*n))/(1 + 4*n) + (3*c*(b*c*d + b^
2*e + a*c*e)*x^(1 + 5*n))/(1 + 5*n) + (c^2*(c*d + 3*b*e)*x^(1 + 6
*n))/(1 + 6*n) + (c^3*e*x^(1 + 7*n))/(1 + 7*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\int d\,dx+\frac{a^2x^{n+1}\left(ae+3bd\right)}{n+1}+\frac{3ax^{2n+1}\left(abe+acd+b^2d\right)}{2n+1}+\frac{c^3ex^{7n+1}}{7n+1}+\frac{c^2x^{6n+1}\left(3be+cd\right)}{6n+1}+\frac{3cx^{5n+1}\left(ace+b\left(be+cd\right)\right)}{5n+1}+\frac{x^{3n+1}\left(3a\left(b^2e+c\left(ae+2bd\right)\right)+b^3d\right)}{3n+1}+\frac{x^{4n+1}\left(b^3e+3c\left(acd+b\left(2ae+bd\right)\right)\right)}{4n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] a**3*Integral(d, x) + a**2*x**(n + 1)*(a*e + 3*b*d)/(n + 1) + 3*a
*x**(2*n + 1)*(a*b*e + a*c*d + b**2*d)/(2*n + 1) + c**3*e*x**(7*n
 + 1)/(7*n + 1) + c**2*x**(6*n + 1)*(3*b*e + c*d)/(6*n + 1) + 3*c
*x**(5*n + 1)*(a*c*e + b*(b*e + c*d))/(5*n + 1) + x**(3*n + 1)*(3
*a*(b**2*e + c*(a*e + 2*b*d)) + b**3*d)/(3*n + 1) + x**(4*n + 1)*
(b**3*e + 3*c*(a*c*d + b*(2*a*e + b*d)))/(4*n + 1)
```

Mathematica [A] time = 0.777617, size = 205, normalized size = 0.94

$$x\left(a^3d+\frac{x^{3n}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{3n+1}+\frac{a^2x^n(ae+3bd)}{n+1}+\frac{3ax^{2n}\left(abe+acd+b^2d\right)}{2n+1}+\frac{3cx^{5n}\left(ace+b^2e+bcd\right)}{5n+1}+\frac{x^{4n}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{4n+1}+\frac{c^2x^{6n}(3be+cd)}{6n+1}+\frac{c^3ex^{7n}}{7n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(a^3*d + (a^2*(3*b*d + a*e)*x^n)/(1 + n) + (3*a*(b^2*d + a*c*d
+ a*b*e)*x^(2*n))/(1 + 2*n) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3
*a^2*c*e)*x^(3*n))/(1 + 3*n) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e +
6*a*b*c*e)*x^(4*n))/(1 + 4*n) + (3*c*(b*c*d + b^2*e + a*c*e)*x^(5
*n))/(1 + 5*n) + (c^2*(c*d + 3*b*e)*x^(6*n))/(1 + 6*n) + (c^3*e*x
^(7*n))/(1 + 7*n))
```

Maple [A] time = 0.027, size = 226, normalized size = 1.

$$a^3dx+\frac{\left(6\,abce+3\,ac^2d+b^3e+3\,b^2cd\right)x\left({e}^{n\ln(x)}\right)^4}{1+4\,n}+\frac{\left(3\,a^2ce+3\,ab^2e+6\,abcd+b^3d\right)x\left({e}^{n\ln(x)}\right)^3}{1+3\,n}+\frac{a^2\left(ae+3\,bd\right)x{e}^{n\ln(x)}}{1+n}+\frac{c^2\left(3\,be+cd\right)x\left({e}^{n\ln(x)}\right)^6}{1+6\,n}+\frac{c^3ex\left({e}^{n\ln(x)}\right)^7}{1+7\,n}+3\,\frac{a\left(abe+acd+b^2d\right)x\left({e}^{n\ln(x)}\right)^2}{1+2\,n}+3\,\frac{c\left(ace+b^2e+bcd\right)x\left({e}^{n\ln(x)}\right)^5}{1+5\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] a^3*d*x+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)/(1+4*n)*x*exp(n*ln(
x))^4+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)/(1+3*n)*x*exp(n*ln(x)
)^3+a^2*(a*e+3*b*d)/(1+n)*x*exp(n*ln(x))+c^2*(3*b*e+c*d)/(1+6*n)*
x*exp(n*ln(x))^6+c^3*e/(1+7*n)*x*exp(n*ln(x))^7+3*a*(a*b*e+a*c*d+
b^2*d)/(1+2*n)*x*exp(n*ln(x))^2+3*c*(a*c*e+b^2*e+b*c*d)/(1+5*n)*x
*exp(n*ln(x))^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279424, size = 1632, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d),x, algorithm="fricas")
```

```
[Out] ((720*c^3*e*n^6 + 1764*c^3*e*n^5 + 1624*c^3*e*n^4 + 735*c^3*e*n^3
 + 175*c^3*e*n^2 + 21*c^3*e*n + c^3*e)*x*x^(7*n) + (840*(c^3*d +
3*b*c^2*e)*n^6 + 2038*(c^3*d + 3*b*c^2*e)*n^5 + 1849*(c^3*d + 3*b
*c^2*e)*n^4 + c^3*d + 3*b*c^2*e + 820*(c^3*d + 3*b*c^2*e)*n^3 + 1
90*(c^3*d + 3*b*c^2*e)*n^2 + 22*(c^3*d + 3*b*c^2*e)*n)*x*x^(6*n)
+ 3*(1008*(b*c^2*d + (b^2*c + a*c^2)*e)*n^6 + 2412*(b*c^2*d + (b^
2*c + a*c^2)*e)*n^5 + 2144*(b*c^2*d + (b^2*c + a*c^2)*e)*n^4 + b*
c^2*d + 925*(b*c^2*d + (b^2*c + a*c^2)*e)*n^3 + 207*(b*c^2*d + (b
^2*c + a*c^2)*e)*n^2 + (b^2*c + a*c^2)*e + 23*(b*c^2*d + (b^2*c +
 a*c^2)*e)*n)*x*x^(5*n) + (1260*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a
*b*c)*e)*n^6 + 2952*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^5
 + 2545*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^4 + 1056*(3*(
b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n^3 + 226*(3*(b^2*c + a*c^2
)*d + (b^3 + 6*a*b*c)*e)*n^2 + 3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b
*c)*e + 24*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*n)*x*x^(4*n)
 + (1680*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^6 + 3796*((b
^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*n^5 + 3112*((b^3 + 6*a*b*c
)*d + 3*(a*b^2 + a^2*c)*e)*n^4 + 1219*((b^3 + 6*a*b*c)*d + 3*(a*b
^2 + a^2*c)*e)*n^3 + 247*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e
)*n^2 + (b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e + 25*((b^3 + 6*a*
b*c)*d + 3*(a*b^2 + a^2*c)*e)*n)*x*x^(3*n) + 3*(2520*(a^2*b*e + (
a*b^2 + a^2*c)*d)*n^6 + 5274*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^5 +
3929*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^4 + a^2*b*e + 1420*(a^2*b*e
+ (a*b^2 + a^2*c)*d)*n^3 + 270*(a^2*b*e + (a*b^2 + a^2*c)*d)*n^2
+ (a*b^2 + a^2*c)*d + 26*(a^2*b*e + (a*b^2 + a^2*c)*d)*n)*x*x^(2*
n) + (5040*(3*a^2*b*d + a^3*e)*n^6 + 8028*(3*a^2*b*d + a^3*e)*n^5
 + 5104*(3*a^2*b*d + a^3*e)*n^4 + 3*a^2*b*d + a^3*e + 1665*(3*a^2
*b*d + a^3*e)*n^3 + 295*(3*a^2*b*d + a^3*e)*n^2 + 27*(3*a^2*b*d +
 a^3*e)*n)*x*x^n + (5040*a^3*d*n^7 + 13068*a^3*d*n^6 + 13132*a^3*
d*n^5 + 6769*a^3*d*n^4 + 1960*a^3*d*n^3 + 322*a^3*d*n^2 + 28*a^3*
d*n + a^3*d)*x)/(5040*n^7 + 13068*n^6 + 13132*n^5 + 6769*n^4 + 19
60*n^3 + 322*n^2 + 28*n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.294883, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d),x, algorithm="giac")
```

```
[Out] Done
```

3.69 $\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=308

$$\frac{x\left(\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^{n+1}}{c(n+1)}$$

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^(1 + n))/(c*(1 + n)) + ((3*c^2
*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3 + ((2*c*d - b*e)*(c^2*d^
2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
])/(c^2*(b - Sqrt[b^2 - 4*a*c])) + ((3*c^2*d^2*e - 3*b*c*d*e^2 +
b^2*e^3 - a*c*e^3 - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d
+ 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 +
n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c^2*(b + Sqrt[b^2 -
 4*a*c]))
```

Rubi [A] time = 1.50681, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{x\left(\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}-ace^3+b^2e^3-3bcde^2+3c^2d^2e\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^{n+1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^(1 + n))/(c*(1 + n)) + ((3*c^2
*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3 + ((2*c*d - b*e)*(c^2*d^
2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
])/(c^2*(b - Sqrt[b^2 - 4*a*c])) + ((3*c^2*d^2*e - 3*b*c*d*e^2 +
b^2*e^3 - a*c*e^3 - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d
+ 3*a*e)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 +
n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c^2*(b + Sqrt[b^2 -
 4*a*c]))
```

Rubi in Sympy [A] time = 177.191, size = 566, normalized size = 1.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*d**3*x*hyper((1, 1/n), (1 + 1/n,), -2*c*x**n/(b + sqrt(-4*a*
c + b**2)))/(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2)) - 2*c*d**3*x*
hyper((1, 1/n), (1 + 1/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/
(-4*a*c + b**2 - b*sqrt(-4*a*c + b**2)) - 6*c*d**2*e*x**(n + 1)*h
yper((1, (n + 1)/n), (2 + 1/n,), -2*c*x**n/(b + sqrt(-4*a*c + b**
2)))/((b + sqrt(-4*a*c + b**2))*(n + 1)*sqrt(-4*a*c + b**2)) + 6*
c*d**2*e*x**(n + 1)*hyper((1, (n + 1)/n), (2 + 1/n,), -2*c*x**n/(
b - sqrt(-4*a*c + b**2)))/((b - sqrt(-4*a*c + b**2))*(n + 1)*sqrt
(-4*a*c + b**2)) - 6*c*d*e**2*x**(2*n + 1)*hyper((1, 2 + 1/n), (3
 + 1/n,), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/((b + sqrt(-4*a*c
+ b**2))*(2*n + 1)*sqrt(-4*a*c + b**2)) + 6*c*d*e**2*x**(2*n + 1)
*hyper((1, 2 + 1/n), (3 + 1/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**
2)))/((b - sqrt(-4*a*c + b**2))*(2*n + 1)*sqrt(-4*a*c + b**2)) -
2*c*e**3*x**(3*n + 1)*hyper((1, 3 + 1/n), (4 + 1/n,), -2*c*x**n/(
b + sqrt(-4*a*c + b**2)))/((b + sqrt(-4*a*c + b**2))*(3*n + 1)*sq
rt(-4*a*c + b**2)) + 2*c*e**3*x**(3*n + 1)*hyper((1, 3 + 1/n), (4
 + 1/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/((b - sqrt(-4*a*c
+ b**2))*(3*n + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 3.90376, size = 455, normalized size = 1.48

$$-\frac{2^{-\frac{n+1}{n}} x \left(\left(b \left(ae^3\sqrt{b^2-4ac}+3acde^2+c^2d^3\right)+c \left(cd^2 \left(d\sqrt{b^2-4ac}-6ae\right)+ae^2 \left(2ae-3d\sqrt{b^2-4ac}\right)\right)-ab^2e^3\right) \left(\frac{c}{-\sqrt{b^2-4a\cdots}}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((x*(-((2^(1 + n^(-1))*c*Sqrt[b^2 - 4*a*c]*(c*d^3*(1 + n) + a*e^
3*x^n))/(1 + n)) + ((-(a*b^2*e^3) + b*(c^2*d^3 + 3*a*c*d*e^2 + a*
Sqrt[b^2 - 4*a*c]*e^3) + c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 6*a*e) +
 a*e^2*(-3*Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*Hypergeometric2F1[-n^(-
1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n
^(-1) + ((a*b^2*e^3 + b*(-(c^2*d^3) - 3*a*c*d*e^2 + a*Sqrt[b^2 -
4*a*c]*e^3) + c*(-(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + c*d^2
*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*Hypergeometric2F1[-n^(-1), -n^(-
1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/(
2^((1 + n)/n)*a*c^2*Sqrt[b^2 - 4*a*c]))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^3}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^3xx^n+\left(3\,cde^2(n+1)-be^3(n+1)\right)x}{c^2(n+1)}-\int -\frac{c^2d^3-\left(3\,cde^2-be^3\right)a+\left(3\,c^2d^2e-3\,bcde^2+b^2e^3-ace^3\right)x^n}{c^3x^{2\,n}+bc^2x^n+ac^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] (c*e^3*x*x^n + (3*c*d*e^2*(n + 1) - b*e^3*(n + 1))*x)/(c^2*(n + 1
)) - integrate(-(c^2*d^3 - (3*c*d*e^2 - b*e^3)*a + (3*c^2*d^2*e -
 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3)*x^n)/(c^3*x^(2*n) + b*c^2*x^n +
 a*c^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{3} x^{3 \, n} + 3 \, d e^{2} x^{2 \, n} + 3 \, d^{2} e x^{n} + d^{3}}{c x^{2 \, n} + b x^{n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c*x
^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e x^{n} + d)^{3}}{c x^{2 \, n} + b x^{n} + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a), x)
```

3.70 $\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=224

$$\frac{x\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d
 + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a
*c])) + ((2*c*d*e - b*e^2 - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a
*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b + Sqrt[b^2 - 4*a*c])
)
```

Rubi [A] time = 0.930949, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{x\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}-be^2+2cde\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (e^2*x)/c + ((2*c*d*e - b*e^2 + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d
 + a*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a
*c])) + ((2*c*d*e - b*e^2 - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a
*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(c*(b + Sqrt[b^2 - 4*a*c])
)
```

Rubi in Sympy [A] time = 120.378, size = 410, normalized size = 1.83

$$-\frac{2cd^2x{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{-4ac+b^2+b\sqrt{-4ac+b^2}}-\frac{2cd^2x{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{-4ac+b^2-b\sqrt{-4ac+b^2}}-\frac{4cdex^{n+1}{}_2F_1\left(\begin{matrix}1,\frac{n+1}{n}\\2+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{\left(b+\sqrt{-4ac+b^2}\right)(n+1)\sqrt{-4ac+b^2}}+\frac{4cdex^{n+1}{}_2F_1\left(\begin{matrix}1,\frac{n+1}{n}\\2+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{\left(b-\sqrt{-4ac+b^2}\right)(n+1)\sqrt{-4ac+b^2}}-\frac{2ce^2x^{2n+1}{}_2F_1\left(\begin{matrix}1,2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{\left(b+\sqrt{-4ac+b^2}\right)(2n+1)\sqrt{-4ac+b^2}}+\frac{2ce^2x^{2n+1}{}_2F_1\left(\begin{matrix}1,2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{\left(b-\sqrt{-4ac+b^2}\right)(2n+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*d**2*x*hyper((1, 1/n), (1 + 1/n,), -2*c*x**n/(b + sqrt(-4*a*
c + b**2)))/(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2)) - 2*c*d**2*x*
hyper((1, 1/n), (1 + 1/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/
(-4*a*c + b**2 - b*sqrt(-4*a*c + b**2)) - 4*c*d*e*x**(n + 1)*hype
r((1, (n + 1)/n), (2 + 1/n,), -2*c*x**n/(b + sqrt(-4*a*c + b**2))
)/((b + sqrt(-4*a*c + b**2))*(n + 1)*sqrt(-4*a*c + b**2)) + 4*c*d
*e*x**(n + 1)*hyper((1, (n + 1)/n), (2 + 1/n,), -2*c*x**n/(b - sq
rt(-4*a*c + b**2)))/((b - sqrt(-4*a*c + b**2))*(n + 1)*sqrt(-4*a*
c + b**2)) - 2*c*e**2*x**(2*n + 1)*hyper((1, 2 + 1/n), (3 + 1/n,)
, -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/((b + sqrt(-4*a*c + b**2))
*(2*n + 1)*sqrt(-4*a*c + b**2)) + 2*c*e**2*x**(2*n + 1)*hyper((1,
 2 + 1/n), (3 + 1/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/((b -
 sqrt(-4*a*c + b**2))*(2*n + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 1.47557, size = 348, normalized size = 1.55

$$2^{-\frac{n+1}{n}} x \left(-\left(cd \left(d\sqrt{b^2-4ac} - 4ae \right) - ae^2\sqrt{b^2-4ac} + b\left(ae^2 + cd^2 \right) \right) \left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n} \right)^{-1/n} {}_2F_1\left(-\frac{1}{n}, -\frac{1}{n}; \frac{n-1}{n}; \frac{b-\sqrt{b^2-4a}}{2cx^n+b-\sqrt{b^2-}} \right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(2^(1 + n^(-1))*c*Sqrt[b^2 - 4*a*c]*d^2 - ((-(a*Sqrt[b^2 - 4*a
*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(c*d^2 + a*e^2))
*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(
Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*Hypergeometric2
F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n))^n^(-1)))/(2^((1 + n)/n)*a*c*Sqrt[b^2 - 4*a*c])
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^2}{a+bx^n+cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{c} - \int -\frac{cd^2 - ae^2 + \left(2\,cde - be^2\right)x^n}{c^2x^{2\,n} + bcx^n + ac}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^n)/(c^2
*x^(2*n) + b*c*x^n + a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2n} + 2dex^n + d^2}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c*x^(2*n) + b*x^n + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a), x)
```

3.71 $\int \frac{d+ex^n}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=154

$$\frac{x\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{x\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b
^2 - 4*a*c]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)])/(b + Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.276203, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}+\frac{x\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b
^2 - 4*a*c]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)])/(b + Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 32.2764, size = 148, normalized size = 0.96

$$\frac{x\left(be-2cd+e\sqrt{-4ac+b^2}\right)\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{-4ac+b^2+b\sqrt{-4ac+b^2}}+\frac{x\left(be-2cd-e\sqrt{-4ac+b^2}\right)\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{-4ac+b^2-b\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] x*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*hyper((1, 1/n), (1 + 1/n,
), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(-4*a*c + b**2 + b*sqrt(-
4*a*c + b**2)) + x*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*hyper((1
, 1/n), (1 + 1/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/(-4*a*c
+ b**2 - b*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.827369, size = 279, normalized size = 1.81

$$\frac{2^{-\frac{n+1}{n}}x\left(-\left(d\sqrt{b^2-4ac}-2ae+bd\right)\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{-1/n}\,{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{n-1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)+\left(-d\sqrt{b^2-4ac}-2ae+b\right.}{a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(2^(1 + n^(-1))*Sqrt[b^2 - 4*a*c]*d - ((b*d + Sqrt[b^2 - 4*a*c
]*d - 2*a*e)*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b -
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1) + ((b*d - Sqrt[b^2 - 4*
a*c]*d - 2*a*e)*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (
b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^
n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/(2^((1 + n)/n)*a*S
qrt[b^2 - 4*a*c])
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{d + e x^{n}}{a + b x^{n} + c x^{2 n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e x^{n} + d}{c x^{2 n} + b x^{n} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e x^{n} + d}{c x^{2 n} + b x^{n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a), x)
```

3.72 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=243

$$-\frac{cx\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)}-\frac{cx\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(\sqrt{b^2-4ac}+b\right)(ae^2-bde+cd^2)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)}$$

```
[Out] -((c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4
*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2))) - (c*(e + (
2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4
*a*c])*(c*d^2 - b*d*e + a*e^2)) + (e^2*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.828948, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{cx\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)}-\frac{cx\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(\sqrt{b^2-4ac}+b\right)(ae^2-bde+cd^2)}+\frac{e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4
*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2))) - (c*(e + (
2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4
*a*c])*(c*d^2 - b*d*e + a*e^2)) + (e^2*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)\,(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(1/((d + e*x**n)*(a + b*x**n + c*x**(2*n))), x)
```

Mathematica [A] time = 2.40775, size = 379, normalized size = 1.56

$$x\left(\frac{2^{-1/n}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{n-1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}+\frac{2^{-1/n}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2a\right.}{}\right.$$
$$\overline{\qquad 2a\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (x*(2*c*d - 2*b*e + (2*a*e^2*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((e*x^n)/d)])/d + ((-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*
e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*Hypergeometric2F1[-n^(-1), -
n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*((c*x^n)/(b - Sqrt[b^2
 - 4*a*c] + 2*c*x^n))^n^(-1)) + ((b*c*d - c*Sqrt[b^2 - 4*a*c]*d -
 b^2*e + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*Hypergeometric2F1[-n^(-
1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2 - 4*a*c]*((c*x^n)/(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^n))^n^(-1))))/(2*a*(c*d^2 + e*(-(b*d) + a*
e)))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n}+bx^n+a)(ex^n+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad+(cex^n+cd+be)x^{2n}+(bd+ae)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d + (c*e*x^n + c*d + b*e)*x^(2*n) + (b*d + a*e)*x^n
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)), x)
```

3.73 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=368

$$-\frac{cx\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}-\frac{cx\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}+\frac{e^2x(2cd-be)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\,(ae^2-bde+cd^2)}$$

```
[Out] -((c*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sq
rt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b
^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2)) - (c*(2*c^2*d^2 + b*(b -
 Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e)
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*
d*e + a*e^2)^2) + (e^2*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2) + (e^
2*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*
(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.36526, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{cx\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}-\frac{cx\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}+\frac{e^2x(2cd-be)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)^2}+\frac{e^2x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -((c*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sq
rt[b^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b
^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2)) - (c*(2*c^2*d^2 + b*(b -
 Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e)
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*
d*e + a*e^2)^2) + (e^2*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^2) + (e^
2*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*
(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(1/((d + e*x**n)**2*(a + b*x**n + c*x**(2*n))), x)
```

Mathematica [B] time = 6.29104, size = 2302, normalized size = 6.26

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] ((a*e^2 - c*d^2*n + b*d*e*n - a*e^2*n)*x)/(a*d^2*(c*d^2 - b*d*e +
 a*e^2)*n) + ((-(a*e^2) + c*d^2*n - b*d*e*n + a*e^2*n)*x)/(a*d^2*
(c*d^2 - b*d*e + a*e^2)*n) + (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*n
*(d + e*x^n)) + (e^2*(-(c*d^2) + b*d*e - a*e^2 + 3*c*d^2*n - 2*b*
d*e*n + a*e^2*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*
x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^2*n) - (2*c^2*d*e*x^(1 + n
)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeometric2F1[-n^(-1), -n^(-1
), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 -
4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 -
 4*a*c])/(2*c) + x^n))^n^(-1))) + Hypergeometric2F1[-n^(-1), -n^(
-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2
- 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2
 - 4*a*c])/(2*c) + x^n))^n^(-1))))/(c*d^2 - b*d*e + a*e^2)^2 + (b
*c*e^2*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeometric2F1[
-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-
b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(
-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))) + Hypergeometric2F
1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-
(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(
-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))))/(c*d^2 - b*d*e
+ a*e^2)^2 - (c^2*d^2*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1),
 (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*
a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)
)^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 -
4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 +
 n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/
(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-
1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c]
)^2/(2*c))))/(c*d^2 - b*d*e + a*e^2)^2 + (2*b*c*d*e*x*((1 - Hyper
geometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*
c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - S
qrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*
c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeomet
ric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2
*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^
2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(
2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(c*d^2 - b*d*e + a*e^2
)^2 - (b^2*e^2*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 +
n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(
2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1
))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])
^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
-(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) +
 x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b
*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*
c))))/(c*d^2 - b*d*e + a*e^2)^2 + (a*c*e^2*x*((1 - Hypergeometric
2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*
(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 -
 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c
) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n
^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b
+ Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c
])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-
```

```
b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(c*d^2 - b*d*e + a*e^2)^2
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2\,(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{cd^4n - bd^3en + ad^2e^2n + (cd^3en - bd^2e^2n + ade^3n)x^n} + \left(cd^2e^2(3n-1) - bde^3(2n-1) + ae^4(n-1)\right)\int \frac{1}{c^2d^6n - 2bcd^5en + b^2d^4e^2n + a^2d^2e^4n + 2(cd^4e^2n - bd^3e^3n)a + (c^2d^5en - \cdots} + \int \frac{c^2d^2 - 2bcde + b^2e^2 - ace^2 - \left(2c^2de - bce^2\right)x^n}{a^3e^4 + 2(cd^2e^2 - bde^3)a^2 + (c^2d^4 - 2bcd^3e + b^2d^2e^2)a + (c^3d^4 - 2bc^2d^3e + b^2cd^2e^2 + a^2ce^4 + 2(c^2d^2e^2 - bcde^3)a)x^{2n} + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^2),x, algorithm="maxima")
```

```
[Out] e^2*x/(c*d^4*n - b*d^3*e*n + a*d^2*e^2*n + (c*d^3*e*n - b*d^2*e^2
*n + a*d*e^3*n)*x^n) + (c*d^2*e^2*(3*n - 1) - b*d*e^3*(2*n - 1) +
 a*e^4*(n - 1))*integrate(1/(c^2*d^6*n - 2*b*c*d^5*e*n + b^2*d^4*
e^2*n + a^2*d^2*e^4*n + 2*(c*d^4*e^2*n - b*d^3*e^3*n)*a + (c^2*d^
5*e*n - 2*b*c*d^4*e^2*n + b^2*d^3*e^3*n + a^2*d*e^5*n + 2*(c*d^3*
e^3*n - b*d^2*e^4*n)*a)*x^n), x) + integrate((c^2*d^2 - 2*b*c*d*e
 + b^2*e^2 - a*c*e^2 - (2*c^2*d*e - b*c*e^2)*x^n)/(a^3*e^4 + 2*(c
*d^2*e^2 - b*d*e^3)*a^2 + (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*a
 + (c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + a^2*c*e^4 + 2*(c^2*
d^2*e^2 - b*c*d*e^3)*a)*x^(2*n) + (b*c^2*d^4 - 2*b^2*c*d^3*e + b^
3*d^2*e^2 + a^2*b*e^4 + 2*(b*c*d^2*e^2 - b^2*d*e^3)*a)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ce^2x^{4n} + be^2x^{3n} + ad^2 + (2cdex^n + cd^2 + 2bde + ae^2)x^{2n} + (bd^2 + 2ade)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(c*e^2*x^(4*n) + b*e^2*x^(3*n) + a*d^2 + (2*c*d*e*x^n
+ c*d^2 + 2*b*d*e + a*e^2)*x^(2*n) + (b*d^2 + 2*a*d*e)*x^n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^2), x)
```

3.74 $\int \frac{1}{(d+ex^n)^3(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=552

$$\frac{e^2x\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^3}-\frac{cx\left(-3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(3b\left(d\sqrt{b^2-4ac}+ae\right)+ae\sqrt{b^2-4ac}+3b^2d\right)-b^2e^3\left(\sqrt{b^2-4ac}+b\right)+2c^3a\cdots\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}-\frac{cx\left(-3c^2de\left(-d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)-b^2e^3\left(b-\sqrt{b^2-4ac}\right)+2\cdots\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}+\frac{e^2x(2cd-be)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2-bde+cd^2\right)^2}+\frac{e^2x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^3\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((c*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*
d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt[b^2 -
4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*x*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2
 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3)) - (c*
(2*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d - S
qrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a
*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(3
*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) +
(e^2*(2*c*d - b*e)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((
e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hypergeometr
ic2F1[3, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^3*(c*d^2 - b*d*e +
 a*e^2))
```

Rubi [A] time = 2.35493, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{e^2x\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^3}-\frac{cx\left(-3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(3b\left(d\sqrt{b^2-4ac}+ae\right)+ae\sqrt{b^2-4ac}+3b^2d\right)-b^2e^3\left(\sqrt{b^2-4ac}+b\right)+2c^3a\cdots\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}-\frac{cx\left(-3c^2de\left(-d\sqrt{b^2-4ac}+2ae+bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)-b^2e^3\left(b-\sqrt{b^2-4ac}\right)+2\cdots\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^3}+\frac{e^2x(2cd-be)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^2\left(ae^2-bde+cd^2\right)^2}+\frac{e^2x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d^3\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```


```
[Out] -((c*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*
d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt[b^2 -
4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*x*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2
 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3)) - (c*
(2*c^3*d^3 - b^2*(b - Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d - S
qrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a
```

```
*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]*e))*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (e^2*(3
*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) +
(e^2*(2*c*d - b*e)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((
e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^2) + (e^2*x*Hypergeometr
ic2F1[3, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^3*(c*d^2 - b*d*e +
 a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^3\,(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(1/((d + e*x**n)**3*(a + b*x**n + c*x**(2*n))), x)
```

Mathematica [B] time = 6.47417, size = 4111, normalized size = 7.45

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] ((-(a*c*d^2*e^2) + a*b*d*e^3 - a^2*e^4 + 7*a*c*d^2*e^2*n - 5*a*b*
d*e^3*n + 3*a^2*e^4*n - 2*c^2*d^4*n^2 + 4*b*c*d^3*e*n^2 - 2*b^2*d
^2*e^2*n^2 - 4*a*c*d^2*e^2*n^2 + 4*a*b*d*e^3*n^2 - 2*a^2*e^4*n^2)
*x)/(2*a*d^3*(c*d^2 - b*d*e + a*e^2)^2*n^2) + ((a*c*d^2*e^2 - a*b
*d*e^3 + a^2*e^4 - 7*a*c*d^2*e^2*n + 5*a*b*d*e^3*n - 3*a^2*e^4*n
+ 2*c^2*d^4*n^2 - 4*b*c*d^3*e*n^2 + 2*b^2*d^2*e^2*n^2 + 4*a*c*d^2
*e^2*n^2 - 4*a*b*d*e^3*n^2 + 2*a^2*e^4*n^2)*x)/(2*a*d^3*(c*d^2 -
b*d*e + a*e^2)^2*n^2) + (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*n*(d
 + e*x^n)^2) + ((-(c*d^2*e^2) + b*d*e^3 - a*e^4 + 6*c*d^2*e^2*n -
 4*b*d*e^3*n + 2*a*e^4*n)*x)/(2*d^2*(c*d^2 - b*d*e + a*e^2)^2*n^2
*(d + e*x^n)) + ((c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e^4 + 2*a
*c*d^2*e^4 - 2*a*b*d*e^5 + a^2*e^6 - 7*c^2*d^4*e^2*n + 12*b*c*d^3
*e^3*n - 5*b^2*d^2*e^4*n - 10*a*c*d^2*e^4*n + 8*a*b*d*e^5*n - 3*a
^2*e^6*n + 12*c^2*d^4*e^2*n^2 - 16*b*c*d^3*e^3*n^2 + 6*b^2*d^2*e^
4*n^2 + 6*a*c*d^2*e^4*n^2 - 6*a*b*d*e^5*n^2 + 2*a^2*e^6*n^2)*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(2*d^3*(c*d
^2 - b*d*e + a*e^2)^3*n^2) - (3*c^3*d^2*e*x^(1 + n)*(x^n)^(n^(-1)
 - (1 + n)/n)*(-(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
-(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) +
 x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c)
+ x^n))^n^(-1))) + Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n
, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c)
 + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c
) + x^n))^n^(-1))))/(c*d^2 - b*d*e + a*e^2)^3 + (3*b*c^2*d*e^2*x^
(1 + n)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeometric2F1[-n^(-1),
-n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[
b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt
[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))) + Hypergeometric2F1[-n^(-1)
, -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqr
t[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sq
rt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))))/(c*d^2 - b*d*e + a*e^2)^
3 - (b^2*c*e^3*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeome
tric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(
2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*
```

```
(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))) + Hypergeo
metric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])
/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c
]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))))/(c*d^2
- b*d*e + a*e^2)^3 + (a*c^2*e^3*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)
/n)*(-(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqr
t[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(S
qrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^
(-1))) + Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + S
qrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/
(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^
n^(-1))))/(c*d^2 - b*d*e + a*e^2)^3 - (c^3*d^3*x*((1 - Hypergeome
tric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(
2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b
^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/
(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F
1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-
(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4
*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c)
+ (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(c*d^2 - b*d*e + a*e^2)^3 +
 (3*b*c^2*d^2*e*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 +
 n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/
(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-
1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c]
)^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
 -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c)
+ x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((
b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2
*c))))/(c*d^2 - b*d*e + a*e^2)^3 - (3*b^2*c*d*e^2*x*((1 - Hyperge
ometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c]
)/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqr
t[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]
))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometri
c2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c
*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2
- 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*
c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(c*d^2 - b*d*e + a*e^2)^
3 + (3*a*c^2*d*e^2*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-
1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c
])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n
^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a
*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)
/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*
c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))
/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2
/(2*c))))/(c*d^2 - b*d*e + a*e^2)^3 + (b^3*e^3*x*((1 - Hypergeome
tric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(
2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b
^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/
(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F
1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-
(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4
*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c)
+ (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(c*d^2 - b*d*e + a*e^2)^3 -
 (2*a*b*c*e^3*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n
)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2
*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1)
)/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^
2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -
(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) +
x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*
(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c
))))/(c*d^2 - b*d*e + a*e^2)^3
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^3\,(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(d+e*x^n)^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^3),x, algorithm="maxima")
```

```
[Out] ((12*n^2 - 7*n + 1)*c^2*d^4*e^2 - 2*(8*n^2 - 6*n + 1)*b*c*d^3*e^3
 + (6*n^2 - 5*n + 1)*b^2*d^2*e^4 + (2*n^2 - 3*n + 1)*a^2*e^6 + 2*
((3*n^2 - 5*n + 1)*c*d^2*e^4 - (3*n^2 - 4*n + 1)*b*d*e^5)*a)*inte
grate(1/2/(c^3*d^9*n^2 - 3*b*c^2*d^8*e*n^2 + 3*b^2*c*d^7*e^2*n^2
- b^3*d^6*e^3*n^2 + a^3*d^3*e^6*n^2 + 3*(c*d^5*e^4*n^2 - b*d^4*e^
5*n^2)*a^2 + 3*(c^2*d^7*e^2*n^2 - 2*b*c*d^6*e^3*n^2 + b^2*d^5*e^4
*n^2)*a + (c^3*d^8*e*n^2 - 3*b*c^2*d^7*e^2*n^2 + 3*b^2*c*d^6*e^3*
n^2 - b^3*d^5*e^4*n^2 + a^3*d^2*e^7*n^2 + 3*(c*d^4*e^5*n^2 - b*d^
3*e^6*n^2)*a^2 + 3*(c^2*d^6*e^3*n^2 - 2*b*c*d^5*e^4*n^2 + b^2*d^4
*e^5*n^2)*a)*x^n), x) + 1/2*((c*d^2*e^3*(6*n - 1) - b*d*e^4*(4*n
- 1) + a*e^5*(2*n - 1))*x*x^n + (c*d^3*e^2*(7*n - 1) - b*d^2*e^3*
(5*n - 1) + a*d*e^4*(3*n - 1))*x)/(c^2*d^8*n^2 - 2*b*c*d^7*e*n^2
+ b^2*d^6*e^2*n^2 + a^2*d^4*e^4*n^2 + 2*(c*d^6*e^2*n^2 - b*d^5*e^
3*n^2)*a + (c^2*d^6*e^2*n^2 - 2*b*c*d^5*e^3*n^2 + b^2*d^4*e^4*n^2
 + a^2*d^2*e^6*n^2 + 2*(c*d^4*e^4*n^2 - b*d^3*e^5*n^2)*a)*x^(2*n)
 + 2*(c^2*d^7*e*n^2 - 2*b*c*d^6*e^2*n^2 + b^2*d^5*e^3*n^2 + a^2*d
^3*e^5*n^2 + 2*(c*d^5*e^3*n^2 - b*d^4*e^4*n^2)*a)*x^n) + integrat
e((c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3 - (3*c^2*d*e
^2 - 2*b*c*e^3)*a - (3*c^3*d^2*e - 3*b*c^2*d*e^2 + b^2*c*e^3 - a*
c^2*e^3)*x^n)/(a^4*e^6 + 3*(c*d^2*e^4 - b*d*e^5)*a^3 + 3*(c^2*d^4
*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e^4)*a^2 + (c^3*d^6 - 3*b*c^2*d^5*
e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*a + (c^4*d^6 - 3*b*c^3*d^5*e +
 3*b^2*c^2*d^4*e^2 - b^3*c*d^3*e^3 + a^3*c*e^6 + 3*(c^2*d^2*e^4 -
 b*c*d*e^5)*a^2 + 3*(c^3*d^4*e^2 - 2*b*c^2*d^3*e^3 + b^2*c*d^2*e^
4)*a)*x^(2*n) + (b*c^3*d^6 - 3*b^2*c^2*d^5*e + 3*b^3*c*d^4*e^2 -
b^4*d^3*e^3 + a^3*b*e^6 + 3*(b*c*d^2*e^4 - b^2*d*e^5)*a^2 + 3*(b*
c^2*d^4*e^2 - 2*b^2*c*d^3*e^3 + b^3*d^2*e^4)*a)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^3 + (3\,cde^2 + be^3)x^{4\,n} + (ce^3x^{2\,n} + 3\,bde^2 + ae^3)x^{3\,n} + (3\,cd^2ex^n + cd^3 + 3\,bd^2e + 3\,ade^2)x^{2\,n} + (bd^3 + 3\,ad^2e)x^n},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^3),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^3 + (3*c*d*e^2 + b*e^3)*x^(4*n) + (c*e^3*x^(2*n)
+ 3*b*d*e^2 + a*e^3)*x^(3*n) + (3*c*d^2*e*x^n + c*d^3 + 3*b*d^2*e
 + 3*a*d*e^2)*x^(2*n) + (b*d^3 + 3*a*d^2*e)*x^n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)(ex^n + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*(e*x^n + d)^3), x)
```

3.75 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=750

$$\frac{x\left(x^n\left(-\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)\right)-abe\left(ae^2+3cd^2\right)-2acd\left(cd^2-3ae^2\right)+b^2cd^3\right)}{acn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$
$$+\frac{e^2x\left(\frac{6cd-3be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{e^2x\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{x\left((1-n)\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-5n)+3cd^2n\right)+4ac^2}{\sqrt{b^2-4ac}}\right.}{acn\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{x\left((1-n)\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)-\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-5n)+3cd^2n\right)+4ac}{\sqrt{b^2-4ac}}\right.}{acn\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^
2) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*
e^2))*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (e^2*
(e + (6*c*d - 3*b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c*(b - Sq
rt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*
c*d*(c*d^2 + 3*a*e^2))*(1 - n) + (b^2*c*d*(3*a*e^2*(1 - 3*n) - c*
d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2)*
(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4
*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*
n) + (e^2*(e - (3*(2*c*d - b*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(c*(b + Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3*c*d^2 -
 a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) - (b^2*c*d*(3*a*e^2*(1
 - 3*n) - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2
 - 3*a*e^2)*(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/
Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b + Sqrt[b^
2 - 4*a*c])*n)
```

Rubi [A] time = 5.68353, antiderivative size = 750, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x\left(x^n\left(-\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)\right)-abe\left(ae^2+3cd^2\right)-2acd\left(cd^2-3ae^2\right)+b^2cd^3\right)}{acn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$
$$+\frac{e^2x\left(\frac{6cd-3be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{e^2x\left(e-\frac{3(2cd-be)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{x\left((1-n)\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-5n)+3cd^2n\right)+4ac^2}{\sqrt{b^2-4ac}}\right.}{acn\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{x\left((1-n)\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)-\frac{-ab^3e^3(1-3n)+b^2cd\left(3ae^2(1-3n)-cd^2(1-n)\right)+2abce\left(ae^2(2-5n)+3cd^2n\right)+4ac}{\sqrt{b^2-4ac}}\right.}{acn\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^
2) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*
e^2))*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) + (e^2*
```

```
(e + (6*c*d - 3*b*e)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(c*(b - Sq
rt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*
c*d*(c*d^2 + 3*a*e^2))*(1 - n) + (b^2*c*d*(3*a*e^2*(1 - 3*n) - c*
d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2 - 3*a*e^2)*
(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/Sqrt[b^2 - 4
*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*
n) + (e^2*(e - (3*(2*c*d - b*e))/Sqrt[b^2 - 4*a*c])*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(c*(b + Sqrt[b^2 - 4*a*c])) + (((a*b^2*e^3 + 2*a*c*e*(3*c*d^2 -
 a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*(1 - n) - (b^2*c*d*(3*a*e^2*(1
 - 3*n) - c*d^2*(1 - n)) - a*b^3*e^3*(1 - 3*n) + 4*a*c^2*d*(c*d^2
 - 3*a*e^2)*(1 - 2*n) + 2*a*b*c*e*(a*e^2*(2 - 5*n) + 3*c*d^2*n))/
Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b + Sqrt[b^
2 - 4*a*c])*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.48779, size = 5537, normalized size = 7.38

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^2d^3+2\,a^2ce^3-\left(6\,c^2d^2e-3\,bcde^2+b^2e^3\right)a\right)xx^n+\left(b^2cd^3+\left(6\,cde^2-be^3\right)a^2-\left(2\,c^2d^3+3\,bcd^2e\right)a\right)x}{a^2b^2cn-4\,a^3c^2n+(ab^2c^2n-4\,a^2c^3n)x^{2\,n}+(ab^3cn-4\,a^2bc^2n)x^n}$$
$$+\int \frac{b^2cd^3(n-1)-\left(6\,cde^2-be^3\right)a^2-\left(2\,c^2d^3(2\,n-1)-3\,bcd^2e\right)a-\left(2\,a^2ce^3(n+1)-bc^2d^3(n-1)+\left(6\,c^2d^2e(n-1)-3\,bc\right.\right.}{a^2b^2cn-4\,a^3c^2n+(ab^2c^2n-4\,a^2c^3n)x^{2\,n}+(ab^3cn-4\,a^2bc^2n)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] ((b*c^2*d^3 + 2*a^2*c*e^3 - (6*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)
*a)*x*x^n + (b^2*c*d^3 + (6*c*d*e^2 - b*e^3)*a^2 - (2*c^2*d^3 + 3
*b*c*d^2*e)*a)*x)/(a^2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a
^2*c^3*n)*x^(2*n) + (a*b^3*c*n - 4*a^2*b*c^2*n)*x^n) + integrate(
(b^2*c*d^3*(n - 1) - (6*c*d*e^2 - b*e^3)*a^2 - (2*c^2*d^3*(2*n -
1) - 3*b*c*d^2*e)*a - (2*a^2*c*e^3*(n + 1) - b*c^2*d^3*(n - 1) +
(6*c^2*d^2*e*(n - 1) - 3*b*c*d*e^2*(n - 1) - b^2*e^3)*a)*x^n)/(a^
2*b^2*c*n - 4*a^3*c^2*n + (a*b^2*c^2*n - 4*a^2*c^3*n)*x^(2*n) + (
a*b^3*c*n - 4*a^2*b*c^2*n)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3\,n} + 3\,de^2x^{2\,n} + 3\,d^2ex^n + d^3}{c^2x^{4\,n} + 2\,abx^n + a^2 + (2\,bcx^n + b^2 + 2\,ac)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^2
*x^(4*n) + 2*a*b*x^n + a^2 + (2*b*c*x^n + b^2 + 2*a*c)*x^(2*n)),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{(cx^{2\,n} + bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.76 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=543

$$-\frac{x\left((1-n)\left(abe^2-4acde+bcd^2\right)-\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4abcden+4ac(1-2n)(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4abcden+4ac(1-2n)(cd^2-ae^2)}{\sqrt{b^2-4ac}}+(1-n)\left(abe^2-4acde+bcd^2\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{an\,(b^2-4ac)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x\left(x^n\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{an\,(b^2-4ac)\,(a+bx^n+cx^{2n})}-\frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*
d*e + a*b*e^2)*x^n))/(a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n)))
- (2*e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (((
b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) - (b^2*(a*e^2*(1 - 3*n) -
c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) + 4*a*b*c*d*e*n)
/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2
 - 4*a*c])*n) - (2*e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c]) - (((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) + (b^2*(a*e^
2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) +
4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)
*(b + Sqrt[b^2 - 4*a*c])*n)
```

Rubi [A] time = 5.10893, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{x\left((1-n)\left(abe^2-4acde+bcd^2\right)-\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4abcden+4ac(1-2n)(cd^2-ae^2)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2(ae^2(1-3n)-cd^2(1-n))+4abcden+4ac(1-2n)(cd^2-ae^2)}{\sqrt{b^2-4ac}}+(1-n)\left(abe^2-4acde+bcd^2\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{an\,(b^2-4ac)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x\left(x^n\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{an\,(b^2-4ac)\,(a+bx^n+cx^{2n})}-\frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2e^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*
d*e + a*b*e^2)*x^n))/(a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n)))
- (2*e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (((
b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) - (b^2*(a*e^2*(1 - 3*n) -
c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) + 4*a*b*c*d*e*n)
/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2
 - 4*a*c])*n) - (2*e^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
```

```
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c]) - (((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*(1 - n) + (b^2*(a*e^
2*(1 - 3*n) - c*d^2*(1 - n)) + 4*a*c*(c*d^2 - a*e^2)*(1 - 2*n) +
4*a*b*c*d*e*n)/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1),
1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)
*(b + Sqrt[b^2 - 4*a*c])*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.41027, size = 4177, normalized size = 7.69

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((-(b^2*d^2) + 2*a*c*d^2 + 2*a*b*d*e - 2*a^2*e^2 + b^2*d^2*n - 4*
a*c*d^2*n)*x)/(a^2*(-b^2 + 4*a*c)*n) + ((b^2*d^2 - 2*a*c*d^2 - 2*
a*b*d*e + 2*a^2*e^2 - b^2*d^2*n + 4*a*c*d^2*n)*x)/(a^2*(-b^2 + 4*
a*c)*n) - (x*(b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e + 2*a^2*e^2 + b*c*d
^2*x^n - 4*a*c*d*e*x^n + a*b*e^2*x^n))/(a*(-b^2 + 4*a*c)*n*(a + b
*x^n + c*x^(2*n))) - (b*c*d^2*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)/n
)*(-(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[
b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqr
t[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-
1))) + Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqr
t[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(S
qrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^
(-1))))/(a*(-b^2 + 4*a*c)) + (4*c*d*e*x^(1 + n)*(x^n)^(n^(-1) - (
1 + n)/n)*(-(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b
 - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n
))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^
n))^n^(-1))) + Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(
-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x
^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) +
x^n))^n^(-1))))/(-b^2 + 4*a*c) - (b*e^2*x^(1 + n)*(x^n)^(n^(-1) -
 (1 + n)/n)*(-(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(
-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x
^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) +
x^n))^n^(-1))) + Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
-(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) +
 x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c)
+ x^n))^n^(-1))))/(-b^2 + 4*a*c) + (b*c*d^2*x^(1 + n)*(x^n)^(n^(-
1) - (1 + n)/n)*(-(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n
, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c)
 + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c
) + x^n))^n^(-1))) + Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)
/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*
c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2
*c) + x^n))^n^(-1))))/(a*(-b^2 + 4*a*c)*n) - (4*c*d*e*x^(1 + n)*(
x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeometric2F1[-n^(-1), -n^(-1),
(-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a
*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*
a*c])/(2*c) + x^n))^n^(-1))) + Hypergeometric2F1[-n^(-1), -n^(-1)
```

```
, (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4
*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 -
4*a*c])/(2*c) + x^n))^n^(-1))))/((-b^2 + 4*a*c)*n) + (b*e^2*x^(1
+ n)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeometric2F1[-n^(-1), -n^
(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2
 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^
2 - 4*a*c])/(2*c) + x^n))^n^(-1))) + Hypergeometric2F1[-n^(-1), -
n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b
^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[
b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))))/((-b^2 + 4*a*c)*n) + (b^2*d
^2*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b -
 Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))
]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b -
 Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) +
(1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[
b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n
/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[
b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(a*(-b
^2 + 4*a*c)) - (4*c*d^2*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1
), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 -
4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^
n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2
- 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1
 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c]
)/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^
(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*
c])^2/(2*c))))/(-b^2 + 4*a*c) - (b^2*d^2*x*((1 - Hypergeometric2F
1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-
(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4
*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c)
+ (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(
-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b +
Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])
/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b
+ Sqrt[b^2 - 4*a*c])^2/(2*c))))/(a*(-b^2 + 4*a*c)*n) + (2*c*d^2*x
*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqr
t[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x
^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqr
t[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 -
 Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2
- 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(
-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2
- 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/((-b^2 + 4
*a*c)*n) + (2*b*d*e*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (
-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*
c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^
n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*
a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n
)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2
*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1)
)/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^
2/(2*c))))/((-b^2 + 4*a*c)*n) - (2*a*e^2*x*((1 - Hypergeometric2F
1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-
(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4
*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c)
+ (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(
-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b +
Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])
/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b
+ Sqrt[b^2 - 4*a*c])^2/(2*c))))/((-b^2 + 4*a*c)*n)
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bcd^2-\left(4\,cde-be^2\right)a\right)xx^n+\left(b^2d^2+2\,a^2e^2-2\,\left(cd^2+bde\right)a\right)x}{a^2b^2n-4\,a^3cn+(ab^2cn-4\,a^2c^2n)x^{2\,n}+(ab^3n-4\,a^2bcn)x^n}-\int-\frac{b^2d^2(n-1)-2\,a^2e^2-2\,\left(cd^2(2\,n-1)-bde\right)a+\left(bcd^2(n-1)-\left(4\,cde(n-1)-be^2(n-1)\right)a\right)x^n}{a^2b^2n-4\,a^3cn+(ab^2cn-4\,a^2c^2n)x^{2\,n}+(ab^3n-4\,a^2bcn)x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] ((b*c*d^2 - (4*c*d*e - b*e^2)*a)*x*x^n + (b^2*d^2 + 2*a^2*e^2 - 2
*(c*d^2 + b*d*e)*a)*x)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^
2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(-(b^2
*d^2*(n - 1) - 2*a^2*e^2 - 2*(c*d^2*(2*n - 1) - b*d*e)*a + (b*c*d
^2*(n - 1) - (4*c*d*e*(n - 1) - b*e^2*(n - 1))*a)*x^n)/(a^2*b^2*n
 - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a
^2*b*c*n)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2\,n}+2\,dex^n+d^2}{c^2x^{4\,n}+2\,abx^n+a^2+(2\,bcx^n+b^2+2\,ac)x^{2\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^2*x^(4*n) + 2*a*b*x^n
 + a^2 + (2*b*c*x^n + b^2 + 2*a*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex^n+d)^2}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="giac")

[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.77 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=362

$$-\frac{cx\left(-b\left(d(1-n)\sqrt{b^2-4ac}-2aen\right)+2a\left(e(1-n)\sqrt{b^2-4ac}+2cd(1-2n)\right)+b^2(-(d-dn))\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\left(b^2-4ac\right)\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{cx\left(b\left(d(1-n)\sqrt{b^2-4ac}+2aen\right)+2a\left(cd(2-4n)-e(1-n)\sqrt{b^2-4ac}\right)+b^2(-d)(1-n)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{an\left(b^2-4ac\right)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4*a
*c)*n*(a + b*x^n + c*x^(2*n))) - (c*(2*a*(2*c*d*(1 - 2*n) + Sqrt[
b^2 - 4*a*c]*e*(1 - n)) - b^2*(d - d*n) - b*(Sqrt[b^2 - 4*a*c]*d*
(1 - n) - 2*a*e*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c
- b*Sqrt[b^2 - 4*a*c])*n) - (c*(2*a*(c*d*(2 - 4*n) - Sqrt[b^2 - 4
*a*c]*e*(1 - n)) - b^2*d*(1 - n) + b*(Sqrt[b^2 - 4*a*c]*d*(1 - n)
 + 2*a*e*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqr
t[b^2 - 4*a*c])*n)
```

Rubi [A] time = 1.38465, antiderivative size = 328, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{cx\left(-(1-n)\sqrt{b^2-4ac}(bd-2ae)+2aben+2acd(2-4n)+b^2(-d)(1-n)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\left(b^2-4ac\right)\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{cx\left((1-n)\sqrt{b^2-4ac}(bd-2ae)+2aben+4acd(1-2n)+b^2(-d)(1-n)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{an\left(b^2-4ac\right)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4*a
*c)*n*(a + b*x^n + c*x^(2*n))) - (c*(2*a*c*d*(2 - 4*n) - b^2*d*(1
 - n) - Sqrt[b^2 - 4*a*c]*(b*d - 2*a*e)*(1 - n) + 2*a*b*e*n)*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*n
) - (c*(4*a*c*d*(1 - 2*n) - b^2*d*(1 - n) + Sqrt[b^2 - 4*a*c]*(b*
d - 2*a*e)*(1 - n) + 2*a*b*e*n)*x*Hypergeometric2F1[1, n^(-1), 1
+ n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(
b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

[Out] Timed out

Mathematica [B] time = 6.26231, size = 3152, normalized size = 8.71

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((-(b^2*d) + 2*a*c*d + a*b*e + b^2*d*n - 4*a*c*d*n)*x)/(a^2*(-b^2
 + 4*a*c)*n) + ((b^2*d - 2*a*c*d - a*b*e - b^2*d*n + 4*a*c*d*n)*x
)/(a^2*(-b^2 + 4*a*c)*n) + (x*(-(b^2*d) + 2*a*c*d + a*b*e - b*c*d
*x^n + 2*a*c*e*x^n))/(a*(-b^2 + 4*a*c)*n*(a + b*x^n + c*x^(2*n)))
 - (b*c*d*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeometric2
F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(
-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^n/
(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))) + Hypergeometri
c2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c
*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]*(x^
n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))))/(a*(-b^2 + 4
*a*c)) + (2*c*e*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hypergeom
etric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/
(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*c]
*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))) + Hyperge
ometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c]
)/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*
c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))))/(-b^2
+ 4*a*c) + (b*c*d*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)/n)*(-(Hyperge
ometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c]
)/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*a*
c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))) + Hyper
geometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*
c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^2 - 4*
a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))))/(a*(
-b^2 + 4*a*c)*n) - (2*c*e*x^(1 + n)*(x^n)^(n^(-1) - (1 + n)/n)*(-
(Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2
 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[b^
2 - 4*a*c]*(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1)))
 + Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^
2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(Sqrt[
b^2 - 4*a*c]*(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1)
)))/((-b^2 + 4*a*c)*n) + (b^2*d*x*((1 - Hypergeometric2F1[-n^(-1)
, -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqr
t[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2
*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - S
qrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(
-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2
 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) +
x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^
2 - 4*a*c])^2/(2*c))))/(a*(-b^2 + 4*a*c)) - (4*c*d*x*((1 - Hyperg
eometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c
])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sq
rt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c
]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometr
ic2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*
c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2
 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2
*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(-b^2 + 4*a*c) - (b^2*d
*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b - S
qrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/
(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b - S
qrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1
 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^
2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(
-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^
2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c])^2/(2*c))))/(a*(-b^2
 + 4*a*c)*n) + (2*c*d*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1),
```

```
(-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-b - Sqrt[b^2 - 4*
a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)
)^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) + (-b - Sqrt[b^2 -
4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 +
 n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sqrt[b^2 - 4*a*c])/
(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n))^n^(-
1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b + Sqrt[b^2 - 4*a*c]
)^2/(2*c))))/((-b^2 + 4*a*c)*n) + (b*e*x*((1 - Hypergeometric2F1[
-n^(-1), -n^(-1), (-1 + n)/n, -(-b - Sqrt[b^2 - 4*a*c])/(2*c*(-(-
b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b - Sqrt[b^2 - 4*a
*c])/(2*c) + x^n))^n^(-1))/((b*(-b - Sqrt[b^2 - 4*a*c]))/(2*c) +
(-b - Sqrt[b^2 - 4*a*c])^2/(2*c)) + (1 - Hypergeometric2F1[-n^(-1
), -n^(-1), (-1 + n)/n, -(-b + Sqrt[b^2 - 4*a*c])/(2*c*(-(-b + Sq
rt[b^2 - 4*a*c])/(2*c) + x^n))]/(x^n/(-(-b + Sqrt[b^2 - 4*a*c])/(
2*c) + x^n))^n^(-1))/((b*(-b + Sqrt[b^2 - 4*a*c]))/(2*c) + (-b +
Sqrt[b^2 - 4*a*c])^2/(2*c))))/((-b^2 + 4*a*c)*n)
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{d+e x^{n}}{\left(a+b x^{n}+c x^{2 n}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd-2\,ace)xx^{n}+\left(b^{2}d-(2\,cd+be)a\right)x}{a^{2}b^{2}n-4\,a^{3}cn+(ab^{2}cn-4\,a^{2}c^{2}n)x^{2\,n}+(ab^{3}n-4\,a^{2}bcn)x^{n}} + \int \frac{b^{2}d(n-1)-(2\,cd(2\,n-1)-be)a+(bcd(n-1)-2\,ace(n-1))x^{n}}{a^{2}b^{2}n-4\,a^{3}cn+(ab^{2}cn-4\,a^{2}c^{2}n)x^{2\,n}+(ab^{3}n-4\,a^{2}bcn)x^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] ((b*c*d - 2*a*c*e)*x*x^n + (b^2*d - (2*c*d + b*e)*a)*x)/(a^2*b^2*
n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*
a^2*b*c*n)*x^n) + integrate((b^2*d*(n - 1) - (2*c*d*(2*n - 1) - b
*e)*a + (b*c*d*(n - 1) - 2*a*c*e*(n - 1))*x^n)/(a^2*b^2*n - 4*a^3
*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n
)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^{n}+d}{c^{2}x^{4\,n}+2\,abx^{n}+a^{2}+\left(2\,bcx^{n}+b^{2}+2\,ac\right)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c^2*x^(4*n) + 2*a*b*x^n + a^2 + (2*b*c*x^n
+ b^2 + 2*a*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.78 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=726

$$-\frac{ce^2x\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}$$
$$-\frac{ce^2x\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)^2}$$
$$-\frac{cx\left((1-n)\left(2ace+b^2(-e)+bcd\right)+\frac{2abce(2-3n)-4ac^2d(1-2n)+b^3(-e)(1-n)+b^2cd(1-n)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)(ae^2-bde+cd^2)}$$
$$+\frac{x\left(cx^n\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d-b^3e+b^2cd\right)}{an\,(b^2-4ac)\,(ae^2-bde+cd^2)\,(a+bx^n+cx^{2n})}$$
$$+\frac{cx\left(b^2(1-n)\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(2ae(2-3n)-d(1-n)\sqrt{b^2-4ac}\right)-2ac\left(e(1-n)\sqrt{b^2-4ac}+2cd(1-2n)\right)+b^3(-\ldots}{an\,(b^2-4ac)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)(ae^2-bde+cd^2)}$$
$$+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\,(ae^2-bde+cd^2)^2}$$

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e +
2*a*c*e)*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*
x^n + c*x^(2*n))) - (c*e^2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*
Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e
+ a*e^2)^2) - (c*((2*a*b*c*e*(2 - 3*n) - 4*a*c^2*d*(1 - 2*n) + b^
2*c*d*(1 - n) - b^3*e*(1 - n))/Sqrt[b^2 - 4*a*c] + (b*c*d - b^2*e
 + 2*a*c*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^
2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) - (c*e^2*(2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(c*d^2 - b*d*e + a*e^2)^2) + (c*(b*c*(2*a*e*(2 - 3*n) - Sqr
t[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4
*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e
)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) + (e^4*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^
2)^2)
```

Rubi [A] time = 4.91619, antiderivative size = 726, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{ce^2x\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$
$$-\frac{ce^2x\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)^2}$$
$$-\frac{cx\left((1-n)\left(2ace+b^2(-e)+bcd\right)+\frac{2abce(2-3n)-4ac^2d(1-2n)+b^3(-e)(1-n)+b^2cd(1-n)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{an\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)\left(ae^2-bde+cd^2\right)}$$
$$+\frac{x\left(cx^n\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d-b^3e+b^2cd\right)}{an\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)\left(a+bx^n+cx^{2n}\right)}$$
$$+\frac{cx\left(b^2(1-n)\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(2ae(2-3n)-d(1-n)\sqrt{b^2-4ac}\right)-2ac\left(e(1-n)\sqrt{b^2-4ac}+2cd(1-2n)\right)+b^3(-\ldots}{an\left(b^2-4ac\right)\left(b\sqrt{b^2-4ac}-4ac+b^2\right)\left(ae^2-bde+cd^2\right)}$$
$$+\frac{e^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)}{d\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e +
2*a*c*e)*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a + b*
x^n + c*x^(2*n))) - (c*e^2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*
Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e
+ a*e^2)^2) - (c*((2*a*b*c*e*(2 - 3*n) - 4*a*c^2*d*(1 - 2*n) + b^
2*c*d*(1 - n) - b^3*e*(1 - n))/Sqrt[b^2 - 4*a*c] + (b*c*d - b^2*e
 + 2*a*c*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^
2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) - (c*e^2*(2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(c*d^2 - b*d*e + a*e^2)^2) + (c*(b*c*(2*a*e*(2 - 3*n) - Sqr
t[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4
*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e
)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n) + (e^4*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^
2)^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^n\right)\left(a+bx^n+cx^{2n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Integral(1/((d + e*x**n)*(a + b*x**n + c*x**(2*n))**2), x)
```

Mathematica [B] time = 7.10621, size = 11767, normalized size = 16.21

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^2),x]

[Out] Result too large to show
```

Maple [F] time = 0.262, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)

[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)),x, algorithm="maxima")

[Out] e^4*integrate(1/(c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + a^2*d*e^4
+ 2*(c*d^3*e^2 - b*d^2*e^3)*a + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*
d^2*e^3 + a^2*e^5 + 2*(c*d^2*e^3 - b*d*e^4)*a)*x^n), x) - ((b*c^2
*d - b^2*c*e + 2*a*c^2*e)*x*x^n + (b^2*c*d - b^3*e - (2*c^2*d - 3
*b*c*e)*a)*x)/(4*a^4*c*e^2*n + (4*c^2*d^2*n - 4*b*c*d*e*n - b^2*e
^2*n)*a^3 - (b^2*c*d^2*n - b^3*d*e*n)*a^2 + (4*a^3*c^2*e^2*n + (4
*c^3*d^2*n - 4*b*c^2*d*e*n - b^2*c*e^2*n)*a^2 - (b^2*c^2*d^2*n -
b^3*c*d*e*n)*a)*x^(2*n) + (4*a^3*b*c*e^2*n + (4*b*c^2*d^2*n - 4*b
^2*c*d*e*n - b^3*e^2*n)*a^2 - (b^3*c*d^2*n - b^4*d*e*n)*a)*x^n) -
 integrate((b^2*c^2*d^3*(n - 1) - 2*b^3*c*d^2*e*(n - 1) + b^4*d*e
^2*(n - 1) + (b*c*e^3*(8*n - 3) - 2*c^2*d*e^2*(4*n - 1))*a^2 + (b
*c^2*d^2*e*(8*n - 5) - 2*c^3*d^3*(2*n - 1) - b^3*e^3*(2*n - 1) -
2*b^2*c*d*e^2*(n - 1))*a + (2*a^2*c^2*e^3*(3*n - 1) + b*c^3*d^3*(
n - 1) - 2*b^2*c^2*d^2*e*(n - 1) + b^3*c*d*e^2*(n - 1) - (b^2*c*e
^3*(2*n - 1) - 2*c^3*d^2*e*(n - 1) + b*c^2*d*e^2*(n - 1))*a)*x^n)
/(4*a^5*c*e^4*n + (8*c^2*d^2*e^2*n - 8*b*c*d*e^3*n - b^2*e^4*n)*a
^4 + 2*(2*c^3*d^4*n - 4*b*c^2*d^3*e*n + b^2*c*d^2*e^2*n + b^3*d*e
^3*n)*a^3 - (b^2*c^2*d^4*n - 2*b^3*c*d^3*e*n + b^4*d^2*e^2*n)*a^2
 + (4*a^4*c^2*e^4*n + (8*c^3*d^2*e^2*n - 8*b*c^2*d*e^3*n - b^2*c*
e^4*n)*a^3 + 2*(2*c^4*d^4*n - 4*b*c^3*d^3*e*n + b^2*c^2*d^2*e^2*n
 + b^3*c*d*e^3*n)*a^2 - (b^2*c^3*d^4*n - 2*b^3*c^2*d^3*e*n + b^4*
c*d^2*e^2*n)*a)*x^(2*n) + (4*a^4*b*c*e^4*n + (8*b*c^2*d^2*e^2*n -
 8*b^2*c*d*e^3*n - b^3*e^4*n)*a^3 + 2*(2*b*c^3*d^4*n - 4*b^2*c^2*
d^3*e*n + b^3*c*d^2*e^2*n + b^4*d*e^3*n)*a^2 - (b^3*c^2*d^4*n - 2
*b^4*c*d^3*e*n + b^5*d^2*e^2*n)*a)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2ex^{3n}+a^2d+(c^2ex^n+c^2d+2\,bce)x^{4n}+(2\,abe+(b^2+2\,ac)d+2\,(bcd+ace)x^n)x^{2n}+(2\,abd+a^2e)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*e*x^(3*n) + a^2*d + (c^2*e*x^n + c^2*d + 2*b*c*e)
*x^(4*n) + (2*a*b*e + (b^2 + 2*a*c)*d + 2*(b*c*d + a*c*e)*x^n)*x^
(2*n) + (2*a*b*d + a^2*e)*x^n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)^2 (ex^n + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)), x)
```

3.79 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=1129

result too large to display

```
[Out] -((x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*
e^2) + 2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b
^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n)))) - (2*c*e^2*(3*c^2*d^2
 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*
c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*
(c*d^2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) + Sqrt
[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 -
7*n) + 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d
*(4*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*a*Sqrt[b^2 -
 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d - Sqrt[b^2
- 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c
 - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (2*c*e^2*(
3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d - 2*Sqrt[b
^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*
n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(
a*e*(5 - 7*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) +
 b*c*(c*d*(4*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) + 3*a*S
qrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d +
Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 +
n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^
2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (
2*e^4*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(
(e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) + (e^4*x*Hypergeometri
c2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e +
a*e^2)^2)
```

Rubi [A] time = 9.80174, antiderivative size = 1129, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{2(2cd-be)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)e^4}{d\left(cd^2-bed+ae^2\right)^3}+\frac{x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)e^4}{d^2\left(cd^2-bed+ae^2\right)^2}$$
$$-\frac{2c\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)e^2}{\left(b^2-\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}$$
$$-\frac{2c\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)e^2}{\left(b^2+\sqrt{b^2-4ac}b-4ac\right)\left(cd^2-bed+ae^2\right)^3}$$
$$+\frac{c\left(e^2(1-n)b^4-e\left(2cd-\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(5-7n)+2\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-n)\right)b^2+c\left(cd\left(4ae(2\right.\right.\right.}{a\left(b^2-4ac\right)\left(b^2\right.}$$
$$+\frac{c\left(e^2(1-n)b^4-e\left(2cd+\sqrt{b^2-4ac}e\right)(1-n)b^3-c\left(e\left(ae(5-7n)-2\sqrt{b^2-4ac}d(1-n)\right)-cd^2(1-n)\right)b^2+c\left(3a\sqrt{b^2-\right.\right.}{a\left(b^2-4ac\right)\left(b^2\right.}$$
$$-\frac{x\left(c\left(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de\right)x^n-b^4e^2-6abc^2de+2b^3cde-b^2c\left(cd^2-4ae^2\right)+2ac^2\left(cd^2-ae^2\right)\right)}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2n\left(bx^n+cx^{2n}+a\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
```

```
[Out] -((x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*
e^2) + 2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b
^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n)))) - (2*c*e^2*(3*c^2*d^2
 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*
c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*
(c*d^2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*n) + Sqrt
[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(a*e*(5 -
7*n) + 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) + b*c*(c*d
*(4*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*a*Sqrt[b^2 -
 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d - Sqrt[b^2
- 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c
 - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) - (2*c*e^2*(
3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d - 2*Sqrt[b
^2 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*(4*a*c^2*(e*(a*e*(1 - 2*
n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - 2*n)) - b^2*c*(e*(
a*e*(5 - 7*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - c*d^2*(1 - n)) +
 b*c*(c*d*(4*a*e*(2 - 3*n) - Sqrt[b^2 - 4*a*c]*d*(1 - n)) + 3*a*S
qrt[b^2 - 4*a*c]*e^2*(1 - n)) + b^4*e^2*(1 - n) - b^3*e*(2*c*d +
Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 +
n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^
2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (
2*e^4*(2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(
(e*x^n)/d)])/(d*(c*d^2 - b*d*e + a*e^2)^3) + (e^4*x*Hypergeometri
c2F1[2, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e +
a*e^2)^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Integral(1/((d + e*x**n)**2*(a + b*x**n + c*x**(2*n))**2), x)
```

Mathematica [B] time = 7.57734, size = 16855, normalized size = 14.93

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2\,n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)

[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)^2),x, algorithm="maxima")

[Out] (c*d^2*e^4*(5*n - 1) - b*d*e^5*(3*n - 1) + a*e^6*(n - 1))*integra
te(1/(c^3*d^8*n - 3*b*c^2*d^7*e*n + 3*b^2*c*d^6*e^2*n - b^3*d^5*e
^3*n + a^3*d^2*e^6*n + 3*(c*d^4*e^4*n - b*d^3*e^5*n)*a^2 + 3*(c^2
*d^6*e^2*n - 2*b*c*d^5*e^3*n + b^2*d^4*e^4*n)*a + (c^3*d^7*e*n -
3*b*c^2*d^6*e^2*n + 3*b^2*c*d^5*e^3*n - b^3*d^4*e^4*n + a^3*d*e^7
*n + 3*(c*d^3*e^5*n - b*d^2*e^6*n)*a^2 + 3*(c^2*d^5*e^3*n - 2*b*c
*d^4*e^4*n + b^2*d^3*e^5*n)*a)*x^n), x) - ((b*c^3*d^3*e - 2*b^2*c
^2*d^2*e^2 + b^3*c*d*e^3 - 4*a^2*c^2*e^4 + (4*c^3*d^2*e^2 - 3*b*c
^2*d*e^3 + b^2*c*e^4)*a)*x*x^(2*n) + (b*c^3*d^4 - b^2*c^2*d^3*e -
 b^3*c*d^2*e^2 + b^4*d*e^3 + 2*(c^2*d*e^3 - 2*b*c*e^4)*a^2 + (2*c
^3*d^3*e + 3*b*c^2*d^2*e^2 - 4*b^2*c*d*e^3 + b^3*e^4)*a)*x*x^n +
(b^2*c^2*d^4 - 2*b^3*c*d^3*e + b^4*d^2*e^2 - 4*a^3*c*e^4 + (2*c^2
*d^2*e^2 + b^2*e^4)*a^2 - 2*(c^3*d^4 - 3*b*c^2*d^3*e + 2*b^2*c*d^
2*e^2)*a)*x)/(4*a^5*c*d^2*e^4*n + (8*c^2*d^4*e^2*n - 8*b*c*d^3*e^
3*n - b^2*d^2*e^4*n)*a^4 + 2*(2*c^3*d^6*n - 4*b*c^2*d^5*e*n + b^2
*c*d^4*e^2*n + b^3*d^3*e^3*n)*a^3 - (b^2*c^2*d^6*n - 2*b^3*c*d^5*
e*n + b^4*d^4*e^2*n)*a^2 + (4*a^4*c^2*d*e^5*n + (8*c^3*d^3*e^3*n
- 8*b*c^2*d^2*e^4*n - b^2*c*d*e^5*n)*a^3 + 2*(2*c^4*d^5*e*n - 4*b
*c^3*d^4*e^2*n + b^2*c^2*d^3*e^3*n + b^3*c*d^2*e^4*n)*a^2 - (b^2*
c^3*d^5*e*n - 2*b^3*c^2*d^4*e^2*n + b^4*c*d^3*e^3*n)*a)*x^(3*n) +
 (4*(c^2*d^2*e^4*n + b*c*d*e^5*n)*a^4 + (8*c^3*d^4*e^2*n - 9*b^2*
c*d^2*e^4*n - b^3*d*e^5*n)*a^3 + 2*(2*c^4*d^6*n - 2*b*c^3*d^5*e*n
 - 3*b^2*c^2*d^4*e^2*n + 2*b^3*c*d^3*e^3*n + b^4*d^2*e^4*n)*a^2 -
 (b^2*c^3*d^6*n - b^3*c^2*d^5*e*n - b^4*c*d^4*e^2*n + b^5*d^3*e^3
*n)*a)*x^(2*n) + (4*a^5*c*d*e^5*n + (8*c^2*d^3*e^3*n - 4*b*c*d^2*
e^4*n - b^2*d*e^5*n)*a^4 + (4*c^3*d^5*e*n - 6*b^2*c*d^3*e^3*n + b
^3*d^2*e^4*n)*a^3 + (4*b*c^3*d^6*n - 9*b^2*c^2*d^5*e*n + 4*b^3*c*
d^4*e^2*n + b^4*d^3*e^3*n)*a^2 - (b^3*c^2*d^6*n - 2*b^4*c*d^5*e*n
 + b^5*d^4*e^2*n)*a)*x^n) + integrate(-(2*a^3*c^2*e^4*(4*n - 1) +
 b^2*c^3*d^4*(n - 1) - 3*b^3*c^2*d^3*e*(n - 1) + 3*b^4*c*d^2*e^2*
(n - 1) - b^5*d*e^3*(n - 1) - 2*(b^2*c*e^4*(7*n - 2) - 2*b*c^2*d*
e^3*(6*n - 1) + 6*c^3*d^2*e^2*n)*a^2 + (b^4*e^4*(3*n - 1) + 4*b*c
^3*d^3*e*(3*n - 2) - 2*c^4*d^4*(2*n - 1) - 2*b^3*c*d*e^3*(n + 1)
- 9*b^2*c^2*d^2*e^2*(n - 1))*a + (b*c^4*d^4*(n - 1) - 3*b^2*c^3*d
^3*e*(n - 1) + 3*b^3*c^2*d^2*e^2*(n - 1) - b^4*c*d*e^3*(n - 1) -
(b*c^2*e^4*(11*n - 3) - 4*c^3*d*e^3*(5*n - 1))*a^2 - (b^2*c^2*d*e
^3*(3*n + 1) - b^3*c*e^4*(3*n - 1) - 4*c^4*d^3*e*(n - 1) + 6*b*c^
3*d^2*e^2*(n - 1))*a)*x^n)/(4*a^6*c*e^6*n + (12*c^2*d^2*e^4*n - 1
2*b*c*d*e^5*n - b^2*e^6*n)*a^5 + 3*(4*c^3*d^4*e^2*n - 8*b*c^2*d^3
*e^3*n + 3*b^2*c*d^2*e^4*n + b^3*d*e^5*n)*a^4 + (4*c^4*d^6*n - 12
*b*c^3*d^5*e*n + 9*b^2*c^2*d^4*e^2*n + 2*b^3*c*d^3*e^3*n - 3*b^4*
d^2*e^4*n)*a^3 - (b^2*c^3*d^6*n - 3*b^3*c^2*d^5*e*n + 3*b^4*c*d^4
*e^2*n - b^5*d^3*e^3*n)*a^2 + (4*a^5*c^2*e^6*n + (12*c^3*d^2*e^4*
n - 12*b*c^2*d*e^5*n - b^2*c*e^6*n)*a^4 + 3*(4*c^4*d^4*e^2*n - 8*
b*c^3*d^3*e^3*n + 3*b^2*c^2*d^2*e^4*n + b^3*c*d*e^5*n)*a^3 + (4*c
^5*d^6*n - 12*b*c^4*d^5*e*n + 9*b^2*c^3*d^4*e^2*n + 2*b^3*c^2*d^3
*e^3*n - 3*b^4*c*d^2*e^4*n)*a^2 - (b^2*c^4*d^6*n - 3*b^3*c^3*d^5*
e*n + 3*b^4*c^2*d^4*e^2*n - b^5*c*d^3*e^3*n)*a)*x^(2*n) + (4*a^5*
b*c*e^6*n + (12*b*c^2*d^2*e^4*n - 12*b^2*c*d*e^5*n - b^3*e^6*n)*a
^4 + 3*(4*b*c^3*d^4*e^2*n - 8*b^2*c^2*d^3*e^3*n + 3*b^3*c*d^2*e^4
*n + b^4*d*e^5*n)*a^3 + (4*b*c^4*d^6*n - 12*b^2*c^3*d^5*e*n + 9*b
^3*c^2*d^4*e^2*n + 2*b^4*c*d^3*e^3*n - 3*b^5*d^2*e^4*n)*a^2 - (b^
3*c^3*d^6*n - 3*b^4*c^2*d^5*e*n + 3*b^5*c*d^4*e^2*n - b^6*d^3*e^3
*n)*a)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2d^2 + (c^2e^2x^{2n} + 2\,c^2dex^n + c^2d^2 + 4\,bcde + (b^2 + 2\,ac)e^2)x^{4n} + 2\,(bce^2x^{2n} + b^2de + abe^2)x^{3n} + (4\,abde + a^2e^2 +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2 + (c^2*e^2*x^(2*n) + 2*c^2*d*e*x^n + c^2*d^2
+ 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*x^(4*n) + 2*(b*c*e^2*x^(2*n) + b
^2*d*e + a*b*e^2)*x^(3*n) + (4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*
d^2 + 2*(b*c*d^2 + 2*a*c*d*e)*x^n)*x^(2*n) + 2*(a*b*d^2 + a^2*d*e
)*x^n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)^2(ex^n + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^2*(e*x^n + d)^2), x)
```

3.80 $\int \frac{(d+ex^n)^3}{(a+bx^n+cx^{2n})^3} dx$

Optimal. Leaf size=1707

result too large to display

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^
2) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*
e^2))*x^n))/(2*a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (
e^2*x*(3*b^2*c*d - 6*a*c^2*d - b^3*e + a*b*c*e + c*(3*b*c*d - b^2
*e - 2*a*c*e)*x^n))/(a*c^2*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n)
)) - (x*(a*b^2*c^2*d*(3*a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) + 4*
a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 4*n) - 2*a*b^5*e^3*n + 2*a^2*b*c
^2*e*(3*c*d^2*(2 - 3*n) - 5*a*e^2*n) - 3*a*b^3*c*e*(c*d^2 - 3*a*e
^2*n) + b^4*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) + c*(4*a^2*c^2*e*(3
*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2
 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*
b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n)))*x^n))/(2*a^2*c^2*(b^2 - 4*a*
c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (e^2*(b*c*(2*a*e*(2 - 5*n) +
3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n) + Sqrt[b^
2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d - Sqrt[b^2 - 4
*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c
- b*Sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e
^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*
a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c
*d^2 - a*e^2*(1 + 2*n))) - (2*a*b^5*e^3*(1 - n)*n - b^4*c*d*(1 -
n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*e^2)*
(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^
2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) +
 a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*
(1 - 19*n + 30*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*c*
(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*n^2) + (e^2*(b*c*(2*a*e*(
2 - 5*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n
) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d + S
qrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b
^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*
c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2
- 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b
^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) + (2*a*b^5*e^3*(1 - n)*n - b^
4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2
- 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*
n^2) - a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n
 - 3*n^2) + a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 -
n) + a*e^2*(1 - 19*n + 30*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(2*a^2*c*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*c])*n^2)
```

Rubi [A] time = 13.9279, antiderivative size = 1707, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

result too large to display

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^
2) - (a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*
e^2))*x^n))/(2*a*c*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (
e^2*x*(3*b^2*c*d - 6*a*c^2*d - b^3*e + a*b*c*e + c*(3*b*c*d - b^2
*e - 2*a*c*e)*x^n))/(a*c^2*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n)
)) - (x*(a*b^2*c^2*d*(3*a*e^2*(1 - 9*n) - 5*c*d^2*(1 - 3*n)) + 4*
a^2*c^3*d*(c*d^2 - 3*a*e^2)*(1 - 4*n) - 2*a*b^5*e^3*n + 2*a^2*b*c
```

```
^2*e*(3*c*d^2*(2 - 3*n) - 5*a*e^2*n) - 3*a*b^3*c*e*(c*d^2 - 3*a*e
^2*n) + b^4*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) + c*(4*a^2*c^2*e*(3
*c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2
 - 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*
b^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n)))*x^n))/(2*a^2*c^2*(b^2 - 4*a*
c)^2*n^2*(a + b*x^n + c*x^(2*n))) + (e^2*(b*c*(2*a*e*(2 - 5*n) +
3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n) + Sqrt[b^
2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d - Sqrt[b^2 - 4
*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b^2 - 4*a*c
- b*Sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*c*d^2 - a*e
^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2 - 7*n) + 3*
a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b^2*c*e*(3*c
*d^2 - a*e^2*(1 + 2*n))) - (2*a*b^5*e^3*(1 - n)*n - b^4*c*d*(1 -
n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2 - 3*a*e^2)*
(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*n^2) - a*e^
2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n - 3*n^2) +
 a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 - n) + a*e^2*
(1 - 19*n + 30*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*c*
(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*n^2) + (e^2*(b*c*(2*a*e*(
2 - 5*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(6*c*d*(1 - 2*n
) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*(3*c*d + S
qrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*c*(b^2 - 4*a*c)*(b
^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) + (((1 - n)*(4*a^2*c^2*e*(3*
c*d^2 - a*e^2)*(1 - 3*n) - 2*a*b^4*e^3*n - 2*a*b*c^2*d*(c*d^2*(2
- 7*n) + 3*a*e^2*n) + b^3*c*d*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - a*b
^2*c*e*(3*c*d^2 - a*e^2*(1 + 2*n))) + (2*a*b^5*e^3*(1 - n)*n - b^
4*c*d*(1 - n)*(c*d^2*(1 - 2*n) + 6*a*e^2*n) - 8*a^2*c^3*d*(c*d^2
- 3*a*e^2)*(1 - 6*n + 8*n^2) + 6*a*b^2*c^2*d*(c*d^2*(1 - 4*n + 3*
n^2) - a*e^2*(1 - 10*n + 15*n^2)) - 4*a^2*b*c^2*e*(3*c*d^2*(1 - n
 - 3*n^2) + a*e^2*(1 - 11*n + 19*n^2)) + a*b^3*c*e*(3*c*d^2*(1 -
n) + a*e^2*(1 - 19*n + 30*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(2*a^2*c*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*c])*n^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**3,x)

[Out] Timed out
```

Mathematica [B] time = 7.79723, size = 13018, normalized size = 7.63

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^3/(a + b*x^n + c*x^(2*n))^3,x]

[Out] Result too large to show
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^3}{(a + bx^n + cx^{2n})^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^3/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*c^2*d^3*(2*n - 1) + 4*a^3*c^2*e^3*(n + 1) + (12*c^3*d^2
*e*(3*n - 1) + b^2*c*e^3*(2*n - 1) - 18*b*c^2*d*e^2*n)*a^2 - (2*b
*c^3*d^3*(7*n - 2) - 3*b^2*c^2*d^2*e)*a)*x*x^(3*n) + (2*b^4*c*d^3
*(2*n - 1) + 2*(b*c*e^3*(3*n + 2) + 6*c^2*d*e^2)*a^3 - (3*b^2*c*d
*e^2*(9*n + 1) - 6*b*c^2*d^2*e*(9*n - 4) - 4*c^3*d^3*(4*n - 1) -
b^3*e^3*(3*n - 1))*a^2 - (b^2*c^2*d^3*(29*n - 9) - 6*b^3*c*d^2*e)
*a)*x*x^(2*n) + (b^5*d^3*(2*n - 1) - 4*a^4*c*e^3*(n - 1) + (b^2*e
^3*(10*n - 1) + 12*c^2*d^2*e*(5*n - 1) - 6*b*c*d*e^2*(5*n - 2))*a
^3 + (3*b^2*c*d^2*e*(4*n - 3) - 3*b^3*d*e^2*(2*n + 1) - 2*b*c^2*d
^3*n)*a^2 - (4*b^3*c*d^3*(3*n - 1) - 3*b^4*d^2*e)*a)*x*x^n + (a*b
^4*d^3*(3*n - 1) - 6*(2*c*d*e^2*(2*n - 1) - b*e^3*n)*a^4 + (4*c^2
*d^3*(6*n - 1) + 6*b*c*d^2*e*(5*n - 2) - 3*b^2*d*e^2*(n + 1))*a^3
 - (b^2*c*d^3*(21*n - 5) + 3*b^3*d^2*e*(n - 1))*a^2)*x)/(a^4*b^4*
n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3
*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3
*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b
^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c
*n^2 + 16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*
b^4*d^3 + 6*(2*c*d*e^2*(2*n - 1) - b*e^3*n)*a^3 + (4*(8*n^2 - 6*n
 + 1)*c^2*d^3 - 6*b*c*d^2*e*(5*n - 2) + 3*b^2*d*e^2*(n + 1))*a^2
- ((16*n^2 - 21*n + 5)*b^2*c*d^3 - 3*b^3*d^2*e*(n - 1))*a + ((2*n
^2 - 3*n + 1)*b^3*c*d^3 + 4*(n^2 - 1)*a^3*c*e^3 + (12*(3*n^2 - 4*
n + 1)*c^2*d^2*e - 18*(n^2 - n)*b*c*d*e^2 + (2*n^2 - 3*n + 1)*b^2
*e^3)*a^2 - (2*(7*n^2 - 9*n + 2)*b*c^2*d^3 - 3*b^2*c*d^2*e*(n - 1
))*a)*x^n)/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2
*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b
^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^{3\,n} + 3\,de^2x^{2\,n} + 3\,d^2ex^n + d^3}{c^3x^{6\,n} + b^3x^{3\,n} + 3\,a^2bx^n + a^3 + 3\,(bc^2x^n + b^2c + ac^2)x^{4\,n} + 3\,(2\,abcx^n + ab^2 + a^2c)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)/(c^3
*x^(6*n) + b^3*x^(3*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + b^2*c
 + a*c^2)*x^(4*n) + 3*(2*a*b*c*x^n + a*b^2 + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**3/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^3}{(cx^{2n} + bx^n + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^3/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.81 $\int \frac{(d+ex^n)^2}{(a+bx^n+cx^{2n})^3} \, dx$

Optimal. Leaf size=1191

result too large to display

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*
d*e + a*b*e^2)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))
^2) + (e^2*x*(b^2 - 2*a*c + b*c*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b
*x^n + c*x^(2*n))) + (x*(2*a*b^3*c*d*e - a*b^2*c*(a*e^2*(1 - 9*n)
 - 5*c*d^2*(1 - 3*n)) - 4*a^2*c^2*(c*d^2 - a*e^2)*(1 - 4*n) - 4*a
^2*b*c^2*d*e*(2 - 3*n) - b^4*(c*d^2*(1 - 2*n) + 2*a*e^2*n) + c*(2
*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n)
 + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n))*x^n))/(2*a^2*c*(
b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) - (e^2*(4*a*c*(1 - 2*
n) - b^2*(1 - n) - b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a
*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)
*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7
*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n)) + (2*a*b^3*c*
d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2*n) - 8*a^2*b
*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n + 8
*n^2) + 2*a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n +
15*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^
2*(b - Sqrt[b^2 - 4*a*c])*n^2) - (e^2*(4*a*c*(1 - 2*n) - b^2*(1 -
 n) + b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c
)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d
*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n
) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n)) - (2*a*b^3*c*d*e*(1 - n) -
 b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2*n) - 8*a^2*b*c^2*d*e*(1 -
 n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 2*a*b
^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)))/Sqr
t[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b
^2 - 4*a*c])*n^2)
```

Rubi [A] time = 8.98719, antiderivative size = 1191, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{\left(-(1-n)b^2-\sqrt{b^2-4ac}(1-n)b+4ac(1-2n)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)e^2}{a\left(b^2-4ac\right)\left(b^2-\sqrt{b^2-4ac}b-4ac\right)n}$$

$$-\frac{\left(-(1-n)b^2+\sqrt{b^2-4ac}(1-n)b+4ac(1-2n)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)e^2}{a\left(b^2-4ac\right)\left(b^2+\sqrt{b^2-4ac}b-4ac\right)n}$$

$$+\frac{x\left(bcx^n+b^2-2ac\right)e^2}{ac\left(b^2-4ac\right)n\left(bx^n+cx^{2n}+a\right)}$$

$$-\frac{\left((1-n)\left(-\left(c(1-2n)d^2+2ae^2n\right)b^3+2acdeb^2+2ac\left(c(2-7n)d^2+ae^2n\right)b-8a^2c^2de(1-3n)\right)+\frac{-(1-n)\left(c(1-2n)d^2+2ae^2n\right)b^4\cdots}{\cdots}\right.\cdots}{2a^2\left(b^2-4ac\right)^2\left(\cdots\right.}$$

$$-\frac{\left((1-n)\left(-\left(c(1-2n)d^2+2ae^2n\right)b^3+2acdeb^2+2ac\left(c(2-7n)d^2+ae^2n\right)b-8a^2c^2de(1-3n)\right)-\frac{-(1-n)\left(c(1-2n)d^2+2ae^2n\right)b^4\cdots}{\cdots}\right.\cdots}{2a^2\left(b^2-4ac\right)^2\left(\cdots\right.}$$

$$+\frac{x\left(c\left(-\left(c(1-2n)d^2+2ae^2n\right)b^3+2acdeb^2+2ac\left(c(2-7n)d^2+ae^2n\right)b-8a^2c^2de(1-3n)\right)x^n+2ab^3cde-ab^2c\left(ae^2(1-\cdots\right.\right.}{2a^2c\left(b^2-4ac\right)^2n^2\left(bx^n+cx^{2n}\cdots\right.}$$

$$+\frac{x\left(\left(bcd^2-4aced+abe^2\right)x^n+b^2d^2-2abde-2a\left(cd^2-ae^2\right)\right)}{2a\left(b^2-4ac\right)n\left(bx^n+cx^{2n}+a\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*
d*e + a*b*e^2)*x^n))/(2*a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))
^2) + (e^2*x*(b^2 - 2*a*c + b*c*x^n))/(a*c*(b^2 - 4*a*c)*n*(a + b
*x^n + c*x^(2*n))) + (x*(2*a*b^3*c*d*e - a*b^2*c*(a*e^2*(1 - 9*n)
 - 5*c*d^2*(1 - 3*n)) - 4*a^2*c^2*(c*d^2 - a*e^2)*(1 - 4*n) - 4*a
^2*b*c^2*d*e*(2 - 3*n) - b^4*(c*d^2*(1 - 2*n) + 2*a*e^2*n) + c*(2
*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n)
 + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n))*x^n))/(2*a^2*c*(
b^2 - 4*a*c)^2*n^2*(a + b*x^n + c*x^(2*n))) - (e^2*(4*a*c*(1 - 2*
n) - b^2*(1 - n) - b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a
*(b^2 - 4*a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)
*(2*a*b^2*c*d*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7
*n) + a*e^2*n) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n)) + (2*a*b^3*c*
d*e*(1 - n) - b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2*n) - 8*a^2*b
*c^2*d*e*(1 - n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n + 8
*n^2) + 2*a*b^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n +
15*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^
2*(b - Sqrt[b^2 - 4*a*c])*n^2) - (e^2*(4*a*c*(1 - 2*n) - b^2*(1 -
 n) + b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c
)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*n) - (((1 - n)*(2*a*b^2*c*d
*e - 8*a^2*c^2*d*e*(1 - 3*n) + 2*a*b*c*(c*d^2*(2 - 7*n) + a*e^2*n
) - b^3*(c*d^2*(1 - 2*n) + 2*a*e^2*n)) - (2*a*b^3*c*d*e*(1 - n) -
 b^4*(1 - n)*(c*d^2*(1 - 2*n) + 2*a*e^2*n) - 8*a^2*b*c^2*d*e*(1 -
 n - 3*n^2) - 8*a^2*c^2*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 2*a*b
^2*c*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(1 - 10*n + 15*n^2)))/Sqr
t[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b
^2 - 4*a*c])*n^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.82098, size = 10910, normalized size = 9.16

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)^2/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.156, size = 0, normalized size = 0.

$$\int \frac{(d + ex^n)^2}{(a + bx^n + cx^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*c^2*d^2*(2*n - 1) + 2*(4*c^3*d*e*(3*n - 1) - 3*b*c^2*e^
2*n)*a^2 - 2*(b*c^3*d^2*(7*n - 2) - b^2*c^2*d*e)*a)*x*x^(3*n) + (
2*b^4*c*d^2*(2*n - 1) + 4*a^3*c^2*e^2 - (b^2*c*e^2*(9*n + 1) - 4*
b*c^2*d*e*(9*n - 4) - 4*c^3*d^2*(4*n - 1))*a^2 - (b^2*c^2*d^2*(29
*n - 9) - 4*b^3*c*d*e)*a)*x*x^(2*n) + (b^5*d^2*(2*n - 1) + 2*(4*c
^2*d*e*(5*n - 1) - b*c*e^2*(5*n - 2))*a^3 + (2*b^2*c*d*e*(4*n - 3
) - b^3*e^2*(2*n + 1) - 2*b*c^2*d^2*n)*a^2 - 2*(2*b^3*c*d^2*(3*n
- 1) - b^4*d*e)*a)*x*x^n + (a*b^4*d^2*(3*n - 1) - 4*a^4*c*e^2*(2*
n - 1) + (4*c^2*d^2*(6*n - 1) + 4*b*c*d*e*(5*n - 2) - b^2*e^2*(n
+ 1))*a^3 - (b^2*c*d^2*(21*n - 5) + 2*b^3*d*e*(n - 1))*a^2)*x)/(a
^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2
- 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2
- 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 -
6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^
4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*((2*n^2 - 3
*n + 1)*b^4*d^2 + 4*a^3*c*e^2*(2*n - 1) + (4*(8*n^2 - 6*n + 1)*c^
2*d^2 - 4*b*c*d*e*(5*n - 2) + b^2*e^2*(n + 1))*a^2 - ((16*n^2 - 2
1*n + 5)*b^2*c*d^2 - 2*b^3*d*e*(n - 1))*a + ((2*n^2 - 3*n + 1)*b^
3*c*d^2 + 2*(4*(3*n^2 - 4*n + 1)*c^2*d*e - 3*(n^2 - n)*b*c*e^2)*a
^2 - 2*((7*n^2 - 9*n + 2)*b*c^2*d^2 - b^2*c*d*e*(n - 1))*a)*x^n)/
(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2
- 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*
a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^{2\,n} + 2\,dex^n + d^2}{c^3x^{6\,n} + b^3x^{3\,n} + 3\,a^2bx^n + a^3 + 3\,(bc^2x^n + b^2c + ac^2)x^{4\,n} + 3\,(2\,abcx^n + ab^2 + a^2c)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)/(c^3*x^(6*n) + b^3*x^(3*
n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + b^2*c + a*c^2)*x^(4*n) +
3*(2*a*b*c*x^n + a*b^2 + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^2}{(cx^{2n} + bx^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^2/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.82 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=713

$$\frac{x\left(cx^n\left(-4a^2ce(1-3n)+ab^2e+2abcd(2-7n)+b^3(-d)(1-2n)\right)-2a^2bce(2-3n)-4a^2c^2d(1-4n)+ab^3e+5ab^2cd(1-3\right.}{2a^2n^2\left(b^2-4ac\right)^2\left(a+bx^n+cx^{2n}\right)}$$
$$+\frac{cx\left(-4a^2c\left(e\left(3n^2-4n+1\right)\sqrt{b^2-4ac}+2cd\left(8n^2-6n+1\right)\right)-2abc\left(2ae\left(-3n^2-n+1\right)-d\left(7n^2-9n+2\right)\sqrt{b^2-4ac}\right)\right.}{2a^2n^2\left(b^2-4ac\right.}$$
$$-\frac{cx\left(-4a^2c\left(e\left(3n^2-4n+1\right)\sqrt{b^2-4ac}-2cd\left(8n^2-6n+1\right)\right)+2abc\left(d\left(7n^2-9n+2\right)\sqrt{b^2-4ac}+2ae\left(-3n^2-n+1\right)\right)\right.}{2a^2n^2\left(b^2-4ac\right.}$$
$$+\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 - 4
*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (x*(a*b^3*e - 4*a^2*c^2*d*(1
 - 4*n) + 5*a*b^2*c*d*(1 - 3*n) - 2*a^2*b*c*e*(2 - 3*n) - b^4*d*(
1 - 2*n) + c*(a*b^2*e + 2*a*b*c*d*(2 - 7*n) - 4*a^2*c*e*(1 - 3*n)
 - b^3*d*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n +
 c*x^(2*n))) + (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e + 6*c*d*(1 - 3*n))*
(1 - n) + b^3*(a*e - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^4
*d*(1 - 3*n + 2*n^2) - 2*a*b*c*(2*a*e*(1 - n - 3*n^2) - Sqrt[b^2
- 4*a*c]*d*(2 - 9*n + 7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 -
 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(
b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*n^2) - (c*(a*b
^2*(Sqrt[b^2 - 4*a*c]*e - 6*c*d*(1 - 3*n))*(1 - n) - b^3*(a*e + S
qrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^4*d*(1 - 3*n + 2*n^2) +
 2*a*b*c*(2*a*e*(1 - n - 3*n^2) + Sqrt[b^2 - 4*a*c]*d*(2 - 9*n +
7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*
(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*n^2)
```

Rubi [A] time = 4.99873, antiderivative size = 713, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{x\left(cx^n\left(-4a^2ce(1-3n)+ab^2e+2abcd(2-7n)+b^3(-d)(1-2n)\right)-2a^2bce(2-3n)-4a^2c^2d(1-4n)+ab^3e+5ab^2cd(1-3\right.}{2a^2n^2\left(b^2-4ac\right)^2\left(a+bx^n+cx^{2n}\right)}$$
$$+\frac{cx\left(-4a^2c\left(e\left(3n^2-4n+1\right)\sqrt{b^2-4ac}+2cd\left(8n^2-6n+1\right)\right)-2abc\left(2ae\left(-3n^2-n+1\right)-d\left(7n^2-9n+2\right)\sqrt{b^2-4ac}\right)\right.}{2a^2n^2\left(b^2-4ac\right.}$$
$$-\frac{cx\left(-4a^2c\left(e\left(3n^2-4n+1\right)\sqrt{b^2-4ac}-2cd\left(8n^2-6n+1\right)\right)+2abc\left(d\left(7n^2-9n+2\right)\sqrt{b^2-4ac}+2ae\left(-3n^2-n+1\right)\right)\right.}{2a^2n^2\left(b^2-4ac\right.}$$
$$+\frac{x\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]
```


```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 - 4
*a*c)*n*(a + b*x^n + c*x^(2*n))^2) + (x*(a*b^3*e - 4*a^2*c^2*d*(1
 - 4*n) + 5*a*b^2*c*d*(1 - 3*n) - 2*a^2*b*c*e*(2 - 3*n) - b^4*d*(
1 - 2*n) + c*(a*b^2*e + 2*a*b*c*d*(2 - 7*n) - 4*a^2*c*e*(1 - 3*n)
 - b^3*d*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*n^2*(a + b*x^n +
 c*x^(2*n))) + (c*(a*b^2*(Sqrt[b^2 - 4*a*c]*e + 6*c*d*(1 - 3*n))*
```

```
(1 - n) + b^3*(a*e - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) - b^4
*d*(1 - 3*n + 2*n^2) - 2*a*b*c*(2*a*e*(1 - n - 3*n^2) - Sqrt[b^2
- 4*a*c]*d*(2 - 9*n + 7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 -
 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(
b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*n^2) - (c*(a*b
^2*(Sqrt[b^2 - 4*a*c]*e - 6*c*d*(1 - 3*n))*(1 - n) - b^3*(a*e + S
qrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^4*d*(1 - 3*n + 2*n^2) +
 2*a*b*c*(2*a*e*(1 - n - 3*n^2) + Sqrt[b^2 - 4*a*c]*d*(2 - 9*n +
7*n^2)) - 4*a^2*c*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) - 2*c*d*
(1 - 6*n + 8*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*n^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)

[Out] Timed out
```

Mathematica [B] time = 6.6777, size = 8593, normalized size = 12.05

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^3,x]

[Out] Result too large to show
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{(a + bx^n + cx^{2n})^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)

[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a^2*c^3*e*(3*n - 1) + b^3*c^2*d*(2*n - 1) - (2*b*c^3*d*(7
*n - 2) - b^2*c^2*e)*a)*x*x^(3*n) + (2*b^4*c*d*(2*n - 1) + 2*(b*c
^2*e*(9*n - 4) + 2*c^3*d*(4*n - 1))*a^2 - (b^2*c^2*d*(29*n - 9) -
 2*b^3*c*e)*a)*x*x^(2*n) + (4*a^3*c^2*e*(5*n - 1) + b^5*d*(2*n -
1) + (b^2*c*e*(4*n - 3) - 2*b*c^2*d*n)*a^2 - (4*b^3*c*d*(3*n - 1)
 - b^4*e)*a)*x*x^n + (a*b^4*d*(3*n - 1) + 2*(2*c^2*d*(6*n - 1) +
b*c*e*(5*n - 2))*a^3 - (b^2*c*d*(21*n - 5) + b^3*e*(n - 1))*a^2)*
x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2
*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c
*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n
^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 -
 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*((2*n^2
 - 3*n + 1)*b^4*d + 2*(2*(8*n^2 - 6*n + 1)*c^2*d - b*c*e*(5*n - 2
))*a^2 - ((16*n^2 - 21*n + 5)*b^2*c*d - b^3*e*(n - 1))*a + ((2*n^
2 - 3*n + 1)*b^3*c*d + 4*(3*n^2 - 4*n + 1)*a^2*c^2*e - (2*(7*n^2
- 9*n + 2)*b*c^2*d - b^2*c*e*(n - 1))*a)*x^n)/(a^3*b^4*n^2 - 8*a^
4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2
 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*
a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^n + d}{c^3x^{6\,n} + b^3x^{3\,n} + 3\,a^2bx^n + a^3 + 3\,(bc^2x^n + b^2c + ac^2)x^{4\,n} + 3\,(2\,abcx^n + ab^2 + a^2c)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a^2*b*x^n + a
^3 + 3*(b*c^2*x^n + b^2*c + a*c^2)*x^(4*n) + 3*(2*a*b*c*x^n + a*b
^2 + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2\,n} + bx^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.83 $\int \frac{1}{(d+ex^n)(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=1708

result too large to display

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e +
2*a*c*e)*x^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a +
b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a
*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n))/(a*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n))) + (x*(2*a^2*b*c^2
*e*(4 - 11*n) - 3*a*b^3*c*e*(2 - 5*n) - 4*a^2*c^3*d*(1 - 4*n) + 5
*a*b^2*c^2*d*(1 - 3*n) - b^4*c*d*(1 - 2*n) + b^5*(e - 2*e*n) - c*
(a*b^2*c*e*(5 - 14*n) - 2*a*b*c^2*d*(2 - 7*n) - 4*a^2*c^2*e*(1 -
3*n) + b^3*c*d*(1 - 2*n) - b^4*e*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4
*a*c)^2*(c*d^2 - b*d*e + a*e^2)*n^2*(a + b*x^n + c*x^(2*n))) - (c
*e^4*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4
*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*e^2*(
b*c*(2*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*
d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*
(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a
*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2
*n) - (c*(a*b^2*c*(Sqrt[b^2 - 4*a*c]*e*(5 - 14*n) - 6*c*d*(1 - 3*
n))*(1 - n) + b^3*c*(a*e*(7 - 18*n) + Sqrt[b^2 - 4*a*c]*d*(1 - 2*
n))*(1 - n) - b^5*e*(1 - 3*n + 2*n^2) + b^4*(c*d - Sqrt[b^2 - 4*a
*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*
n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a
*c]*d*(2 - 9*n + 7*n^2) + 2*a*e*(3 - 13*n + 13*n^2)))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c
*d^2 - b*d*e + a*e^2)*n^2) - (c*e^4*(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2
 - b*d*e + a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 -
4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*
(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n
))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (c*(a*b^2*c*(Sqrt[b^2 - 4*a*
c]*e*(5 - 14*n) + 6*c*d*(1 - 3*n))*(1 - n) - b^3*c*(a*e*(7 - 18*n
) - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^5*e*(1 - 3*n + 2*n
^2) - b^4*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c
^2*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^
2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) - 2*a*e*(3
 - 13*n + 13*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n^2) + (e^6*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d
^2 - b*d*e + a*e^2)^3)
```

Rubi [A] time = 14.9069, antiderivative size = 1708, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

result too large to display

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e +
2*a*c*e)*x^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*n*(a +
b*x^n + c*x^(2*n))^2) + (e^2*x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a
*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^n))/(a*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n))) + (x*(2*a^2*b*c^2
```

```
*e*(4 - 11*n) - 3*a*b^3*c*e*(2 - 5*n) - 4*a^2*c^3*d*(1 - 4*n) + 5
*a*b^2*c^2*d*(1 - 3*n) - b^4*c*d*(1 - 2*n) + b^5*(e - 2*e*n) - c*
(a*b^2*c*e*(5 - 14*n) - 2*a*b*c^2*d*(2 - 7*n) - 4*a^2*c^2*e*(1 -
3*n) + b^3*c*d*(1 - 2*n) - b^4*e*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4
*a*c)^2*(c*d^2 - b*d*e + a*e^2)*n^2*(a + b*x^n + c*x^(2*n))) - (c
*e^4*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4
*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^3) + (c*e^2*(
b*c*(2*a*e*(2 - 3*n) + Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 2*a*c*(2*c*
d*(1 - 2*n) - Sqrt[b^2 - 4*a*c]*e*(1 - n)) - b^3*e*(1 - n) + b^2*
(c*d - Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a
*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2
*n) - (c*(a*b^2*c*(Sqrt[b^2 - 4*a*c]*e*(5 - 14*n) - 6*c*d*(1 - 3*
n))*(1 - n) + b^3*c*(a*e*(7 - 18*n) + Sqrt[b^2 - 4*a*c]*d*(1 - 2*
n))*(1 - n) - b^5*e*(1 - 3*n + 2*n^2) + b^4*(c*d - Sqrt[b^2 - 4*a
*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c^2*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*
n + 3*n^2) - 2*c*d*(1 - 6*n + 8*n^2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a
*c]*d*(2 - 9*n + 7*n^2) + 2*a*e*(3 - 13*n + 13*n^2)))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c
*d^2 - b*d*e + a*e^2)*n^2) - (c*e^4*(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2
 - b*d*e + a*e^2)^3) + (c*e^2*(b*c*(2*a*e*(2 - 3*n) - Sqrt[b^2 -
4*a*c]*d*(1 - n)) - 2*a*c*(2*c*d*(1 - 2*n) + Sqrt[b^2 - 4*a*c]*e*
(1 - n)) - b^3*e*(1 - n) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - n
))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(c*d^2 - b*d*e + a*e^2)^2*n) + (c*(a*b^2*c*(Sqrt[b^2 - 4*a*
c]*e*(5 - 14*n) + 6*c*d*(1 - 3*n))*(1 - n) - b^3*c*(a*e*(7 - 18*n
) - Sqrt[b^2 - 4*a*c]*d*(1 - 2*n))*(1 - n) + b^5*e*(1 - 3*n + 2*n
^2) - b^4*(c*d + Sqrt[b^2 - 4*a*c]*e)*(1 - 3*n + 2*n^2) - 4*a^2*c
^2*(Sqrt[b^2 - 4*a*c]*e*(1 - 4*n + 3*n^2) + 2*c*d*(1 - 6*n + 8*n^
2)) - 2*a*b*c^2*(Sqrt[b^2 - 4*a*c]*d*(2 - 9*n + 7*n^2) - 2*a*e*(3
 - 13*n + 13*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)*n^2) + (e^6*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d*(c*d
^2 - b*d*e + a*e^2)^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^n)(a + bx^n + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Integral(1/((d + e*x**n)*(a + b*x**n + c*x**(2*n))**3), x)
```

Mathematica [B] time = 8.9555, size = 43535, normalized size = 25.49

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.435, size = 0, normalized size = 0.

$$\int \frac{1}{(d+e x^n)\left(a+b x^n+c x^{2n}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)),x, algorithm="maxima")
```

```
[Out] e^6*integrate(1/(c^3*d^7 - 3*b*c^2*d^6*e + 3*b^2*c*d^5*e^2 - b^3*
d^4*e^3 + a^3*d*e^6 + 3*(c*d^3*e^4 - b*d^2*e^5)*a^2 + 3*(c^2*d^5*
e^2 - 2*b*c*d^4*e^3 + b^2*d^3*e^4)*a + (c^3*d^6*e - 3*b*c^2*d^5*e
^2 + 3*b^2*c*d^4*e^3 - b^3*d^3*e^4 + a^3*e^7 + 3*(c*d^2*e^5 - b*d
*e^6)*a^2 + 3*(c^2*d^4*e^3 - 2*b*c*d^3*e^4 + b^2*d^2*e^5)*a)*x^n)
, x) - 1/2*((4*a^3*c^4*e^3*(7*n - 1) - b^3*c^4*d^3*(2*n - 1) + 2*
b^4*c^3*d^2*e*(2*n - 1) - b^5*c^2*d*e^2*(2*n - 1) - (b^2*c^3*e^3*
(26*n - 5) - 4*c^5*d^2*e*(3*n - 1) - 10*b*c^4*d*e^2*n)*a^2 - (b^2
*c^4*d^2*e*(28*n - 9) - 2*b*c^5*d^3*(7*n - 2) - 2*b^3*c^3*d*e^2*(
5*n - 2) - b^4*c^2*e^3*(4*n - 1))*a)*x*x^(3*n) - (2*b^4*c^3*d^3*(
2*n - 1) - 4*b^5*c^2*d^2*e*(2*n - 1) + 2*b^6*c*d*e^2*(2*n - 1) -
2*(b*c^3*e^3*(37*n - 6) - 2*c^4*d*e^2*(8*n - 1))*a^3 - (2*b*c^4*d
^2*e*(25*n - 8) + 3*b^2*c^3*d*e^2*(5*n + 1) - 11*b^3*c^2*e^3*(5*n
 - 1) - 4*c^5*d^3*(4*n - 1))*a^2 - (b^2*c^4*d^3*(29*n - 9) - 2*b^
3*c^3*d^2*e*(29*n - 10) + 3*b^4*c^2*d*e^2*(7*n - 3) + 2*b^5*c*e^3
*(4*n - 1))*a)*x*x^(2*n) + (4*a^4*c^3*e^3*(9*n - 1) - b^5*c^2*d^3
*(2*n - 1) + 2*b^6*c*d^2*e*(2*n - 1) - b^7*d*e^2*(2*n - 1) + (b^2
*c^2*e^3*(14*n - 3) - 2*b*c^3*d*e^2*(13*n - 2) + 4*c^4*d^2*e*(5*n
 - 1))*a^3 - (b^4*c*e^3*(24*n - 5) - b^3*c^2*d*e^2*(20*n - 1) - 2
*b*c^4*d^3*n + 3*b^2*c^3*d^2*e)*a^2 - (3*b^4*c^2*d^2*e*(8*n - 3)
- b^6*e^3*(4*n - 1) - 4*b^3*c^3*d^3*(3*n - 1) - 4*b^5*c*d*e^2*(2*
n - 1))*a)*x*x^n + (2*(b*c^2*e^3*(29*n - 4) - 2*c^3*d*e^2*(10*n -
 1))*a^4 + (2*b*c^3*d^2*e*(29*n - 6) - 4*c^4*d^3*(6*n - 1) - 6*b^
3*c*e^3*(6*n - 1) - b^2*c^2*d*e^2*(n - 3))*a^3 - (b^3*c^2*d^2*e*(
43*n - 11) - b^2*c^3*d^3*(21*n - 5) - b^4*c*d*e^2*(17*n - 5) - b^
5*e^3*(5*n - 1))*a^2 - (b^4*c^2*d^3*(3*n - 1) - 2*b^5*c*d^2*e*(3*
n - 1) + b^6*d*e^2*(3*n - 1))*a)*x)/(16*a^8*c^2*e^4*n^2 + 8*(4*c^
3*d^2*e^2*n^2 - 4*b*c^2*d*e^3*n^2 - b^2*c*e^4*n^2)*a^7 + (16*c^4*
d^4*n^2 - 32*b*c^3*d^3*e*n^2 + 16*b^3*c*d*e^3*n^2 + b^4*e^4*n^2)*
a^6 - 2*(4*b^2*c^3*d^4*n^2 - 8*b^3*c^2*d^3*e*n^2 + 3*b^4*c*d^2*e^
2*n^2 + b^5*d*e^3*n^2)*a^5 + (b^4*c^2*d^4*n^2 - 2*b^5*c*d^3*e*n^2
 + b^6*d^2*e^2*n^2)*a^4 + (16*a^6*c^4*e^4*n^2 + 8*(4*c^5*d^2*e^2*
n^2 - 4*b*c^4*d*e^3*n^2 - b^2*c^3*e^4*n^2)*a^5 + (16*c^6*d^4*n^2
- 32*b*c^5*d^3*e*n^2 + 16*b^3*c^3*d*e^3*n^2 + b^4*c^2*e^4*n^2)*a^
4 - 2*(4*b^2*c^5*d^4*n^2 - 8*b^3*c^4*d^3*e*n^2 + 3*b^4*c^3*d^2*e^
2*n^2 + b^5*c^2*d*e^3*n^2)*a^3 + (b^4*c^4*d^4*n^2 - 2*b^5*c^3*d^3
*e*n^2 + b^6*c^2*d^2*e^2*n^2)*a^2)*x^(4*n) + 2*(16*a^6*b*c^3*e^4*
n^2 + 8*(4*b*c^4*d^2*e^2*n^2 - 4*b^2*c^3*d*e^3*n^2 - b^3*c^2*e^4*
n^2)*a^5 + (16*b*c^5*d^4*n^2 - 32*b^2*c^4*d^3*e*n^2 + 16*b^4*c^2*
d*e^3*n^2 + b^5*c*e^4*n^2)*a^4 - 2*(4*b^3*c^4*d^4*n^2 - 8*b^4*c^3
*d^3*e*n^2 + 3*b^5*c^2*d^2*e^2*n^2 + b^6*c*d*e^3*n^2)*a^3 + (b^5*
c^3*d^4*n^2 - 2*b^6*c^2*d^3*e*n^2 + b^7*c*d^2*e^2*n^2)*a^2)*x^(3*
n) + (32*a^7*c^3*e^4*n^2 + 64*(c^4*d^2*e^2*n^2 - b*c^3*d*e^3*n^2)
*a^6 + 2*(16*c^5*d^4*n^2 - 32*b*c^4*d^3*e*n^2 + 16*b^2*c^3*d^2*e^
```

```
2*n^2 - 3*b^4*c*e^4*n^2)*a^5 - (12*b^4*c^2*d^2*e^2*n^2 - 12*b^5*c
*d*e^3*n^2 - b^6*e^4*n^2)*a^4 - 2*(3*b^4*c^3*d^4*n^2 - 6*b^5*c^2*
d^3*e*n^2 + 2*b^6*c*d^2*e^2*n^2 + b^7*d*e^3*n^2)*a^3 + (b^6*c^2*d
^4*n^2 - 2*b^7*c*d^3*e*n^2 + b^8*d^2*e^2*n^2)*a^2)*x^(2*n) + 2*(1
6*a^7*b*c^2*e^4*n^2 + 8*(4*b*c^3*d^2*e^2*n^2 - 4*b^2*c^2*d*e^3*n^
2 - b^3*c*e^4*n^2)*a^6 + (16*b*c^4*d^4*n^2 - 32*b^2*c^3*d^3*e*n^2
 + 16*b^4*c*d*e^3*n^2 + b^5*e^4*n^2)*a^5 - 2*(4*b^3*c^3*d^4*n^2 -
 8*b^4*c^2*d^3*e*n^2 + 3*b^5*c*d^2*e^2*n^2 + b^6*d*e^3*n^2)*a^4 +
 (b^5*c^2*d^4*n^2 - 2*b^6*c*d^3*e*n^2 + b^7*d^2*e^2*n^2)*a^3)*x^n
) - integrate(-1/2*((2*n^2 - 3*n + 1)*b^4*c^3*d^5 - 3*(2*n^2 - 3*
n + 1)*b^5*c^2*d^4*e + 3*(2*n^2 - 3*n + 1)*b^6*c*d^3*e^2 - (2*n^2
 - 3*n + 1)*b^7*d^2*e^3 + 2*(2*(24*n^2 - 10*n + 1)*c^3*d*e^4 - (4
8*n^2 - 29*n + 4)*b*c^2*e^5)*a^4 + (8*(12*n^2 - 8*n + 1)*c^4*d^3*
e^2 - 12*(16*n^2 - 13*n + 2)*b*c^3*d^2*e^3 + (48*n^2 - 59*n + 11)
*b^2*c^2*d*e^4 + 6*(8*n^2 - 6*n + 1)*b^3*c*e^5)*a^3 + (4*(8*n^2 -
 6*n + 1)*c^5*d^5 - 2*(48*n^2 - 41*n + 8)*b*c^4*d^4*e + 2*(24*n^2
 - 19*n + 5)*b^2*c^3*d^3*e^2 + 2*(32*n^2 - 39*n + 7)*b^3*c^2*d^2*
e^3 - (42*n^2 - 53*n + 11)*b^4*c*d*e^4 - (6*n^2 - 5*n + 1)*b^5*e^
5)*a^2 - ((16*n^2 - 21*n + 5)*b^2*c^4*d^5 - 16*(3*n^2 - 4*n + 1)*
b^3*c^3*d^4*e + 3*(14*n^2 - 19*n + 5)*b^4*c^2*d^3*e^2 - 2*(2*n^2
- 3*n + 1)*b^5*c*d^2*e^3 - 2*(3*n^2 - 4*n + 1)*b^6*d*e^4)*a + ((2
*n^2 - 3*n + 1)*b^3*c^4*d^5 - 3*(2*n^2 - 3*n + 1)*b^4*c^3*d^4*e +
 3*(2*n^2 - 3*n + 1)*b^5*c^2*d^3*e^2 - (2*n^2 - 3*n + 1)*b^6*c*d^
2*e^3 - 4*(15*n^2 - 8*n + 1)*a^4*c^3*e^5 - (8*(5*n^2 - 6*n + 1)*c
^4*d^2*e^3 - 2*(9*n^2 - 11*n + 2)*b*c^3*d*e^4 - (42*n^2 - 31*n +
5)*b^2*c^2*e^5)*a^3 - (4*(3*n^2 - 4*n + 1)*c^5*d^4*e + 12*(n^2 -
n)*b*c^4*d^3*e^2 - 2*(32*n^2 - 39*n + 7)*b^2*c^3*d^2*e^3 + 9*(4*n
^2 - 5*n + 1)*b^3*c^2*d*e^4 + (6*n^2 - 5*n + 1)*b^4*c*e^5)*a^2 -
(2*(7*n^2 - 9*n + 2)*b*c^5*d^5 - (42*n^2 - 55*n + 13)*b^2*c^4*d^4
*e + 12*(3*n^2 - 4*n + 1)*b^3*c^3*d^3*e^2 - (2*n^2 - 3*n + 1)*b^4
*c^2*d^2*e^3 - 2*(3*n^2 - 4*n + 1)*b^5*c*d*e^4)*a)*x^n)/(16*a^8*c
^2*e^6*n^2 + 8*(6*c^3*d^2*e^4*n^2 - 6*b*c^2*d*e^5*n^2 - b^2*c*e^6
*n^2)*a^7 + (48*c^4*d^4*e^2*n^2 - 96*b*c^3*d^3*e^3*n^2 + 24*b^2*c
^2*d^2*e^4*n^2 + 24*b^3*c*d*e^5*n^2 + b^4*e^6*n^2)*a^6 + (16*c^5*
d^6*n^2 - 48*b*c^4*d^5*e*n^2 + 24*b^2*c^3*d^4*e^2*n^2 + 32*b^3*c^
2*d^3*e^3*n^2 - 21*b^4*c*d^2*e^4*n^2 - 3*b^5*d*e^5*n^2)*a^5 - (8*
b^2*c^4*d^6*n^2 - 24*b^3*c^3*d^5*e*n^2 + 21*b^4*c^2*d^4*e^2*n^2 -
 2*b^5*c*d^3*e^3*n^2 - 3*b^6*d^2*e^4*n^2)*a^4 + (b^4*c^3*d^6*n^2
- 3*b^5*c^2*d^5*e*n^2 + 3*b^6*c*d^4*e^2*n^2 - b^7*d^3*e^3*n^2)*a^
3 + (16*a^7*c^3*e^6*n^2 + 8*(6*c^4*d^2*e^4*n^2 - 6*b*c^3*d*e^5*n^
2 - b^2*c^2*e^6*n^2)*a^6 + (48*c^5*d^4*e^2*n^2 - 96*b*c^4*d^3*e^3
*n^2 + 24*b^2*c^3*d^2*e^4*n^2 + 24*b^3*c^2*d*e^5*n^2 + b^4*c*e^6*
n^2)*a^5 + (16*c^6*d^6*n^2 - 48*b*c^5*d^5*e*n^2 + 24*b^2*c^4*d^4*
e^2*n^2 + 32*b^3*c^3*d^3*e^3*n^2 - 21*b^4*c^2*d^2*e^4*n^2 - 3*b^5
*c*d*e^5*n^2)*a^4 - (8*b^2*c^5*d^6*n^2 - 24*b^3*c^4*d^5*e*n^2 + 2
1*b^4*c^3*d^4*e^2*n^2 - 2*b^5*c^2*d^3*e^3*n^2 - 3*b^6*c*d^2*e^4*n
^2)*a^3 + (b^4*c^4*d^6*n^2 - 3*b^5*c^3*d^5*e*n^2 + 3*b^6*c^2*d^4*
e^2*n^2 - b^7*c*d^3*e^3*n^2)*a^2)*x^(2*n) + (16*a^7*b*c^2*e^6*n^2
 + 8*(6*b*c^3*d^2*e^4*n^2 - 6*b^2*c^2*d*e^5*n^2 - b^3*c*e^6*n^2)*
a^6 + (48*b*c^4*d^4*e^2*n^2 - 96*b^2*c^3*d^3*e^3*n^2 + 24*b^3*c^2
*d^2*e^4*n^2 + 24*b^4*c*d*e^5*n^2 + b^5*e^6*n^2)*a^5 + (16*b*c^5*
d^6*n^2 - 48*b^2*c^4*d^5*e*n^2 + 24*b^3*c^3*d^4*e^2*n^2 + 32*b^4*
c^2*d^3*e^3*n^2 - 21*b^5*c*d^2*e^4*n^2 - 3*b^6*d*e^5*n^2)*a^4 - (
8*b^3*c^4*d^6*n^2 - 24*b^4*c^3*d^5*e*n^2 + 21*b^5*c^2*d^4*e^2*n^2
 - 2*b^6*c*d^3*e^3*n^2 - 3*b^7*d^2*e^4*n^2)*a^3 + (b^5*c^3*d^6*n^
2 - 3*b^6*c^2*d^5*e*n^2 + 3*b^7*c*d^4*e^2*n^2 - b^8*d^3*e^3*n^2)*
a^2)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^3d + (c^3ex^n + c^3d)x^{6n} + (3\,bc^2ex^{2n} + 3\,(b^2c + ac^2)d + (b^3 + 6\,abc)e + 3\,(bc^2d + ac^2e)x^n)x^{4n} + (3\,b^2cex^{2n} + b^3d + \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)),x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*d + (c^3*e*x^n + c^3*d)*x^(6*n) + (3*b*c^2*e*x^(2
```

```
*n) + 3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e + 3*(b*c^2*d + a*c^
2*e)*x^n)*x^(4*n) + (3*b^2*c*e*x^(2*n) + b^3*d + 3*a*b^2*e)*x^(3*
n) + 3*(a^2*b*e + (a*b^2 + a^2*c)*d + (2*a*b*c*d + a^2*c*e)*x^n)*
x^(2*n) + (3*a^2*b*d + a^3*e)*x^n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^{2n} + bx^n + a)^3(ex^n + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)), x)
```

3.84 $\int \frac{1}{(d+ex^n)^2(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=2446

result too large to display

```
[Out] -(x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e
^2) + 2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^
3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n))^2) - (e^2*x*(5*b^3*c*d
*e - 14*a*b*c^2*d*e - 2*b^4*e^2 - b^2*c*(3*c*d^2 - 7*a*e^2) + 2*a
*c^2*(3*c*d^2 - a*e^2) + c*(5*b^2*c*d*e - 8*a*c^2*d*e - 2*b^3*e^2
 - b*c*(3*c*d^2 - 5*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2)^3*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*(a*e^2*(13
 - 37*n) - 5*c*d^2*(1 - 3*n)) - b^4*c*(a*e^2*(7 - 17*n) - c*d^2*(
1 - 2*n)) - 4*a^2*b*c^3*d*e*(4 - 11*n) + 6*a*b^3*c^2*d*e*(2 - 5*n
) + 4*a^2*c^3*(c*d^2 - a*e^2)*(1 - 4*n) - 2*b^5*c*d*e*(1 - 2*n) +
 b^6*e^2*(1 - 2*n) + c*(2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 -
7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2
*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n)
 + b^5*e^2*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e
 + a*e^2)^2*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(10*c^2*d^2 + 3
*b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d + 3*Sqrt[b^2 - 4*a*
c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*
(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4*a*c^2*(e*(a*e*(1 - 2*n) +
2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - 2*n)) - b^2*c*(e*(a
*e*(9 - 13*n) + 5*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - n))
 + b*c*(c*d*(4*a*e*(5 - 8*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 5
*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^3*e*(5*
c*d - 2*Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*
a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^
3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*
c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2*d*e*(5 - 14
*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^5*e^2*(
1 - 2*n))*(1 - n) - (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*n))*
(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^
2) - 8*a^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*
e*(3 - 13*n + 13*n^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a
*b^2*c^2*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(9 - 38*n + 35*n^2)))
/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sq
rt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) - (c*e^4*(10*c^2*
d^2 + 3*b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d - 3*Sqrt[b^2
 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4
*a*c])*(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4*a*c^2*(e*(a*e*(1 -
2*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - 2*n)) - b^2*
c*(e*(a*e*(9 - 13*n) - 5*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(
1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1 -
n)) + 5*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^
3*e*(5*c*d + 2*Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b
^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n))
 - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2*d*e*
(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^
5*e^2*(1 - 2*n))*(1 - n) + (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 -
 2*n))*(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n
 + 2*n^2) - 8*a^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b
*c^3*d*e*(3 - 13*n + 13*n^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2
) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(9 - 38*n + 35
*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*
(b + Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) + (3*e^6*(
2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)
/d)])/(d*(c*d^2 - b*d*e + a*e^2)^4) + (e^6*x*Hypergeometric2F1[2,
 n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^
3)
```

Rubi [A] time = 24.8268, antiderivative size = 2446, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

result too large to display

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] -(x*(2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e
^2) + 2*a*c^2*(c*d^2 - a*e^2) + c*(2*b^2*c*d*e - 4*a*c^2*d*e - b^
3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^n))/(2*a*(b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)^2*n*(a + b*x^n + c*x^(2*n))^2) - (e^2*x*(5*b^3*c*d
*e - 14*a*b*c^2*d*e - 2*b^4*e^2 - b^2*c*(3*c*d^2 - 7*a*e^2) + 2*a
*c^2*(3*c*d^2 - a*e^2) + c*(5*b^2*c*d*e - 8*a*c^2*d*e - 2*b^3*e^2
 - b*c*(3*c*d^2 - 5*a*e^2))*x^n))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2)^3*n*(a + b*x^n + c*x^(2*n))) - (x*(a*b^2*c^2*(a*e^2*(13
 - 37*n) - 5*c*d^2*(1 - 3*n)) - b^4*c*(a*e^2*(7 - 17*n) - c*d^2*(
1 - 2*n)) - 4*a^2*b*c^3*d*e*(4 - 11*n) + 6*a*b^3*c^2*d*e*(2 - 5*n
) + 4*a^2*c^3*(c*d^2 - a*e^2)*(1 - 4*n) - 2*b^5*c*d*e*(1 - 2*n) +
 b^6*e^2*(1 - 2*n) + c*(2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 -
7*n)) - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2
*d*e*(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n)
 + b^5*e^2*(1 - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*(c*d^2 - b*d*e
 + a*e^2)^2*n^2*(a + b*x^n + c*x^(2*n))) - (c*e^4*(10*c^2*d^2 + 3
*b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d + 3*Sqrt[b^2 - 4*a*
c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*
(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4*a*c^2*(e*(a*e*(1 - 2*n) +
2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - 2*n)) - b^2*c*(e*(a
*e*(9 - 13*n) + 5*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - n))
 + b*c*(c*d*(4*a*e*(5 - 8*n) + 3*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 5
*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^3*e*(5*
c*d - 2*Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*
a*c)*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^
3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n)) - b^3*
c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2*d*e*(5 - 14
*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^5*e^2*(
1 - 2*n))*(1 - n) - (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 - 2*n))*
(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n + 2*n^
2) - 8*a^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b*c^3*d*
e*(3 - 13*n + 13*n^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2) + 2*a
*b^2*c^2*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(9 - 38*n + 35*n^2)))
/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sq
rt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) - (c*e^4*(10*c^2*
d^2 + 3*b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(5*b*d - 3*Sqrt[b^2
 - 4*a*c]*d + a*e))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4
*a*c])*(c*d^2 - b*d*e + a*e^2)^4) + (c*e^2*(4*a*c^2*(e*(a*e*(1 -
2*n) - 2*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(1 - 2*n)) - b^2*
c*(e*(a*e*(9 - 13*n) - 5*Sqrt[b^2 - 4*a*c]*d*(1 - n)) - 3*c*d^2*(
1 - n)) + b*c*(c*d*(4*a*e*(5 - 8*n) - 3*Sqrt[b^2 - 4*a*c]*d*(1 -
n)) + 5*a*Sqrt[b^2 - 4*a*c]*e^2*(1 - n)) + 2*b^4*e^2*(1 - n) - b^
3*e*(5*c*d + 2*Sqrt[b^2 - 4*a*c]*e)*(1 - n))*x*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b
^2 - 4*a*c)*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e +
a*e^2)^3*n) + (c*((2*a*b*c^2*(a*e^2*(4 - 13*n) - c*d^2*(2 - 7*n))
 - b^3*c*(2*a*e^2*(3 - 8*n) - c*d^2*(1 - 2*n)) + 2*a*b^2*c^2*d*e*
(5 - 14*n) - 8*a^2*c^3*d*e*(1 - 3*n) - 2*b^4*c*d*e*(1 - 2*n) + b^
5*e^2*(1 - 2*n))*(1 - n) + (b^4*c*(4*a*e^2*(2 - 5*n) - c*d^2*(1 -
 2*n))*(1 - n) + 2*b^5*c*d*e*(1 - 3*n + 2*n^2) - b^6*e^2*(1 - 3*n
 + 2*n^2) - 8*a^2*c^3*(c*d^2 - a*e^2)*(1 - 6*n + 8*n^2) + 8*a^2*b
*c^3*d*e*(3 - 13*n + 13*n^2) - 2*a*b^3*c^2*d*e*(7 - 25*n + 18*n^2
) + 2*a*b^2*c^2*(3*c*d^2*(1 - 4*n + 3*n^2) - a*e^2*(9 - 38*n + 35
*n^2)))/Sqrt[b^2 - 4*a*c])*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*
```

```
(b + Sqrt[b^2 - 4*a*c])*(c*d^2 - b*d*e + a*e^2)^2*n^2) + (3*e^6*(
2*c*d - b*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)
/d)])/(d*(c*d^2 - b*d*e + a*e^2)^4) + (e^6*x*Hypergeometric2F1[2,
 n^(-1), 1 + n^(-1), -((e*x^n)/d)])/(d^2*(c*d^2 - b*d*e + a*e^2)^
3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Integral(1/((d + e*x**n)**2*(a + b*x**n + c*x**(2*n))**3), x)
```

Mathematica [B] time = 9.88568, size = 56566, normalized size = 23.13

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.608, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^n)^2 (a+bx^n+cx^{2n})^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int(1/(d+e*x^n)^2/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)^2),x, algorithm="maxima")
```

```
[Out] (c*d^2*e^6*(7*n - 1) - b*d*e^7*(4*n - 1) + a*e^8*(n - 1))*integra
te(1/(c^4*d^10*n - 4*b*c^3*d^9*e*n + 6*b^2*c^2*d^8*e^2*n - 4*b^3*
c*d^7*e^3*n + b^4*d^6*e^4*n + a^4*d^2*e^8*n + 4*(c*d^4*e^6*n - b*
d^3*e^7*n)*a^3 + 6*(c^2*d^6*e^4*n - 2*b*c*d^5*e^5*n + b^2*d^4*e^6
*n)*a^2 + 4*(c^3*d^8*e^2*n - 3*b*c^2*d^7*e^3*n + 3*b^2*c*d^6*e^4*
n - b^3*d^5*e^5*n)*a + (c^4*d^9*e*n - 4*b*c^3*d^8*e^2*n + 6*b^2*c
^2*d^7*e^3*n - 4*b^3*c*d^6*e^4*n + b^4*d^5*e^5*n + a^4*d*e^9*n +
```

4*(c*d^3*e^7*n - b*d^2*e^8*n)*a^3 + 6*(c^2*d^5*e^5*n - 2*b*c*d^4*
e^6*n + b^2*d^3*e^7*n)*a^2 + 4*(c^3*d^7*e^3*n - 3*b*c^2*d^6*e^4*n
 + 3*b^2*c*d^5*e^5*n - b^3*d^4*e^6*n)*a)*x^n), x) + 1/2*((b^3*c^5
*d^5*e*(2*n - 1) - 3*b^4*c^4*d^4*e^2*(2*n - 1) + 3*b^5*c^3*d^3*e^
3*(2*n - 1) - b^6*c^2*d^2*e^4*(2*n - 1) + 32*a^4*c^4*e^6*n + 2*(b
*c^4*d*e^5*(33*n - 4) - 4*c^5*d^2*e^4*(11*n - 1) - 8*b^2*c^3*e^6*
n)*a^3 + 2*(b^2*c^4*d^2*e^4*(29*n - 1) - 3*b^3*c^3*d*e^5*(7*n - 1
) - 4*c^6*d^4*e^2*(3*n - 1) + 6*b*c^5*d^3*e^3*(n - 1) + b^4*c^2*e
^6*n)*a^2 - (3*b^3*c^4*d^3*e^3*(12*n - 5) + 2*b*c^6*d^5*e*(7*n -
2) - b^5*c^2*d*e^5*(6*n - 1) - 14*b^2*c^5*d^4*e^2*(3*n - 1) - 2*b
^4*c^3*d^2*e^4*(n - 2))*a)*x*x^(4*n) + (b^3*c^5*d^6*(2*n - 1) - b
^4*c^4*d^5*e*(2*n - 1) - 3*b^5*c^3*d^4*e^2*(2*n - 1) + 5*b^6*c^2*
d^3*e^3*(2*n - 1) - 2*b^7*c*d^2*e^4*(2*n - 1) - 4*(c^4*d*e^5*(8*n
 - 1) - 16*b*c^3*e^6*n)*a^4 + (b^2*c^3*d*e^5*(163*n - 21) - 6*b*c
^4*d^2*e^4*(27*n - 2) - 8*c^5*d^3*e^3*(5*n - 1) - 32*b^3*c^2*e^6*
n)*a^3 - (b^4*c^2*d*e^5*(89*n - 13) - b^3*c^3*d^2*e^4*(77*n + 5)
- 2*b^2*c^4*d^3*e^3*(50*n - 19) + 8*b*c^5*d^4*e^2*(9*n - 2) + 4*c
^6*d^5*e*(2*n - 1) - 4*b^5*c*e^6*n)*a^2 - (b^4*c^3*d^3*e^3*(73*n
- 29) - b^3*c^4*d^4*e^2*(51*n - 16) - b^2*c^5*d^5*e*(13*n - 5) -
b^5*c^2*d^2*e^4*(11*n - 10) + 2*b*c^6*d^6*(7*n - 2) - 2*b^6*c*d*e
^5*(6*n - 1))*a)*x*x^(3*n) + (2*b^4*c^4*d^6*(2*n - 1) - 5*b^5*c^3
*d^5*e*(2*n - 1) + 3*b^6*c^2*d^4*e^2*(2*n - 1) + b^7*c*d^3*e^3*(2
*n - 1) - b^8*d^2*e^4*(2*n - 1) + 64*a^5*c^3*e^6*n - 2*(2*c^4*d^2
e^4(34*n - 3) - b*c^3*d*e^5*(23*n - 2))*a^4 + (b^2*c^3*d^2*e^4*
(81*n - 11) + b^3*c^2*d*e^5*(48*n - 7) - 8*b*c^4*d^3*e^3*(18*n -
1) + 8*c^5*d^4*e^2*(n + 1) - 12*b^4*c*e^6*n)*a^3 - (2*b*c^5*d^5*e
*(43*n - 14) + b^4*c^2*d^2*e^4*(21*n - 10) + 2*b^5*c*d*e^5*(20*n
- 3) - 5*b^3*c^3*d^3*e^3*(19*n - 2) - 4*c^6*d^6*(4*n - 1) - 10*b^
2*c^4*d^4*e^2*(4*n - 3) - 2*b^6*e^6*n)*a^2 - (b^4*c^3*d^4*e^2*(39
*n - 19) + b^2*c^5*d^6*(29*n - 9) + b^5*c^2*d^3*e^3*(25*n - 6) -
3*b^3*c^4*d^5*e*(25*n - 9) - b^7*d*e^5*(6*n - 1) - 6*b^6*c*d^2*e^
4*(2*n - 1))*a)*x*x^(2*n) + (b^5*c^3*d^6*(2*n - 1) - 3*b^6*c^2*d^
5*e*(2*n - 1) + 3*b^7*c*d^4*e^2*(2*n - 1) - b^8*d^3*e^3*(2*n - 1)
 - 4*(c^3*d*e^5*(10*n - 1) - 16*b*c^2*e^6*n)*a^5 + (b^2*c^2*d*e^5
*(115*n - 13) - 2*b*c^3*d^2*e^4*(55*n - 4) - 8*c^4*d^3*e^3*(7*n -
 1) - 32*b^3*c*e^6*n)*a^4 - (b^4*c*d*e^5*(55*n - 7) - 3*b^3*c^2*d
^2*e^4*(35*n - 2) + 2*b^2*c^3*d^3*e^3*(8*n + 7) + 4*c^5*d^5*e*(4*
n - 1) + 8*b*c^4*d^4*e^2*(n - 1) - 4*b^5*e^6*n)*a^3 + (b^3*c^3*d^
4*e^2*(41*n - 26) - b^5*c*d^2*e^4*(31*n - 1) - b^2*c^4*d^5*e*(23*
n - 11) + b^4*c^2*d^3*e^3*(8*n + 15) + b^6*d*e^5*(7*n - 1) - 2*b*
c^5*d^6*n)*a^2 + (3*b^4*c^3*d^5*e*(13*n - 5) - 3*b^5*c^2*d^4*e^2*
(13*n - 6) + b^6*c*d^3*e^3*(9*n - 7) - 4*b^3*c^4*d^6*(3*n - 1) +
3*b^7*d^2*e^4*n)*a)*x*x^n + (32*a^6*c^2*e^6*n - 4*(c^3*d^2*e^4*(1
0*n - 1) + 4*b^2*c*e^6*n)*a^5 + (b^2*c^2*d^2*e^4*(115*n - 13) - 1
2*b*c^3*d^3*e^3*(13*n - 1) + 48*c^4*d^4*e^2*n + 2*b^4*e^6*n)*a^4
+ (b^3*c^2*d^3*e^3*(57*n + 1) - b^4*c*d^2*e^4*(55*n - 7) - 4*b*c^
4*d^5*e*(23*n - 5) + 6*b^2*c^3*d^4*e^2*(11*n - 4) + 4*c^5*d^6*(6*
n - 1))*a^3 + (b^3*c^3*d^5*e*(65*n - 17) - b^2*c^4*d^6*(21*n - 5)
 - 6*b^4*c^2*d^4*e^2*(10*n - 3) + b^5*c*d^3*e^3*(9*n - 5) + b^6*d
^2*e^4*(7*n - 1))*a^2 + (b^4*c^3*d^6*(3*n - 1) - 3*b^5*c^2*d^5*e*
(3*n - 1) + 3*b^6*c*d^4*e^2*(3*n - 1) - b^7*d^3*e^3*(3*n - 1))*a)
*x)/(16*a^9*c^2*d^2*e^6*n^2 + 8*(6*c^3*d^4*e^4*n^2 - 6*b*c^2*d^3*
e^5*n^2 - b^2*c*d^2*e^6*n^2)*a^8 + (48*c^4*d^6*e^2*n^2 - 96*b*c^3
*d^5*e^3*n^2 + 24*b^2*c^2*d^4*e^4*n^2 + 24*b^3*c*d^3*e^5*n^2 + b^
4*d^2*e^6*n^2)*a^7 + (16*c^5*d^8*n^2 - 48*b*c^4*d^7*e*n^2 + 24*b^
2*c^3*d^6*e^2*n^2 + 32*b^3*c^2*d^5*e^3*n^2 - 21*b^4*c*d^4*e^4*n^2
 - 3*b^5*d^3*e^5*n^2)*a^6 - (8*b^2*c^4*d^8*n^2 - 24*b^3*c^3*d^7*e
*n^2 + 21*b^4*c^2*d^6*e^2*n^2 - 2*b^5*c*d^5*e^3*n^2 - 3*b^6*d^4*e
^4*n^2)*a^5 + (b^4*c^3*d^8*n^2 - 3*b^5*c^2*d^7*e*n^2 + 3*b^6*c*d^
6*e^2*n^2 - b^7*d^5*e^3*n^2)*a^4 + (16*a^7*c^4*d*e^7*n^2 + 8*(6*c
^5*d^3*e^5*n^2 - 6*b*c^4*d^2*e^6*n^2 - b^2*c^3*d*e^7*n^2)*a^6 + (
48*c^6*d^5*e^3*n^2 - 96*b*c^5*d^4*e^4*n^2 + 24*b^2*c^4*d^3*e^5*n^
2 + 24*b^3*c^3*d^2*e^6*n^2 + b^4*c^2*d*e^7*n^2)*a^5 + (16*c^7*d^7
*e*n^2 - 48*b*c^6*d^6*e^2*n^2 + 24*b^2*c^5*d^5*e^3*n^2 + 32*b^3*c
^4*d^4*e^4*n^2 - 21*b^4*c^3*d^3*e^5*n^2 - 3*b^5*c^2*d^2*e^6*n^2)*
a^4 - (8*b^2*c^6*d^7*e*n^2 - 24*b^3*c^5*d^6*e^2*n^2 + 21*b^4*c^4*
d^5*e^3*n^2 - 2*b^5*c^3*d^4*e^4*n^2 - 3*b^6*c^2*d^3*e^5*n^2)*a^3
+ (b^4*c^5*d^7*e*n^2 - 3*b^5*c^4*d^6*e^2*n^2 + 3*b^6*c^3*d^5*e^3*
n^2 - b^7*c^2*d^4*e^4*n^2)*a^2)*x^(5*n) + (16*(c^4*d^2*e^6*n^2 +
2*b*c^3*d*e^7*n^2)*a^7 + 8*(6*c^5*d^4*e^4*n^2 + 6*b*c^4*d^3*e^5*n
^2 - 13*b^2*c^3*d^2*e^6*n^2 - 2*b^3*c^2*d*e^7*n^2)*a^6 + (48*c^6*
d^6*e^2*n^2 - 168*b^2*c^4*d^4*e^4*n^2 + 72*b^3*c^3*d^3*e^5*n^2 +

49*b^4*c^2*d^2*e^6*n^2 + 2*b^5*c*d*e^7*n^2)*a^5 + (16*c^7*d^8*n^2
 - 16*b*c^6*d^7*e*n^2 - 72*b^2*c^5*d^6*e^2*n^2 + 80*b^3*c^4*d^5*e
^3*n^2 + 43*b^4*c^3*d^4*e^4*n^2 - 45*b^5*c^2*d^3*e^5*n^2 - 6*b^6*
c*d^2*e^6*n^2)*a^4 - (8*b^2*c^6*d^8*n^2 - 8*b^3*c^5*d^7*e*n^2 - 2
7*b^4*c^4*d^6*e^2*n^2 + 40*b^5*c^3*d^5*e^3*n^2 - 7*b^6*c^2*d^4*e^
4*n^2 - 6*b^7*c*d^3*e^5*n^2)*a^3 + (b^4*c^5*d^8*n^2 - b^5*c^4*d^7
*e*n^2 - 3*b^6*c^3*d^6*e^2*n^2 + 5*b^7*c^2*d^5*e^3*n^2 - 2*b^8*c*
d^4*e^4*n^2)*a^2)*x^(4*n) + (32*a^8*c^3*d*e^7*n^2 + 32*(3*c^4*d^3
*e^5*n^2 - 2*b*c^3*d^2*e^6*n^2)*a^7 + 2*(48*c^5*d^5*e^3*n^2 - 48*
b*c^4*d^4*e^4*n^2 - 8*b^3*c^2*d^2*e^6*n^2 - 3*b^4*c*d*e^7*n^2)*a^
6 + (32*c^6*d^7*e*n^2 - 96*b^2*c^4*d^5*e^3*n^2 + 16*b^3*c^3*d^4*e
^4*n^2 + 30*b^4*c^2*d^3*e^5*n^2 + 20*b^5*c*d^2*e^6*n^2 + b^6*d*e^
7*n^2)*a^5 + (32*b*c^6*d^8*n^2 - 96*b^2*c^5*d^7*e*n^2 + 48*b^3*c^
4*d^6*e^2*n^2 + 46*b^4*c^3*d^5*e^3*n^2 - 6*b^5*c^2*d^4*e^4*n^2 -
21*b^6*c*d^3*e^5*n^2 - 3*b^7*d^2*e^6*n^2)*a^4 - (16*b^3*c^5*d^8*n
^2 - 42*b^4*c^4*d^7*e*n^2 + 24*b^5*c^3*d^6*e^2*n^2 + 11*b^6*c^2*d
^5*e^3*n^2 - 6*b^7*c*d^4*e^4*n^2 - 3*b^8*d^3*e^5*n^2)*a^3 + (2*b^
5*c^4*d^8*n^2 - 5*b^6*c^3*d^7*e*n^2 + 3*b^7*c^2*d^6*e^2*n^2 + b^8
*c*d^5*e^3*n^2 - b^9*d^4*e^4*n^2)*a^2)*x^(3*n) + (32*(c^3*d^2*e^6
*n^2 + b*c^2*d*e^7*n^2)*a^8 + 16*(6*c^4*d^4*e^4*n^2 - 6*b^2*c^2*d
^2*e^6*n^2 - b^3*c*d*e^7*n^2)*a^7 + 2*(48*c^5*d^6*e^2*n^2 - 48*b*
c^4*d^5*e^3*n^2 - 48*b^2*c^3*d^4*e^4*n^2 + 24*b^3*c^2*d^3*e^5*n^2
 + 21*b^4*c*d^2*e^6*n^2 + b^5*d*e^7*n^2)*a^6 + (32*c^6*d^8*n^2 -
64*b*c^5*d^7*e*n^2 + 16*b^3*c^3*d^5*e^3*n^2 + 46*b^4*c^2*d^4*e^4*
n^2 - 24*b^5*c*d^3*e^5*n^2 - 5*b^6*d^2*e^6*n^2)*a^5 - (16*b^3*c^4
*d^7*e*n^2 - 30*b^4*c^3*d^6*e^2*n^2 + 6*b^5*c^2*d^5*e^3*n^2 + 11*
b^6*c*d^4*e^4*n^2 - 3*b^7*d^3*e^5*n^2)*a^4 - (6*b^4*c^4*d^8*n^2 -
 20*b^5*c^3*d^7*e*n^2 + 21*b^6*c^2*d^6*e^2*n^2 - 6*b^7*c*d^5*e^3*
n^2 - b^8*d^4*e^4*n^2)*a^3 + (b^6*c^3*d^8*n^2 - 3*b^7*c^2*d^7*e*n
^2 + 3*b^8*c*d^6*e^2*n^2 - b^9*d^5*e^3*n^2)*a^2)*x^(2*n) + (16*a^
9*c^2*d*e^7*n^2 + 8*(6*c^3*d^3*e^5*n^2 - 2*b*c^2*d^2*e^6*n^2 - b^
2*c*d*e^7*n^2)*a^8 + (48*c^4*d^5*e^3*n^2 - 72*b^2*c^2*d^3*e^5*n^2
 + 8*b^3*c*d^2*e^6*n^2 + b^4*d*e^7*n^2)*a^7 + (16*c^5*d^7*e*n^2 +
 48*b*c^4*d^6*e^2*n^2 - 168*b^2*c^3*d^5*e^3*n^2 + 80*b^3*c^2*d^4*
e^4*n^2 + 27*b^4*c*d^3*e^5*n^2 - b^5*d^2*e^6*n^2)*a^6 + (32*b*c^5
*d^8*n^2 - 104*b^2*c^4*d^7*e*n^2 + 72*b^3*c^3*d^6*e^2*n^2 + 43*b^
4*c^2*d^5*e^3*n^2 - 40*b^5*c*d^4*e^4*n^2 - 3*b^6*d^3*e^5*n^2)*a^5
 - (16*b^3*c^4*d^8*n^2 - 49*b^4*c^3*d^7*e*n^2 + 45*b^5*c^2*d^6*e^
2*n^2 - 7*b^6*c*d^5*e^3*n^2 - 5*b^7*d^4*e^4*n^2)*a^4 + 2*(b^5*c^3
*d^8*n^2 - 3*b^6*c^2*d^7*e*n^2 + 3*b^7*c*d^6*e^2*n^2 - b^8*d^5*e^
3*n^2)*a^3)*x^n) + integrate(1/2*((2*n^2 - 3*n + 1)*b^4*c^4*d^6 -
 4*(2*n^2 - 3*n + 1)*b^5*c^3*d^5*e + 6*(2*n^2 - 3*n + 1)*b^6*c^2*
d^4*e^2 - 4*(2*n^2 - 3*n + 1)*b^7*c*d^3*e^3 + (2*n^2 - 3*n + 1)*b
^8*d^2*e^4 - 4*(24*n^2 - 10*n + 1)*a^5*c^3*e^6 + (4*(48*n^2 - 2*n
 - 1)*c^4*d^2*e^4 - 4*(96*n^2 - 29*n + 2)*b*c^3*d*e^5 + (240*n^2
- 115*n + 13)*b^2*c^2*e^6)*a^4 + (4*(32*n^2 - 18*n + 1)*c^5*d^4*e
^2 - 8*(48*n^2 - 37*n + 4)*b*c^4*d^3*e^3 + (288*n^2 - 337*n + 49)
*b^2*c^3*d^2*e^4 + 2*(32*n^2 + 29*n - 7)*b^3*c^2*d*e^5 - (102*n^2
 - 55*n + 7)*b^4*c*e^6)*a^3 + (4*(8*n^2 - 6*n + 1)*c^6*d^6 - 4*(3
2*n^2 - 29*n + 6)*b*c^5*d^5*e + (128*n^2 - 137*n + 39)*b^2*c^4*d^
4*e^2 + 8*(8*n^2 - 7*n - 1)*b^3*c^3*d^3*e^3 - 4*(37*n^2 - 43*n +
6)*b^4*c^2*d^2*e^4 + 4*(10*n^2 - 16*n + 3)*b^5*c*d*e^5 + (12*n^2
- 7*n + 1)*b^6*e^6)*a^2 - ((16*n^2 - 21*n + 5)*b^2*c^5*d^6 - 2*(3
2*n^2 - 43*n + 11)*b^3*c^4*d^5*e + 2*(44*n^2 - 61*n + 17)*b^4*c^3
*d^4*e^2 - 20*(2*n^2 - 3*n + 1)*b^5*c^2*d^3*e^3 - (8*n^2 - 7*n -
1)*b^6*c*d^2*e^4 + 2*(4*n^2 - 5*n + 1)*b^7*d*e^5)*a + ((2*n^2 - 3
*n + 1)*b^3*c^5*d^6 - 4*(2*n^2 - 3*n + 1)*b^4*c^4*d^5*e + 6*(2*n^
2 - 3*n + 1)*b^5*c^3*d^4*e^2 - 4*(2*n^2 - 3*n + 1)*b^6*c^2*d^3*e^
3 + (2*n^2 - 3*n + 1)*b^7*c*d^2*e^4 - 2*(4*(35*n^2 - 12*n + 1)*c^
4*d*e^5 - (81*n^2 - 37*n + 4)*b*c^3*e^6)*a^4 - 2*(8*(7*n^2 - 8*n
+ 1)*c^5*d^3*e^3 - (83*n^2 - 97*n + 14)*b*c^4*d^2*e^4 - (44*n^2 +
 7*n - 3)*b^2*c^3*d*e^5 + 3*(15*n^2 - 8*n + 1)*b^3*c^2*e^6)*a^3 -
 (8*(3*n^2 - 4*n + 1)*c^6*d^5*e - 2*(11*n^2 - 19*n + 8)*b*c^5*d^4
e^2 - 4(22*n^2 - 23*n + 1)*b^2*c^4*d^3*e^3 + (136*n^2 - 159*n +
 23)*b^3*c^3*d^2*e^4 - 2*(16*n^2 - 27*n + 5)*b^4*c^2*d*e^5 - (12*
n^2 - 7*n + 1)*b^5*c*e^6)*a^2 - 2*((7*n^2 - 9*n + 2)*b*c^6*d^6 -
(28*n^2 - 37*n + 9)*b^2*c^5*d^5*e + 2*(19*n^2 - 26*n + 7)*b^3*c^4
*d^4*e^2 - 8*(2*n^2 - 3*n + 1)*b^4*c^3*d^3*e^3 - 5*(n^2 - n)*b^5*
c^2*d^2*e^4 + (4*n^2 - 5*n + 1)*b^6*c*d*e^5)*a)*x^n)/(16*a^9*c^2*
e^8*n^2 + 8*(8*c^3*d^2*e^6*n^2 - 8*b*c^2*d*e^7*n^2 - b^2*c*e^8*n^
2)*a^8 + (96*c^4*d^4*e^4*n^2 - 192*b*c^3*d^3*e^5*n^2 + 64*b^2*c^2
*d^2*e^6*n^2 + 32*b^3*c*d*e^7*n^2 + b^4*e^8*n^2)*a^7 + 4*(16*c^5*

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d^6*e^2*n^2 - 48*b*c^4*d^5*e^3*n^2 + 36*b^2*c^3*d^4*e^4*n^2 + 8*b
^3*c^2*d^3*e^5*n^2 - 11*b^4*c*d^2*e^6*n^2 - b^5*d*e^7*n^2)*a^6 +
2*(8*c^6*d^8*n^2 - 32*b*c^5*d^7*e*n^2 + 32*b^2*c^4*d^6*e^2*n^2 +
16*b^3*c^3*d^5*e^3*n^2 - 37*b^4*c^2*d^4*e^4*n^2 + 10*b^5*c*d^3*e^
5*n^2 + 3*b^6*d^2*e^6*n^2)*a^5 - 4*(2*b^2*c^5*d^8*n^2 - 8*b^3*c^4
*d^7*e*n^2 + 11*b^4*c^3*d^6*e^2*n^2 - 5*b^5*c^2*d^5*e^3*n^2 - b^6
*c*d^4*e^4*n^2 + b^7*d^3*e^5*n^2)*a^4 + (b^4*c^4*d^8*n^2 - 4*b^5*
c^3*d^7*e*n^2 + 6*b^6*c^2*d^6*e^2*n^2 - 4*b^7*c*d^5*e^3*n^2 + b^8
*d^4*e^4*n^2)*a^3 + (16*a^8*c^3*e^8*n^2 + 8*(8*c^4*d^2*e^6*n^2 -
8*b*c^3*d*e^7*n^2 - b^2*c^2*e^8*n^2)*a^7 + (96*c^5*d^4*e^4*n^2 -
192*b*c^4*d^3*e^5*n^2 + 64*b^2*c^3*d^2*e^6*n^2 + 32*b^3*c^2*d*e^7
*n^2 + b^4*c*e^8*n^2)*a^6 + 4*(16*c^6*d^6*e^2*n^2 - 48*b*c^5*d^5*
e^3*n^2 + 36*b^2*c^4*d^4*e^4*n^2 + 8*b^3*c^3*d^3*e^5*n^2 - 11*b^4
*c^2*d^2*e^6*n^2 - b^5*c*d*e^7*n^2)*a^5 + 2*(8*c^7*d^8*n^2 - 32*b
*c^6*d^7*e*n^2 + 32*b^2*c^5*d^6*e^2*n^2 + 16*b^3*c^4*d^5*e^3*n^2
- 37*b^4*c^3*d^4*e^4*n^2 + 10*b^5*c^2*d^3*e^5*n^2 + 3*b^6*c*d^2*e
^6*n^2)*a^4 - 4*(2*b^2*c^6*d^8*n^2 - 8*b^3*c^5*d^7*e*n^2 + 11*b^4
*c^4*d^6*e^2*n^2 - 5*b^5*c^3*d^5*e^3*n^2 - b^6*c^2*d^4*e^4*n^2 +
b^7*c*d^3*e^5*n^2)*a^3 + (b^4*c^5*d^8*n^2 - 4*b^5*c^4*d^7*e*n^2 +
 6*b^6*c^3*d^6*e^2*n^2 - 4*b^7*c^2*d^5*e^3*n^2 + b^8*c*d^4*e^4*n^
2)*a^2)*x^(2*n) + (16*a^8*b*c^2*e^8*n^2 + 8*(8*b*c^3*d^2*e^6*n^2
- 8*b^2*c^2*d*e^7*n^2 - b^3*c*e^8*n^2)*a^7 + (96*b*c^4*d^4*e^4*n^
2 - 192*b^2*c^3*d^3*e^5*n^2 + 64*b^3*c^2*d^2*e^6*n^2 + 32*b^4*c*d
*e^7*n^2 + b^5*e^8*n^2)*a^6 + 4*(16*b*c^5*d^6*e^2*n^2 - 48*b^2*c^
4*d^5*e^3*n^2 + 36*b^3*c^3*d^4*e^4*n^2 + 8*b^4*c^2*d^3*e^5*n^2 -
11*b^5*c*d^2*e^6*n^2 - b^6*d*e^7*n^2)*a^5 + 2*(8*b*c^6*d^8*n^2 -
32*b^2*c^5*d^7*e*n^2 + 32*b^3*c^4*d^6*e^2*n^2 + 16*b^4*c^3*d^5*e^
3*n^2 - 37*b^5*c^2*d^4*e^4*n^2 + 10*b^6*c*d^3*e^5*n^2 + 3*b^7*d^2
*e^6*n^2)*a^4 - 4*(2*b^3*c^5*d^8*n^2 - 8*b^4*c^4*d^7*e*n^2 + 11*b
^5*c^3*d^6*e^2*n^2 - 5*b^6*c^2*d^5*e^3*n^2 - b^7*c*d^4*e^4*n^2 +
b^8*d^3*e^5*n^2)*a^3 + (b^5*c^4*d^8*n^2 - 4*b^6*c^3*d^7*e*n^2 + 6
*b^7*c^2*d^6*e^2*n^2 - 4*b^8*c*d^5*e^3*n^2 + b^9*d^4*e^4*n^2)*a^2
)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3e^2x^{5n} + a^3d^2 + (c^3e^2x^{2n} + 2c^3dex^n + c^3d^2)x^{6n} + (3bc^2e^2x^{3n} + 3(b^2c + ac^2)d^2 + 2(b^3 + 6abc)de + 3(ab^2 + a^2c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*e^2*x^(5*n) + a^3*d^2 + (c^3*e^2*x^(2*n) + 2*c^3*
d*e*x^n + c^3*d^2)*x^(6*n) + (3*b*c^2*e^2*x^(3*n) + 3*(b^2*c + a*
c^2)*d^2 + 2*(b^3 + 6*a*b*c)*d*e + 3*(a*b^2 + a^2*c)*e^2 + 3*(2*b
*c^2*d*e + (b^2*c + a*c^2)*e^2)*x^(2*n) + 3*(b*c^2*d^2 + 2*a*c^2*
d*e)*x^n)*x^(4*n) + (b^3*d^2 + 6*a*b^2*d*e + 3*a^2*b*e^2 + 6*(b^2
*c*d*e + a*b*c*e^2)*x^(2*n))*x^(3*n) + (6*a^2*b*d*e + a^3*e^2 + 3
*(a*b^2 + a^2*c)*d^2 + 6*(a*b*c*d^2 + a^2*c*d*e)*x^n)*x^(2*n) + (
3*a^2*b*d^2 + 2*a^3*d*e)*x^n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x**n)**2/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{2n} + bx^n + a\right)^3 \left(ex^n + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^3*(e*x^n + d)^2), x)
```

3.85 $\int (d+ex^n)\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=292

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{1}{2},-\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -1/
2, -1/2, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (
d*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 +
n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.842371, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{1}{2},-\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -1/
2, -1/2, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + (
d*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 +
n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 80.1638, size = 262, normalized size = 0.9

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(\frac{n+1}{n},-\frac{1}{2},-\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] d*x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, -1/2, -1/2, 1 + 1
/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a
*c + b**2)))/(sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2
*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)) + e*x**(n + 1)*sqrt(a + b
*x**n + c*x**(2*n))*appellf1((n + 1)/n, -1/2, -1/2, 2 + 1/n, -2*c
*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b**
2)))/((n + 1)*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2
*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.25912, size = 3778, normalized size = 12.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]*(((2*c*d + 4*c*d*n + b*e*n)*x)/(2*c*(
1 + n)*(1 + 2*n)) + (e*x^(1 + n))/(1 + 2*n)) - (2*a^2*b*d*n*x^(1
+ n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b -
Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)^2*(a + x^n*(b
+ c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2
+ n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^
(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
llF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (4*a^3*e*n*x^(
1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b
- Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)^2*(a + x^n*(
b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-1), 1/2, 1/2,
2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[2 +
n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Ap
pellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (2*a^2*b^2*e
*n*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
)/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n)^2*(a
 + x^n*(b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-1), 1/
2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*Appel
lF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*
a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) - (4*a
^2*b*d*n^2*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + n
)^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-
1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])
*AppellF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^
2 - 4*a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
 - (4*a^3*e*n^2*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^
(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1
 + n)^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1 +
```

```
n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a
*c])*AppellF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqr
t[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
))) + (a^2*b^2*e*n^2*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2,
2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*
a*c])*(1 + n)^2*(a + x^n*(b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*Appel
lF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b
^2 - 4*a*c])*AppellF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] +
(b - Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])]))) - (4*a^3*d*n*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 +
 2*n)*(a + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*
AppellF1[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^
2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] - 4*a*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))
+ (2*a^3*b*e*n*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
)/(c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + 2*n)*(a
 + x^n*(b + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1
[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*
c])*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a
*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (8*a^3
*d*n^2*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((b -
Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 + 2*n)*(a + x^n*(b
+ c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1
), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*
AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*A
ppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int (d + ex^n) \sqrt{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2\,n} + bx^{n} + a}(ex^{n} + d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex^{n})\sqrt{a + bx^{n} + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d + e*x**n)*sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^{2\,n} + bx^{n} + a}(ex^{n} + d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(e*x^n + d), x)
```

3.86 $\int (d + ex^n) \left(a + bx^n + cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=294

$$\frac{adx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -
3/2, -3/2, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) +
 (a*d*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2,
1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.829324, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{adx\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}+\frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}F_1\left(1+\frac{1}{n};-\frac{3}{2},-\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*e*x^(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[1 + n^(-1), -
3/2, -3/2, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*Sqrt[1 + (2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) +
 (a*d*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2,
1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 84.9607, size = 265, normalized size = 0.9

$$\frac{adx\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}+\frac{aex^{n+1}\sqrt{a+bx^n+cx^{2n}}\operatorname{appellf_1}\left(\frac{n+1}{n},-\frac{3}{2},-\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] a*d*x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, -3/2, -3/2, 1 +
 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4
*a*c + b**2)))/(sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt
(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)) + a*e*x**(n + 1)*sqrt(a
 + b*x**n + c*x**(2*n))*appellf1((n + 1)/n, -3/2, -3/2, 2 + 1/n,
-2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c +
 b**2)))/((n + 1)*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sq
rt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.915, size = 10587, normalized size = 36.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (d + ex^n) \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} (ex^n + d) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2\,n} + bx^n + a\right)^{\frac{3}{2}}(ex^n + d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(e*x^n + d), x)
```

3.87 $\int \frac{d+ex^n}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=292

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2
, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/((1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*
x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/Sqrt[a + b*x^n + c*x^(2*n)]
```

Rubi [A] time = 0.836251, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2
, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/((1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (d*
x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/Sqrt[a + b*x^n + c*x^(2*n)]
```

Rubi in Sympy [A] time = 89.8877, size = 258, normalized size = 0.88

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf_1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,\mathrm{appellf_1}\left(\frac{n+1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] d*x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, 1/2, 1/2, 1 + 1/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(a*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2
*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)) + e*x**(n + 1)*sqrt(a + b
*x**n + c*x**(2*n))*appellf1((n + 1)/n, 1/2, 1/2, 2 + 1/n, -2*c*x
**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b**2)
))/(a*(n + 1)*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2
*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.860791, size = 688, normalized size = 2.36

$$ax\left(-\sqrt{b^2-4ac}+b+2cx^n\right)\left(\sqrt{b^2-4ac}+b+2cx^n\right)\left(\frac{d(n+1)^2F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\right.}{nx^n\left(-\left(\sqrt{b^2-4ac}+b\right)\right)F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+nx^n\left(\sqrt{b^2-4ac}-b\right)}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (a*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)*(-((e*(1 + 2*n)*x^n*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^
(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])])/(-4*(a + 2*a*n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 +
n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-
1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (d*(1 + n)^2*App
ellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(-((b + Sqrt[b^2 - 4*
a*c])*n*x^n*AppellF1[1 + n^(-1), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) +
(-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 3/2, 1/2, 2 +
 n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])] + 4*a*(1 + n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])))/(c*(1 + n)*(a + x^n*(b + c*x^n))^(3/2))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int (d+ex^n)\frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n+d}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^n}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d + e*x**n)/sqrt(a + b*x**n + c*x**(2*n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/sqrt(c*x^(2*n) + b*x^n + a), x)
```

3.88 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{3/2}}\, dx$

Optimal. Leaf size=298

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(1+\frac{1}{n};\frac{3}{2},\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 3/2, 3/2
, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(a*(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (
d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.856491, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(1+\frac{1}{n};\frac{3}{2},\frac{3}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 3/2, 3/2
, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(a*(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) + (
d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(a*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 87.6691, size = 262, normalized size = 0.88

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{n+1}{n},\frac{3}{2},\frac{3}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a^2(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] d*x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, 3/2, 3/2, 1 + 1/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(a**2*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqr
t(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)) + e*x**(n + 1)*sqrt(a
+ b*x**n + c*x**(2*n))*appellf1((n + 1)/n, 3/2, 3/2, 2 + 1/n, -2*
c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b*
*2)))/(a**2*(n + 1)*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*
sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.26113, size = 3012, normalized size = 10.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (2*x*(-(b^2*d) + 2*a*c*d + a*b*e - b*c*d*x^n + 2*a*c*e*x^n))/(a*(
-b^2 + 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)]) - (8*a*b*c*d*(1 + 2*
n)*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
)/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])
*n*(1 + n)*(a + x^n*(b + c*x^n))^(3/2)*(-4*(a + 2*a*n)*AppellF1[1
 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*x^n*((b + Sqrt[b^2 -
4*a*c])*AppellF1[2 + n^(-1), 1/2, 3/2, 3 + n^(-1), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 3/2, 1/2, 3 + n^(-1), (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c
])]))) + (16*a^2*c*e*(1 + 2*n)*x^(1 + n)*(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1[1 + n^(-1),
1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a
*c])*(b + Sqrt[b^2 - 4*a*c])*n*(1 + n)*(a + x^n*(b + c*x^n))^(3/2
)*(-4*(a + 2*a*n)*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] + n*x^n*((b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 1/2, 3/2
, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1
), 3/2, 1/2, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (4*a*b^2*d*(1 + n)*x*(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*Appe
llF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - S
qrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n))^(
3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 1/2, 3/2
, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1
+ n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*AppellF1[n^(
-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (16*a^2*c*d*(1 + n)*x*(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*Ap
pellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b -
 Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n))
^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 1/2, 3
/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
 + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[
1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*AppellF1[n
^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) - (8*a*b^2*d*(1 + n)*x*(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*A
ppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b
- Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(a + x^n*(b + c*x^
n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1), 1/2
, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
```

```
(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*Appell
F1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*AppellF
1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (16*a^2*c*d*(1 + n)*x*
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)
*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(a + x^n*(b +
c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-1),
 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n*Ap
pellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*App
ellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])) + (8*a^2*b*e*(1 + n)
*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n)*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a
*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*n*(a + x^n*(b
 + c*x^n))^(3/2)*((b + Sqrt[b^2 - 4*a*c])*n*x^n*AppellF1[1 + n^(-
1), 1/2, 3/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (-b + Sqrt[b^2 - 4*a*c])*n*x^n
*AppellF1[1 + n^(-1), 3/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*a*(1 + n)*
AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (d+e x^n) \left(a+b x^n+c x^{2n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e x^n + d}{(c x^{2n} + b x^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

3.89 $\int \frac{d+ex^n}{(a+bx^n+cx^{2n})^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{5}{2},\frac{5}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(1+\frac{1}{n};\frac{5}{2},\frac{5}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 5/2, 5/2
, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(a^2*(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) +
 (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 5/2, 5/2, 1 + n^(-1
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(a^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi [A] time = 0.843464, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{dx\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{n};\frac{5}{2},\frac{5}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}+\frac{ex^{n+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(1+\frac{1}{n};\frac{5}{2},\frac{5}{2};2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2(n+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

```
[Out] (e*x^(1 + n)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 5/2, 5/2
, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(a^2*(1 + n)*Sqrt[a + b*x^n + c*x^(2*n)]) +
 (d*x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 5/2, 5/2, 1 + n^(-1
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(a^2*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rubi in Sympy [A] time = 103.334, size = 262, normalized size = 0.88

$$\frac{dx\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{1}{n},\frac{5}{2},\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a^3\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}+\frac{ex^{n+1}\sqrt{a+bx^n+cx^{2n}}\,\text{appellf}_1\left(\frac{n+1}{n},\frac{5}{2},\frac{5}{2},2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{a^3(n+1)\sqrt{\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(5/2),x)
```

```
[Out] d*x*sqrt(a + b*x**n + c*x**(2*n))*appellf1(1/n, 5/2, 5/2, 1 + 1/n
, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c
```

```
 + b**2)))/(a**3*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*sqr
t(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)) + e*x**(n + 1)*sqrt(a
+ b*x**n + c*x**(2*n))*appellf1((n + 1)/n, 5/2, 5/2, 2 + 1/n, -2*
c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b*
*2)))/(a**3*(n + 1)*sqrt(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)*
sqrt(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.60549, size = 8781, normalized size = 29.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)/(a + b*x^n + c*x^(2*n))^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (d+ex^n)\left(a+bx^n+cx^{2\,n}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

```
[Out] int((d+e*x^n)/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n+d}{(cx^{2\,n}+bx^n+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)/(a+b*x**n+c*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^n + d}{(cx^{2n} + bx^n + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)/(c*x^(2*n) + b*x^n + a)^(5/2), x)
```

3.90 $\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \,dx$

Optimal. Leaf size=29

$$\text{Int}\left((d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p ,x\right)$$

```
[Out] Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

Rubi [A] time = 0.0259954, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left((d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p ,x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] Defer[Int][(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**q*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Integral((d + e*x**n)**q*(a + b*x**n + c*x**(2*n))**p, x)
```

Mathematica [A] time = 0.321258, size = 0, normalized size = 0.

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2n}\right)^p \,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] Integrate[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x]
```

Maple [A] time = 0.192, size = 0, normalized size = 0.

$$\int (d+ex^n)^q \left(a+bx^n+cx^{2\,n}\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)^q*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^{n} + a\right)^{p}(ex^{n} + d)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2n} + bx^{n} + a\right)^{p}(ex^{n} + d)^{q}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**q*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^{n} + a\right)^{p}(ex^{n} + d)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(e*x^n + d)^q, x)
```

3.91 $\int (d+ex^n)^3 \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=606

$$d^3x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{3d^2ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}+\frac{3de^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2n+1}+\frac{e^3x^{3n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(3+\frac{1}{n};-p,-p;4+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3n+1}$$

```
[Out] (3*d^2*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1),
 -p, -p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (3*d*
e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[2 + n^(-1), -p
, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^3*x
^(1 + 3*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[3 + n^(-1), -p, -p,
 4 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])])/((1 + 3*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d^3*x*(a +
 b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 1.3927, antiderivative size = 606, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$d^3x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{3d^2ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}+\frac{3de^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2n+1}+\frac{e^3x^{3n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(3+\frac{1}{n};-p,-p;4+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3n+1}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```


```
[Out] (3*d^2*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1),
 -p, -p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (3*d*
e^2*x^(1 + 2*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[2 + n^(-1), -p
, -p, 3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
```

```
 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^3*x
^(1 + 3*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[3 + n^(-1), -p, -p,
 4 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])])/((1 + 3*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d^3*x*(a +
 b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 165.131, size = 520, normalized size = 0.86

$$d^3x\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)+\frac{3d^2ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{n+1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+}}\right.}{n+1}+\frac{3de^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx}{b+\sqrt{-4a}}\right.}{2n+1}+\frac{e^3x^{3n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+}}\right.}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**3*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] d**3*x*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(
b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*
appellf1(1/n, -p, -p, 1 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)
), -2*c*x**n/(b + sqrt(-4*a*c + b**2))) + 3*d**2*e*x**(n + 1)*(2*
c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b + sqrt(-
4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*appellf1((
n + 1)/n, -p, -p, 2 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -
2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(n + 1) + 3*d*e**2*x**(2*n +
1)*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b +
sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*appe
llf1(2 + 1/n, -p, -p, 3 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)
), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(2*n + 1) + e**3*x**(3*n
+ 1)*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b
+ sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*ap
pellf1(3 + 1/n, -p, -p, 4 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**
2)), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(3*n + 1)
```

Mathematica [B] time = 24.4185, size = 2025, normalized size = 3.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)^3*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (3*2^(-1 - p)*c*(b + Sqrt[b^2 - 4*a*c])*d^2*e*(1 + 2*n)*x^(1 + n)
*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/c)^(1 + p)*(-2*a + (-b + Sqrt
[b^2 - 4*a*c])*x^n)^2*(a + x^n*(b + c*x^n))^(-1 + p)*AppellF1[1 +
 n^(-1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*(1
 + n)*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p*(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^n)*(-2*(a + 2*a*n)*AppellF1[1 + n^(-1), -p, -p, 2 + n
```

```
^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] + n*p*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^
(-1), 1 - p, -p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*App
ellF1[2 + n^(-1), -p, 1 - p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (3*2^(-1 - p
)*c*(b + Sqrt[b^2 - 4*a*c])*d*e^2*(1 + 3*n)*x^(1 + 2*n)*((b - Sqr
t[b^2 - 4*a*c] + 2*c*x^n)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a
*c])*x^n)^2*(a + x^n*(b + c*x^n))^(-1 + p)*AppellF1[2 + n^(-1), -
p, -p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*(1 + 2*n)*((
b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)*(-2*(a + 3*a*n)*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])] + n*p*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[3 + n^(-1), 1
- p, -p, 4 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[3
+ n^(-1), -p, 1 - p, 4 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (2^(-1 - p)*c*(b + S
qrt[b^2 - 4*a*c])*e^3*(1 + 4*n)*x^(1 + 3*n)*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*
(a + x^n*(b + c*x^n))^(-1 + p)*AppellF1[3 + n^(-1), -p, -p, 4 + n
^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*(1 + 3*n)*((b - Sqrt[b^2
 - 4*a*c])/(2*c) + x^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(-2*(
a + 4*a*n)*AppellF1[3 + n^(-1), -p, -p, 4 + n^(-1), (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p*
x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[4 + n^(-1), 1 - p, -p, 5 +
 n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[4 + n^(-1), -p
, 1 - p, 5 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])]))) - (2^(-1 - 2*p)*(b + Sqrt[b^2 - 4*
a*c])*d^3*(1 + n)*x*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*((b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/c)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
c)^(-1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n*(b +
 c*x^n))^(-1 + p)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c
*(-b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p
*((b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p*((-b + Sqrt[b^2 - 4*a*c]
)*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, -p, 2 + n^(-1), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (b
 + Sqrt[b^2 - 4*a*c])*n*p*x^n*AppellF1[1 + n^(-1), -p, 1 - p, 2 +
 n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt
[b^2 - 4*a*c])] - 2*a*(1 + n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])]))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int (d+e x^n)^3 \left(a+b x^n+c x^{2 n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)

[Out] int((d+e*x^n)^3*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e x^n+d)^3 \left(c x^{2 n}+b x^n+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^3*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^{3\,n} + 3\,de^2x^{2\,n} + 3\,d^2ex^n + d^3\right)\left(cx^{2\,n} + bx^n + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^3*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x
^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**3*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^3*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.92 $\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=447

$$d^2x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{2dex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}+\frac{e^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2n+1}$$

```
[Out] (2*d*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -
p, -p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
- 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^2*x^
(1 + 2*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[2 + n^(-1), -p, -p,
3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d^2*x*(a +
b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.99276, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$d^2x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{2dex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}+\frac{e^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(2+\frac{1}{n};-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (2*d*e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -
p, -p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
- 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e^2*x^
(1 + 2*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[2 + n^(-1), -p, -p,
3 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/((1 + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d^2*x*(a +
b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
/((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 123.616, size = 382, normalized size = 0.85

$$d^2x\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)$$
$$+\frac{2dex^{n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{n+1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{n+1}$$
$$+\frac{e^2x^{2n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(2+\frac{1}{n},-p,-p,3+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] d**2*x*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(
b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*
appellf1(1/n, -p, -p, 1 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)
), -2*c*x**n/(b + sqrt(-4*a*c + b**2))) + 2*d*e*x**(n + 1)*(2*c*x
**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b + sqrt(-4*a
*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*appellf1((n +
 1)/n, -p, -p, 2 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c
*x**n/(b + sqrt(-4*a*c + b**2)))/(n + 1) + e**2*x**(2*n + 1)*(2*c
*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b + sqrt(-4
*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*appellf1(2
+ 1/n, -p, -p, 3 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c
*x**n/(b + sqrt(-4*a*c + b**2)))/(2*n + 1)
```

Mathematica [B] time = 6.26088, size = 1522, normalized size = 3.4

$$\frac{2^{-p}c\left(b+\sqrt{b^2-4ac}\right)de(2n+1)\left(x^n+\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{-p}\left(\frac{2cx^n+b-\sqrt{b^2-4ac}}{c}\right)^{p+1}\left(\left(\sqrt{b^2\ldots\right.\right.}{\left(\sqrt{b^2-4ac}-b\right)(n+1)\left(2cx^n+b+\sqrt{b^2-4ac}\right)\left(npx^n\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(2+\frac{1}{n};1-p,-p;3+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac-b}}\ldots\right.\right.\right.}$$
$$+\frac{2^{-p-1}c\left(b+\sqrt{b^2-4ac}\right)e^2(3n+1)\left(x^n+\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{-p}\left(\frac{2cx^n+b-\sqrt{b^2-4ac}}{c}\right)^{p+1}\left(\left(\sqrt{b\ldots\right.\right.}{\left(\sqrt{b^2-4ac}-b\right)(2n+1)\left(2cx^n+b+\sqrt{b^2-4ac}\right)\left(npx^n\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(3+\frac{1}{n};1-p,-p;4+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}\ldots}\right.\right.\right.}$$
$$-\frac{2^{-2p-1}\left(b+\sqrt{b^2-4ac}\right)d^2(n+1)\left(x^n+\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{-p}\left(x^n+\frac{b+\sqrt{b^2-4ac}}{2c}\right)^{-p}\left(-2cx^n-b+\sqrt{b^2-4ac}\right)\left(\frac{2cx^n+b-\sqrt{b^2-4ac}}{c}\right)^{p}\left(\ldots\right.}{c\left(\sqrt{b^2-4ac}-b\right)\left(\left(\sqrt{b^2-4ac}-b\right)npF_1\left(1+\frac{1}{n};1-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac-b}}\right)x^n-\left(b+\sqrt{b^2-4ac}\right)np\ldots\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (c*(b + Sqrt[b^2 - 4*a*c])*d*e*(1 + 2*n)*x^(1 + n)*((b - Sqrt[b^2
 - 4*a*c] + 2*c*x^n)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*
x^n)^2*(a + x^n*(b + c*x^n))^(-1 + p)*AppellF1[1 + n^(-1), -p, -p
, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])])/(2^p*(-b + Sqrt[b^2 - 4*a*c])*(1 + n)*((b -
 Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)*(-2*(a + 2*a*n)*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c]
)] + n*p*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 1 - p
, -p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n
^(-1), -p, 1 - p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))) + (2^(-1 - p)*c*(b + Sqrt
[b^2 - 4*a*c])*e^2*(1 + 3*n)*x^(1 + 2*n)*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a
```

```
+ x^n*(b + c*x^n))^(-1 + p)*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*(1 + 2*n)*((b - Sqrt[b^2 -
4*a*c])/(2*c) + x^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(-2*(a +
 3*a*n)*AppellF1[2 + n^(-1), -p, -p, 3 + n^(-1), (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p*x^n
*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[3 + n^(-1), 1 - p, -p, 4 + n^
(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[3 + n^(-1), -p, 1
 - p, 4 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])]))) - (2^(-1 - 2*p)*(b + Sqrt[b^2 - 4*a*c
])*d^2*(1 + n)*x*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/c)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/c)^
(-1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n*(b + c*
x^n))^(-1 + p)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(-
b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p*((
b + Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p*((-b + Sqrt[b^2 - 4*a*c])*n
*p*x^n*AppellF1[1 + n^(-1), 1 - p, -p, 2 + n^(-1), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (b +
Sqrt[b^2 - 4*a*c])*n*p*x^n*AppellF1[1 + n^(-1), -p, 1 - p, 2 + n^
(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])] - 2*a*(1 + n)*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])]))
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)

[Out] int((d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^2 \left(cx^{2n} + bx^n + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="maxima")

[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^{2n} + 2\,dex^n + d^2\right)\left(cx^{2n} + bx^n + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="fricas")

[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^
p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.93 $\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=288

$$dx\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}$$

```
[Out] (e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -
p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d*x*(a + b
*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.620041, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$dx\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1}{n};-p,-p;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)+\frac{ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(1+\frac{1}{n};-p,-p;2+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```


```
[Out] (e*x^(1 + n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[1 + n^(-1), -p, -
p, 2 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (d*x*(a + b
*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
((1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]))^p)
```


Rubi in Sympy [A] time = 80.8878, size = 245, normalized size = 0.85

$$dx\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)+\frac{ex^{n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{n+1}{n},-p,-p,2+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] d*x*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b +
 sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*app
ellf1(1/n, -p, -p, 1 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)),
-2*c*x**n/(b + sqrt(-4*a*c + b**2))) + e*x**(n + 1)*(2*c*x**n/(b
- sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b + sqrt(-4*a*c + b*
*2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*appellf1((n + 1)/n,
-p, -p, 2 + 1/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(
b + sqrt(-4*a*c + b**2)))/(n + 1)
```

Mathematica [B] time = 1.36053, size = 902, normalized size = 3.13

$$2^{-2p-1}\left(b+\sqrt{b^2-4ac}\right)x\left(x^n+\frac{b-\sqrt{b^2-4ac}}{2c}\right)^{-p}\left(\frac{2cx^n+b-\sqrt{b^2-4ac}}{c}\right)^p\left(\left(\sqrt{b^2-4ac}-b\right)x^n-2a\right)^2\left((cx^n+b)x^n+a\right)^{p-1}\left(\frac{\quad}{npx^n(}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (2^(-1 - 2*p)*(b + Sqrt[b^2 - 4*a*c])*x*((b - Sqrt[b^2 - 4*a*c] +
 2*c*x^n)/c)^p*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n*(
b + c*x^n))^(-1 + p)*((2^p*e*(1 + 2*n)*x^n*(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^n)*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(-2*
(a + 2*a*n)*AppellF1[1 + n^(-1), -p, -p, 2 + n^(-1), (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p
*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), 1 - p, -p, 3
+ n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[2 + n^(-1), -
p, 1 - p, 3 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
n)/(-b + Sqrt[b^2 - 4*a*c])])) - (d*(1 + n)^2*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^n)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/c)^p*AppellF1[
n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(((b + Sqrt[b^2 - 4*a*c])/(2*
c) + x^n)^p*((-b + Sqrt[b^2 - 4*a*c])*n*p*x^n*AppellF1[1 + n^(-1)
, 1 - p, -p, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*n*p*x^n
*AppellF1[1 + n^(-1), -p, 1 - p, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*a*(1 + n)
*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))))/((-b + Sqrt[b^2
- 4*a*c])*(1 + n)*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n))
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^n + d)\left(cx^{2n} + bx^n + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)\left(cx^{2n} + bx^n + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p, x)
```

3.94 $\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{d+ex^n}, x\right)$$

```
[Out] Unintegrable[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]
```

Rubi [A] time = 0.0282734, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{d+ex^n}, x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]

[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)

[Out] Integral((a + b*x**n + c*x**(2*n))**p/(d + e*x**n), x)
```

Mathematica [A] time = 0.0986101, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{d+ex^n} dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n),x]

[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n), x]
```

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2\,n})^p}{d+ex^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{ex^n + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + bx^n + a)^p}{ex^n + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d),x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p}{ex^n + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d), x)
```

3.95 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2},x\right)$$

```
[Out] Unintegrable[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Rubi [A] time = 0.0275941, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\mathrm{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]

[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)

[Out] Integral((a + b*x**n + c*x**(2*n))**p/(d + e*x**n)**2, x)
```

Mathematica [A] time = 0.165419, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2,x]

[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^2, x]
```

Maple [A] time = 0.084, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^p}{\left(ex^n + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2n} + bx^n + a\right)^p}{e^2x^{2n} + 2\,dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(e^2*x^(2*n) + 2*d*e*x^n + d^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^p}{\left(ex^n + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^2, x)
```

3.96 $\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3},x\right)$$

```
[Out] Unintegrable[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Rubi [A] time = 0.02769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\mathrm{Int}\left(\frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] Defer[Int][(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**p/(d + e*x**n)**3, x)
```

Mathematica [A] time = 1.01462, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3,x]
```

```
[Out] Integrate[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^3, x]
```

Maple [A] time = 0.11, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n+cx^{2\,n})^p}{(d+ex^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^p}{\left(ex^n + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2n} + bx^n + a\right)^p}{e^3x^{3n} + 3\,de^2x^{2n} + 3\,d^2ex^n + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p/(e^3*x^(3*n) + 3*d*e^2*x^(2*n)
 + 3*d^2*e*x^n + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^p}{\left(ex^n + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p/(e*x^n + d)^3, x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```